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Nonlinear analysis of cable systems with point based iterative procedure

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Geometric nonlinear static analysis of structural systems with cable elements is carried out using point based iterative procedure. In all sub systems as cable systems, constituted for each node having at least one degree of freedom that is idealized by finite elements, successive calculations are performed. In the analysis part, based on finite element displacement method, to the maximum number of unknown displacements required for each sub-system calculation is limited with three. Tangent stiffness matrix, including pre-stressed internal forces as well as varying geometries with respect to different external force applications, is utilized. The convergence procedure is adapted into the method to prevent excessive displacements through the calculations. In the present study, a computer program has been developed to present a very effective calculation method. Different numerical applications have been considered and the results were compared with the literature results.

Key words: Cable systems, nonlinear analysis, point based iterative procedure, convergence procedure.

INTRODUCTION

Structural analysis of systems having cable elements is relatively complex than other structural systems. The reasons can be summarizd as follows:

1. Displacements are large as a reason of flexibility.
2. The system cannot work for shear, axial pressure forces and bending moments.
3. Pre-stress is applied to the system in order to increase the rigidity.
4. Divergence of calculation occurs rather frequently.

Therefore, cable systems do not exhibit typical nonlinear behavior. Consequently, in the analysis of such systems, iterative methods that can handle geometric nonlinearities are necessary.

Many researchers studied these types of systems in the past. Differential equations (Sinclair and Hodder, 1981), flexibility (O'Brien, 1967), energy (Monforton and El-Hakim, 1980; Pietrzak, 1977), dynamic relaxation (Lewis et al., 1984), and rigidity (Baron and Venkatesan, 1971) methods are used commonly for cabled structures. In summary, nonlinear analysis is conducted in two main steps. In the first step, equilibrium equations that involve unknown internal forces or displacements are developed. In the next step, a numerical methodology is applied to solve the equations of the equilibrium.

In the recent years, the finite element stiffness method based calculation methods are commonly preferred (Desai et al., 1988; Eisenloffel and Adeli, 1994). Geometric nonlinear numerical analysis based methods comprises of successive iterative process. Newton-Raphson numerical method, in which entire the external forces are applied altogether, is utilized in many studies. In the analysis, in which the forces against the equilibrium are considered, a divergence problem is often encountered. In order to avoid this, gradually increasing external forces are applied to the bearing system by increasing it step by step. In the both methods, tangent stiffness which varies at each step is used as a result of nonlinear load-displacement relationship.

In the present study, a simple and effective approach is presented to analyze pre-stressed cable systems. In the method, which is referred to as point-based iterative procedure, sub-systems consisting of nodal points are successively calculated instead of calculating the entire system. Palkowski and Kozlowska (1988) adopted the only elastic axial rigidity to the cross method in which frame structures are applied. In this study, tangent

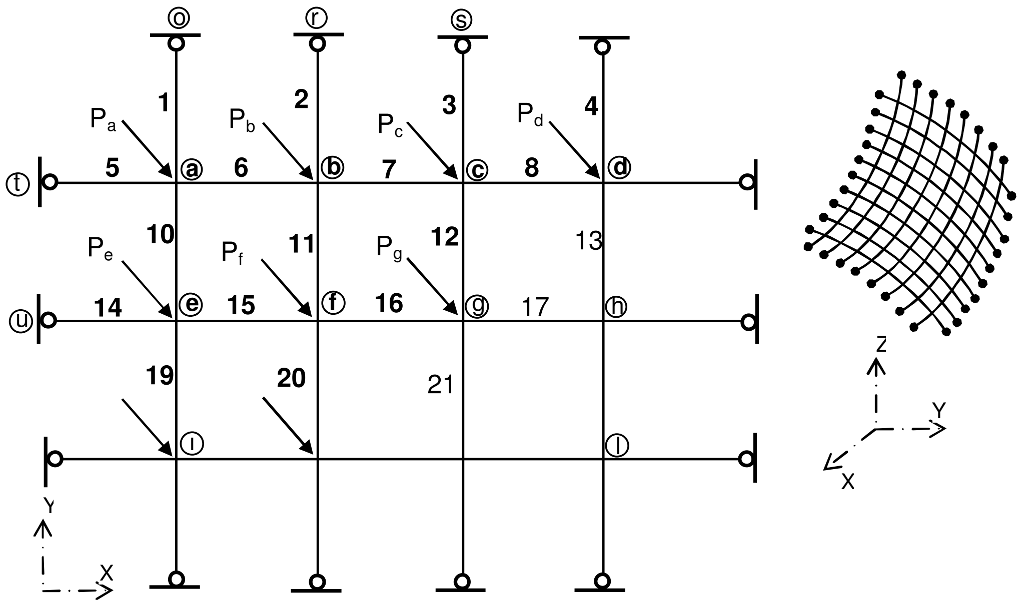


Figure 1. Sample idealized cable system.

