

Full Length Research Paper

## Effect of fiber size on elastic constants of hybrid elliptical fiber reinforced lamina

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Accepted 3 June, 2013

**Micromechanical analysis of an fiber reinforced polymer (FRP) lamina consisting of unequal sizes of unidirectional continuous elliptical fibers of two different materials arranged in hexagonal pattern is the present subject. The objective is to observe the change in Young's moduli and Poisson's ratios due to change in volume fractions and ellipse aspect ratio 'a' of individual fibers in a Representative Volume Element (RVE) without altering the total reinforcement share. A three-dimensional finite element model is developed and solved. Two different fiber materials (T-300 and S-Glass) are embedded in hexagonal pattern in a polymer matrix. The problem is simulated in ANSYS software. The converged results are validated using rule of mixtures which works exactly for longitudinal Young's modulus. It is observed that the longitudinal Young's modulus increases with increase in volume fraction of T-300 fiber and does not change due to 'a'. The transverse moduli decrease with increase in size of T-300 fiber and affected by variation of 'a'. Increasing in 'a' increases  $E_2$  and decreases  $E_3$ .**

**Key words:** Hybrid, fiber reinforced polymer (FRP), finite element method (FEM), micromechanics, elliptical fiber.

### INTRODUCTION

The mechanical properties of an fiber reinforced polymer (FRP) composite depend upon fiber material, its size, shape and arrangement in matrix. Authors of this present work intend to obtain a wide range of mechanical properties of a unidirectional continuous fiber reinforced composite lamina by selecting two different fibers of elliptical cross-section arranged in hexagonal pattern with a provision to change ellipse aspect ratio and size of individual fibers. The brief review of literature relevant to the present topic is presented in the following paragraphs.

Noteworthy works of earlier researchers Eshelby (1957), Hashin (1962), Hill (1963, 1965), Hashin and Shtrikman (1963a) and Hashin and Rosen (1964).

Hashin and Shtrikman (1963b) used variational principles to obtain upper and lower bounds for the effective elastic moduli as well as the effective electrical and thermal conductivities of multiphase composites with quasi-isotropic global characteristics. Later on, Milton (1981, 1982) obtained higher-order bounds for the elastic, electromagnetic, and transport properties of two-component macroscopically homogenous and isotropic composites given the properties of the individual constituents. More recently, Drugan and Willis (1996) and Drugan (2003), employed the Hashin-Shtrikman variational principles to analyze two-phase composites with random microstructure. A numerical implementation of this work was carried out by Segurado and Llorca

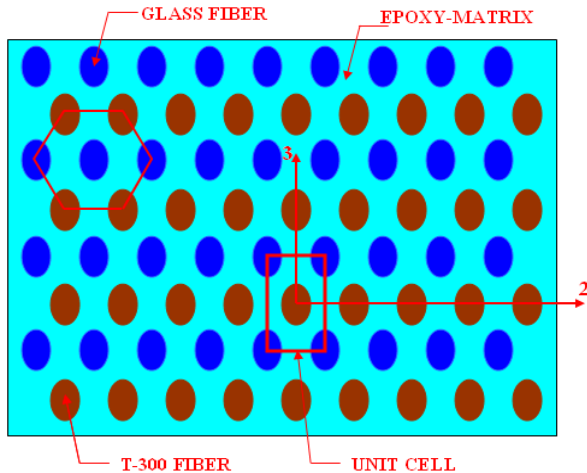


Figure 1. Hybrid composite lamina.

(2002). Other significant early results can be found in the work of Budiansky (1965) and Russel (1973). Mori and Tanaka (1973) in their micromechanical approach obtained closed-form expressions for the elastic properties of two-phase composites. Ying and Kin (2002) proposed a simple life prediction model for the hybrid composite.

Most recently Srinivasa Sai et al. (2010, 2011, 2012, 2013a and b) performed micromechanical analysis of hybrid elliptical fiber reinforced composite for the prediction of elastic properties and fiber-matrix interface stresses. The effects of fiber volume fraction and ellipse aspect ratio on the behaviour of hybrid lamina were studied. The present research work is the extension of Srinivasa Sai et al. (2013b) to study the effect of individual fiber volume fractions on Young's moduli and Poisson's ratios of hybrid composite for a constant total fiber percentage of 50.