stiffness matrix of the finite element displacement method is used. A sub system is constituted for a node having at least one degree of freedom. Neighboring nodes of sub-systems consisting of elements that are only connected to node are taken into consideration as the points that are fully supported. As a result, maximum three equilibrium equations are developed for the analysis of three dimensional systems. Therefore the number of the calculated displacements is limited to 3. In this regard, there is no need for the constitution of the global stiffness matrix of the entire system.

Especially, for the analysis of cable systems having high nonlinearity degree, the problems of divergence or slow convergence are usually encountered. This situation occurs by the continuous increase in displacement values. In this study, a simple convergence procedure is adapted in to the method by checking the displacement increments after each iteration step and also checking those ones that decreasing excessive.

PROCEDURE OF POINT BASED ITERATIVE ANALYSIS

In the proposed method, which involves nonlinear geometric analysis, calculations are based on direct stiffness rules of the finite element matrix displacement method. The cable system is idealized by linear finite elements connecting nodes in the systems. Behaviour of the material is assumed linearly-elastic, homogeneous and isotropic.

Tangent stiffness matrix, which consists of the sum of elastic and geometric stiffness, is used for the development of equilibrium equations. Cables are only subjected to axial tensile forces. In addition, elements of cross-section areas are assumed constant and external forces are applied only on nodal points.

Calculation steps of the method are explained on illustration cable net system depicted in Figure 1. In Figure 1, a, b, c,… stand for nodes, 1, 2, 3,… stand for cable elements and P_a, P_b, P_c,… represent the external forces applied by the nodes on the idealized bearing system with finite elements. At the beginning, displacements and external forces of the elements is equal to zero, if the system has not been pre-tensioned. If pre-stress exists, pre-stress forces are considered as the internal forces at the beginning.

Iterative calculation begins from a random node having at least, one degree of freedom. Fully restraint support nodes, such as r and s, are not required to be computed. Calculations successively continue with respect to the sequence of nodal points. For example, a sub system is constituted for the calculation of node "a" (Figure 2a). The sub system of node "a" is formed by cable elements 1, 5, 6 and 10. Neighboring points of the sub system are b, e, o and t which fully restraint its support. Consequently, only P_a exhibits external forces applied on point "a" in this sub-system.

The sub-system of node "a", having 3 degree of freedom, $\Delta X_{\mathbf{a}}$, $\Delta Y_{\mathbf{a}}$ and $\Delta Z_{\mathbf{a}}$ displacements occurred with respect to applied load P_a. Equilibrium equations can be calculated using a standard methodology. In this study, the finite element displacement procedure is preferred for its advantages:

$$Q_a = K_{Ta} .\Delta_a \rightarrow \begin{bmatrix} Q_{Xa} \\ Q_{Ya} \\ Q_{Za} \end{bmatrix} = \mathbf{K}_{Ta}. \begin{bmatrix} \Delta X_a \\ \Delta Y_a \\ \Delta Z_a \end{bmatrix} \tag{1}$$

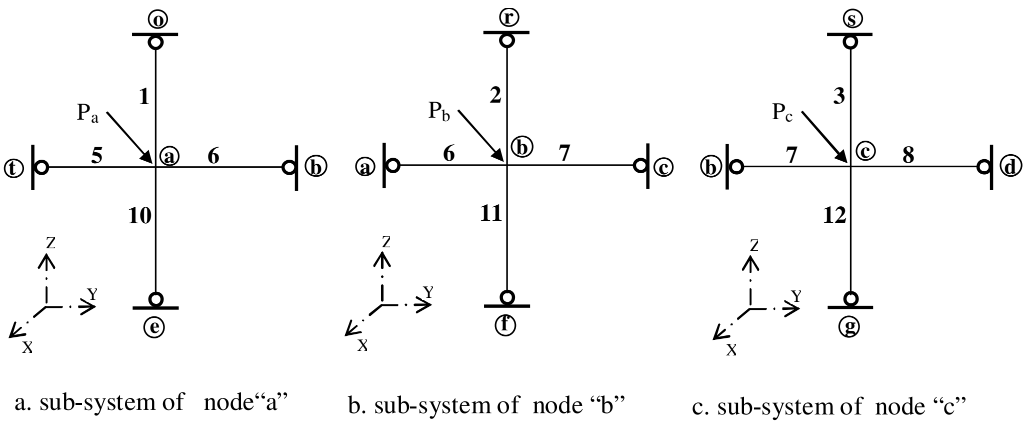


Figure 2. Sub-systems, constituted for a, b and c nodes, respectively.

Where, Δ_a is X, Y and Z directional global unknown displacements of point "a". K_{Ta} is the global tangent stiffness matrix, which consists of the sum of global stiffness of the each element of this node's sub-system. The sum of the elastic (K_E) and geometric (K_G) stiffness matrixes give the global tangent stiffness matrix of each element for only the node of element:

$\mathbf{K}_T = \mathbf{K}_E + \mathbf{K}_G =$

$$\frac{EA}{L}\begin{bmatrix} l^2 & lm & \ln \\ ml & m^2 & mn \\ nl & nm & n^2 \end{bmatrix} + \frac{F}{L}\begin{bmatrix} (m^2+n^2) & -lm & -\ln \\ -ml & (l^2+n^2) & -mn \\ -nl & -nm & (m^2+l^2) \end{bmatrix} \quad (2)$$

where E is the modulus of elasticity, A is cross sectional area, L is the length of the element and F is axial internal force of the cable element. l, m and n are direction cosine values of the angles between local x axes and global X, Y and Z axis, respectively. Geometric stiffness matrix (K_G), which depends on the axial force and the length of the element, is zero at the beginning, if no pre-stress exists. In case of pre-stress, it is considered in the system by means of geometrical stiffness matrix. $\underline{Q}_a$ is unbalanced load vector and is presented by:

$$Q_a = P_a - F_a \quad (3)$$

where, P_a is external force vector applied to the node "a". F_a is the internal force vector of the elements in the sub-system. If no pre-stress exits, F_a is zero at the beginning. At the successive iteration steps or if a pre-stress exists, the reactions of internal forces at the node "a" is accounted as external forces to the node "a". Global displacements (Δ_a) of node "a" are obtained by the solution of linear Equation (1):

$$\Delta_a = [\, \Delta X_a , \Delta Y_a , \Delta Z_a \,]^t \quad (4)$$

As can be derived from Equation (4), the maximum number of unknown displacements for the calculation of a sub-system constituted by cable elements with axial forces is limited with three. In this step, geometry and internal force distribution of the system is changed. New location of the node "a" is calculated by the consideration of obtained displacements as follows:

$$(X_a , Y_a , Z_a)^{new} = (X_a , Y_a , Z_a)^{old} + (\Delta X_a , \Delta Y_a , \Delta Z_a) \quad (5)$$

Where, X_a , Y_a and Z_a are global coordinates of the processing node "a". Superscript "new" is the coordinates obtained after point-based calculation step. Superscript "old" exhibits previous coordinates. With respect to the displacements, relevant variations in axial internal forces of cable elements 1, 5, 6 and 10 are calculated by multiplication as follows:

$$\mathbf{\Delta F}^t_{(1,5,6,10)} = \mathbf{K}_{(1,5,6,10)} \cdot \mathbf{\Delta}_a \quad (6)$$

For example, the variation in the end force of element 6 can be determined as:

$$\mathbf{\Delta F}^t_6 = K_6 \cdot \mathbf{\Delta}_a \quad (7)$$

Where, ΔF^t_6, K_6 and Δ_a are global end forces, stiffness matrix and end displacements of the element 6, respectively. Similarly, the procedure is repeated for all elements connected to node "a". Therefore, new axial internal forces are found as:

$$(F_{(1,5,6,10)})^{new} = (F_{(1,5,6,10)})^{old} + \Delta F_{(1,5,6,10)} \quad (8)$$

Axial force of the element is calculated by the vectoral sum of end forces in X, Y and Z directions. For example, the axial force of the element 6 is:

$$F_6^{new} = F_6^{old} + \Delta F_6 \quad (9)$$

Analogous computations are made for elements 1, 5, 10, which are connected to the node. In a situation where the axial force is compressive, the axial force of the element is assumed to be zero. Therefore, a cable element is restricted in order to expose compression. The method presented in this study can be use for geometric nonlinear analysis of truss systems. But, in the truss analysis, zero compressive load assumption is invalid.