## METHODOLOGY

### Hexagonal array of unit cells

Hexagonal arrangement of elliptical fibers of two different materials in alternative rows in a matrix is shown in Figure 1. It is assumed that the fiber and matrix materials are linearly elastic. A unit cell is adopted for the analysis. The fiber volume fraction  $V_f$  is calculated as the cross-sectional area of the fibers relative to the total cross sectional area of the unit cell since third dimension is same for all constituents for the unidirectional continuous fiber reinforced composite. A material coordinate system 1-2-3 is assigned as shown in Figure 1 for the analysis of the problem. 1) is the longitudinal direction of the fiber, 2) the in-plane transverse direction and 3) through-thickness or out-of-plane transverse direction.

### Finite element model

One-eighth portion of the unit cell (that is, one quarter in the cross

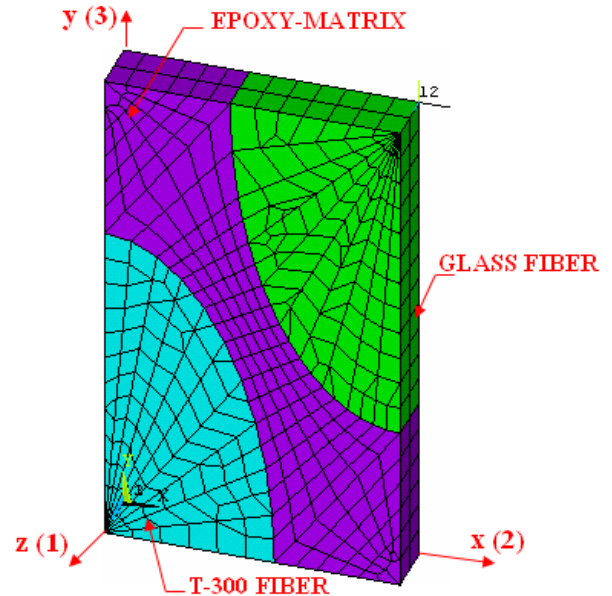


Figure 2. FE mesh on one-eighth portion of the unit cell (one quarter in cross section and half in length direction).

section and one-half in the longitudinal direction of the fiber) is modeled by taking the advantage of symmetry in geometry, material and loading (Figure 2). The element named SOLID 95 in ANSYS software is used for finite element generation. This is a quadratic brick element having 20 nodes with 3- degrees of freedom (linear displacement components) at each node. This element uses three-dimensional elasticity theory while preparing element matrices. The dimensions of FE model are taken as 50, 86.6 and 10 units in x- y- and z- directions respectively. The dimensions of ellipse are obtained according to the fiber volume fraction and the ellipse aspect ratio ('a'= axis length in 3-direction by axis length in 2-direction).

### Boundary conditions and loading

Three perpendicular planes on negative side of coordinate axes are constrained to restrict displacement in normal directions. The opposite faces are constrained to have uniform normal displacement. A uni-axial external load is applied once at a time with respect to the assigned material coordinates in order to use simple Hooke's law to determine Young's moduli. Corresponding Poisson's ratios are also calculated.

### Materials

One of the fibers that is, T-300 is orthotropic and other S-Glass fiber and matrix materials are isotropic. The mechanical properties of the constituent materials used in the present analysis are given in Table 1.

## RESULTS

The finite element software ANSYS is successfully executed for the analysis. The elastic properties are

**Table 1.** Mechanical properties of the constituent materials.

Property	T-300 fiber	S-Glass fiber	HM polymer matrix
$E_1$ (GPa)	220.60	85.50	5.17
$E_2$ (GPa)	13.79	85.50	5.17
$E_3$ (GPa)	13.79	85.50	5.17
$\nu_{12}$	0.20	0.20	0.35
$\nu_{23}$	0.25	0.20	0.35
$\nu_{13}$	0.20	0.20	0.35
$G_{12}$ (GPa)	8.96	35.62	1.91
$G_{23}$ (GPa)	4.83	35.62	1.91
$G_{13}$ (GPa)	8.96	35.62	1.91

**Table 2.** Validation of  $E_1$  (MPa).

$V_{f1} + V_{f2}$	ROM	FEM	% Variation
5+45	52090	52129	0.076
10+40	58845	58884	0.067
15+35	65600	65640	0.061
20+30	72355	72396	0.056
25+25	79110	79150	0.051
35+15	85865	85905	0.046
40+10	92620	92660	0.043
45+5	99375	99413	0.039

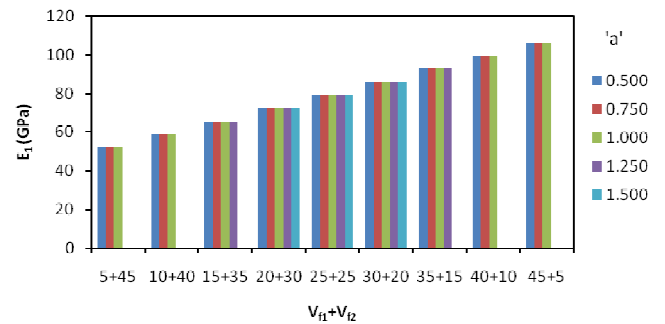
evaluated using the normal strains in 1, 2 and 3 directions calculated from the normal deformations of the unit cell obtained from finite element analysis. The displacements in x, y and z directions,  $U_x$ ,  $U_y$  and  $U_z$  respectively of the finite element model are obtained from the finite element solutions. The corresponding normal strains are determined from the displacements. The longitudinal Young's moduli and Poisson's ratios due to the longitudinal load are determined from the following expressions.

$$E_1 = \sigma_1 / \epsilon_1 \quad \nu_{12} = - \epsilon_2 / \epsilon_1 \quad \nu_{13} = - \epsilon_3 / \epsilon_1$$

where 1(z), 2(x) and 3(y) are longitudinal, in-plane transverse and out-of-plane transverse directions respectively of the composite lamina. Remaining properties are obtained in similar fashion for in-plane and out-of-plane transverse loads. The results are obtained for hybrid lamina with varying volume fractions of individual fibers for a total fiber volume fraction of 50%.

### Validation

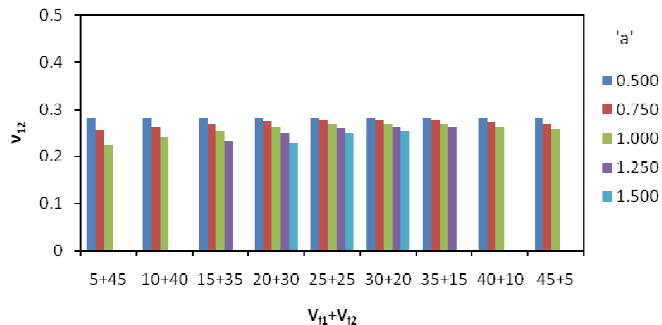
The converged finite element results are validated for the longitudinal Young's modulus using rule of mixtures (ROM) which is an exact theoretical approach for unidirectional continuous fiber composites. The results are presented in Table 2 for various arrangements of fibers for 'a'=0.5.  $V_{f1}$  and  $V_{f2}$  are the volume fractions of T-300 and S-Glass fibers respectively. A very close agreement is observed between the FE and analytical results.