At this stage, the computations made for system of node "a" is completed. Next, new sub-system is considered for node "b". Neighboring nodes are assumed to be fully supported. In Figure 2b, sub system of node "b" is depicted. The nodal sub-system consists of elements 2, 6, 7 and 11, which are connected to the node. Neighboring nodes a, c, r and f are supported. P_b is the external force that is applied to the system on point "b". Previously mentioned calculations are repeated for this node. Firstly, equilibrium equations are establishes as follows:

$$Q_b = K_{Tb} \,.\, \Delta_b \tag{10}$$

Where, Q_b is the unbalanced load vector on node "b" and was found by the addition of external forces (P_b) and the reactions of internal forces at the node "b" (F_b):

$$Q_b = P_b - F_b \tag{11}$$

K_{Tb} is the tangent stiffness matrix of sub-system "b". It should be noted that, in order to find out the new location of point "b", revised coordinates of node "a" should be accounted in the calculations. Therefore, previous calculations are systematically considered in the successive computations. Similar rule is applied in the internal force calculations.

Considering the boundary conditions and solving 3 equilibrium equations, the following displacements are computed for node "b". New coordinates are computed for this node:

$$\mathbf{\Delta_b} = [\ \Delta X_{\mathbf{b}}\ ,\ \Delta Y_{\mathbf{b}}\ ,\ \Delta Z_{\mathbf{b}}\ ]^{\mathbf{t}} \tag{12}$$

with respect to displacements, new coordinates and internal forces was determined by the additions:

$$(\ X_b\ ,\ Y_b\ ,\ Z_b\ )^{new} = (\ X_b\ ,\ Y_b\ ,\ Z_b\ )^{old} + (\Delta X_b\ ,\ \Delta Y_b\ ,\ \Delta Z_b\ ) \tag{13}$$

$$(\ F_{(2,\,6,\,7,\,11)}\ )^{new} = (F_{(2,\,6,\,7,\,11)}\ )^{old} + \Delta F_{(2,\,6,\,7,\,11)} \tag{14}$$

For example, the internal force of element 6 located in "a" and "b" sub-systems:

$$F_6^{\ new} = F_6^{old} + \Delta F_6 \tag{15}$$

It should be noted that, the value of $F_6^{\ old}$ in this equation and the value of $F_6^{\ new}$ in Equation (9) are the same with respect to local axis.

After the calculation of new axial forces in this sub-system, the next node (for example, node "c") is processed. Sub-system of node "c" is given in Figure 2c and the aforementioned explained procedure is applied for node "c".

The first iteration step is terminated after the completion of similar computations for each node having at least one degree of freedom. The second iteration step begins and the procedure is repeated for each node. Numerical analysis is continued until a predefined error criterion is reached. Error criterion or tolerance of iterative calculation vector is determined by equation (9):

$$\boldsymbol{\xi}^n_t = [F_1^{\ n},\ F_2^{\ n},\ F_3^{\ n},\ldots\ldots,F_i^{\ n}]^{\ t} - [F_1^{\ n-1},\ F_2^{\ n-1},\ F_3^{\ n-1},\ldots\ldots,\ F_i^{\ n-1}] \tag{16}$$

where, n is iteration number and i is the number of elements in the entire system. ξ^n is column matrix expressing error amount of the n^{th} iteration step. Basically, it is the absolute difference between the axial forces calculated at n^{th} and $(n-1)^{th}$ steps. Different tolerance values are calculated for different elements. Consequently, calculated values for each element must be smaller than predefined tolerance value:

$$\xi^n <= \xi^0 \tag{17}$$

where, ξ^0 predefined maximum tolerance vector, indicating the global sensitivity. ξ^0 tolerance vector is determined by the user and different values are obtained for various units.

Using iterative procedure as mentioned, results are obtained without computing large matrixes. In addition to cable elements, systems involving axial compression elements or complex structures which are made of cable and truss elements can be solved with the presented approach. Maximum number of equations is limited to 3 for any sub system. This is independent from the number of nodes and elements of the entire system.