**Figure 3.** Variation of  $E_1$  with respect to varying fiber volume fractions for different 'a'.

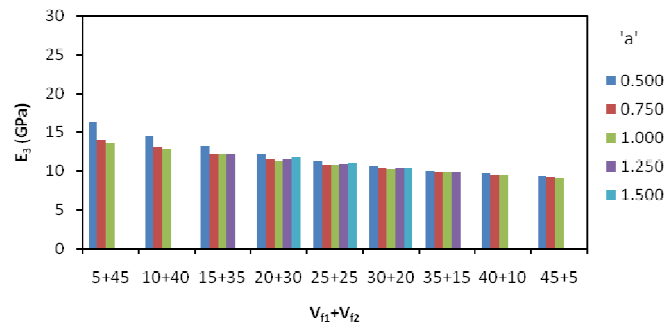
### Analysis

It can be observed that  $E_1$  linearly increases with respect to change in individual fiber volume fractions but not affected by 'a' (Figure 3). Which is expected since rule of mixtures does not take shape of fiber into account.  $\nu_{12}$  decreases and  $\nu_{13}$  increases with 'a' at larger rate for lower volume fractions of T-300 fiber and at smaller rate for smaller volume fractions of S-Glass fiber as shown in Figures 4 and 5. At 'a'=0.5 there is no considerable variation in these Poisson's ratios with respect to change in  $V_f$  of fibers. However,  $\nu_{12}$  increases up to certain extent followed by a slight drop for other values of 'a', whereas  $\nu_{13}$  variation is opposite to that of  $\nu_{12}$ .

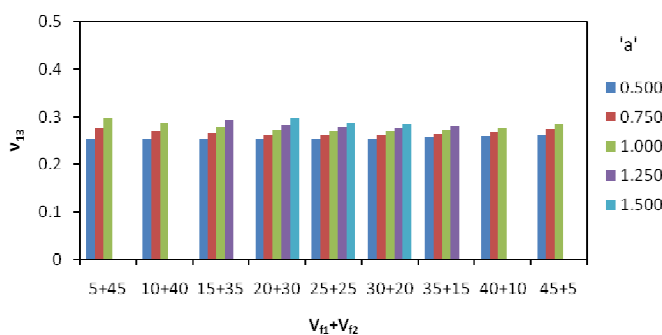
Figures 6 and 7 show the variation of  $E_2$  and  $E_3$  respectively.  $E_2$  and  $E_3$  decrease with increasing in size of T-300 fiber and decrease of S-Glass fiber. This is obvious since stiffness of glass fiber is greater than transverse stiffness of T-300 fiber.  $E_2$  increases and  $E_3$  decreases with 'a' at larger rate for small volume of T-300 fiber. Increase in 'a' increases reinforcement effect in 2-direction and decreases in 3-direction due to increase in projected area of ellipse normal to 2-direction. Figures 8 and 9 show the variation of Poisson's ratios  $\nu_{21}$  and  $\nu_{31}$ . These properties decrease with volume fraction changes. With respect to 'a',  $\nu_{21}$  increases at smaller volume fractions of T-300 fiber. At other places and for all values of 'a' these Poisson's ratios are not changing.



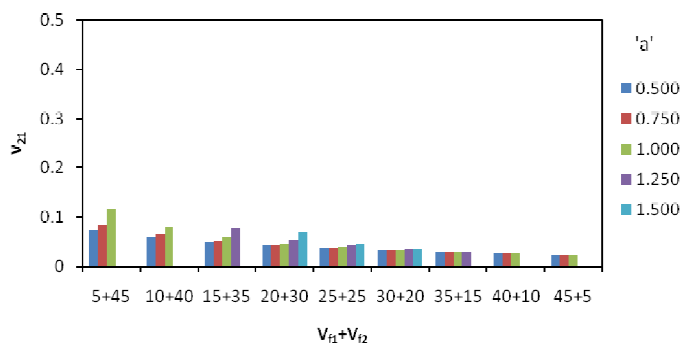
**Figure 4.** Variation of  $v_{12}$  with respect to varying fiber volume fractions for different 'a'.



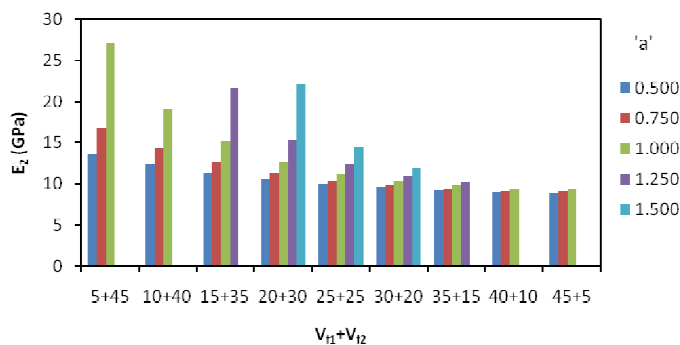
**Figure 7.** Variation of  $E_3$  with respect to varying fiber volume fractions for different 'a'.



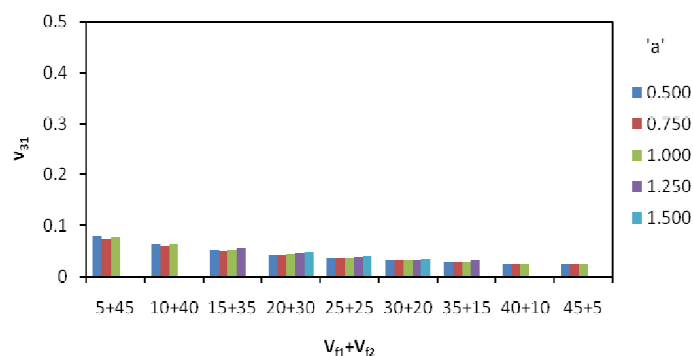
**Figure 5.** Variation of  $v_{13}$  with respect to varying fiber volume fractions for different 'a'.



**Figure 8.** Variation of  $v_{21}$  with respect to varying fiber volume fractions for different 'a'.



**Figure 6.** Variation of  $E_2$  with respect to varying fiber volume fractions for different 'a'.

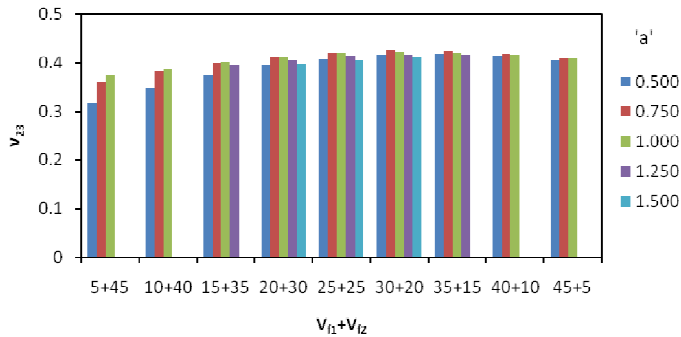


**Figure 9.** Variation of  $v_{31}$  with respect to varying fiber volume fractions for different 'a'.

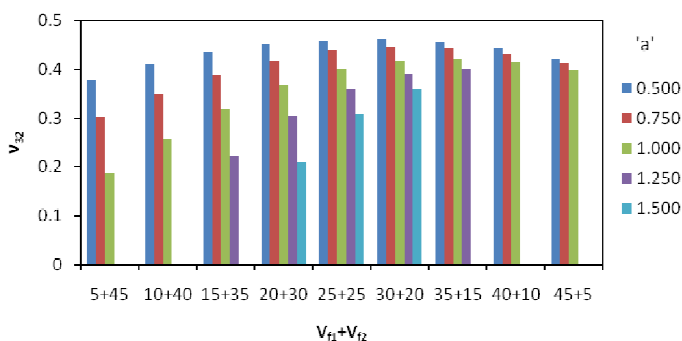
considerably. Figures 10 and 11 show the variation of Poisson's ratios  $v_{23}$  and  $v_{32}$ . These properties increases up to certain extent and decreases with respect to fiber volume fraction changes.  $v_{23}$  increases with 'a' at lower volumes of T-300 fiber followed by negligible changes, whereas  $v_{32}$  decreases with 'a'. The reasons for the variation of Poisson's ratios are similar to that explained for variation of Young's moduli since the strains are taken from the same load cases.

### Conclusions

Young's moduli and Poisson's ratios are predicted for an elliptical hybrid fiber reinforced composite lamina using three-dimensional finite element analysis. The effect of change in volume fractions of individual fibers and fiber cross-sectional aspect ratio on these elastic properties is studied. It is observed that this study gives a scope to



**Figure 10.** Variation of  $v_{23}$  with respect to varying fiber volume fractions for different 'a'.



**Figure 11.** Variation of  $v_{32}$  with respect to varying fiber volume fractions for different 'a'.

obtain a wide range of elastic properties for selection of material as per design requirements.

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