THE CONVERGENCE PROCEDURE

In common practice, divergence and slowly convergence problems are mostly encountered in iterative process conducted for the geometrical analysis of structural systems. Namely, solution divergence results in higher deformation or resonance increase, which is avoided in target construction designs. The divergence problem, which is commonly encountered in the analysis of cable systems, can be overcome by several numerical control codes applied through the iterations. Incremental load procedure, in which external forces are treated by the increments applied step by step, is usually preferred by analyzers. On the other hand, another problem is the

Table 1. Comparable displacement results at mid span of pre-stressed single cable.

Uniformly vertical distributed load (N/m)	Displacement at mid span point , Δ_Y (m)			
	Jayaraman and Knudson (1981)	Desai et al. (1988)	Ozdemir (1979)	Point based Iterative procedure
3.50	3.343	3.341	3.343	3.344
10.50	5.948	5.944	5.867	5.952
17.50	7.437	7.432	7.315	7.440
24.50	8.535	8.528	8.407	8.537
31.50	9.427	9.419	9.347	9.428

slow convergence of the solution, which consequently increases the amount of computational duration. In order to avoid this handicap, successive displacements are estimated from the displacements calculated at previous steps. Therefore, number of iteration steps is minimized (Kar and Okazaki, 1973).

Similarly, divergence problems are also observed during the studies carried out in this investigation. In detail, divergence increased slowly, for systems that are pre-stressed and/or subject to homogeneous loadings. Furthermore, if the geometry of the structural system is not symmetrical, then divergence speed is increased.

Analogous to the simple methodology implemented here, a simple convergence procedure is adapted to the method. In essence, the displacements tending to increase is prevented. Namely, increasing displacement values obtained at the iteration steps are divided by a constant (η) that is greater than 1. The formulation of this process is given for the i^{th} step of iteration where the divergence began:

$$(\Delta^i)^{corrected} = (\Delta^i)^{obtained} / \eta^i \quad (18)$$

In the formulation, $(\Delta^i)^{obtained}$ is the excessive displacement (maximum value in the all of them) obtained by the solution of equilibrium (Equations 1 and 10).

$(\Delta^i)^{corrected}$ is the actual displacement value that is decreased by η^i parameter. The η parameter is automatically adjusted with respect to the extent of the divergence. The variation is controlled by the software routine. Therefore, η correction parameter is automatically determined according to the results gathered from previous iteration steps. As a consequence of the convergence procedure, the number of iterations is increased naturally. However, recursive calculations are reduced by the procedure drastically. In addition, the advances in computer technology make this drawback unimportant factor.

PROPOSED METHODOLOGY

To demonstrate the proposed methodology, a computer program has been developed for the geometric nonlinear analysis of cable system using the mentioned nodal iterative and convergence methodology. The simple flow chart of the code, which work within the basis of the finite element displacement method and direct rigidity principles, is given in Figure 3.

Four applications obtained from the literature are solved by the proposed method. The values of the result are compared and evaluated. These numerical applications include 2D and 3D pre-tensioned cable systems. Symmetry is used for all problems if present. The relationships between iteration number with displacement and tolerance value are shown in diagrams for the applications.

First, a single cable, pre-tensioned between two horizontal nodes that are loaded linearly in a uniformly distributed vertical load, has been analyzed. Cable system and main structural properties are shown in Figure 4. Bearing system is idealized by 20 straight cable elements. Value of uniformly distributed load q is changed from 3.50 - 31.50 N/m by 7.00 N/m. Equivalence concentrated loads are applied at the points. Cable is pre-tensioned and value of initial stress is taken (138000 KN/m^2).

The comparable results (Desai et al., 1988; Jayaraman and Knudson, 1981; Ozdemir, 1979) of vertical displacements are given in Table 1. Correction parameter value is taken because divergence problem does not occur in this application. Computation duration is approximately 2 s for 200 - 250 iterative steps, in 3.0 GHz computer. Maximum tolerance value is selected to be 0.001 (N-m units) for all of the loading conditions.

The relationships between iteration number with displacement for each loaded condition and calculated tolerance value for q=31.50 N/m load are shown in Figure 5.

In the presented application, some data have been changed. Pre-stress is canceled and 2000 N vertical single load is applied at mid span node (Figure 6). So, the present rigid system is transformed into bearing system which has high nonlinearity degree. The uniform distributed vertical load is 3.5 N/m on the single cable. Bearing system is idealized by 20 straight cable elements and main structural properties are shown in Figure 6.

When the cable system is calculated by the point based iterative procedure presented in this study, divergence problem occurred during the analysis. In order to solve this problem, correction parameter is used as a constant value of 70. The displacement values are divided by correction

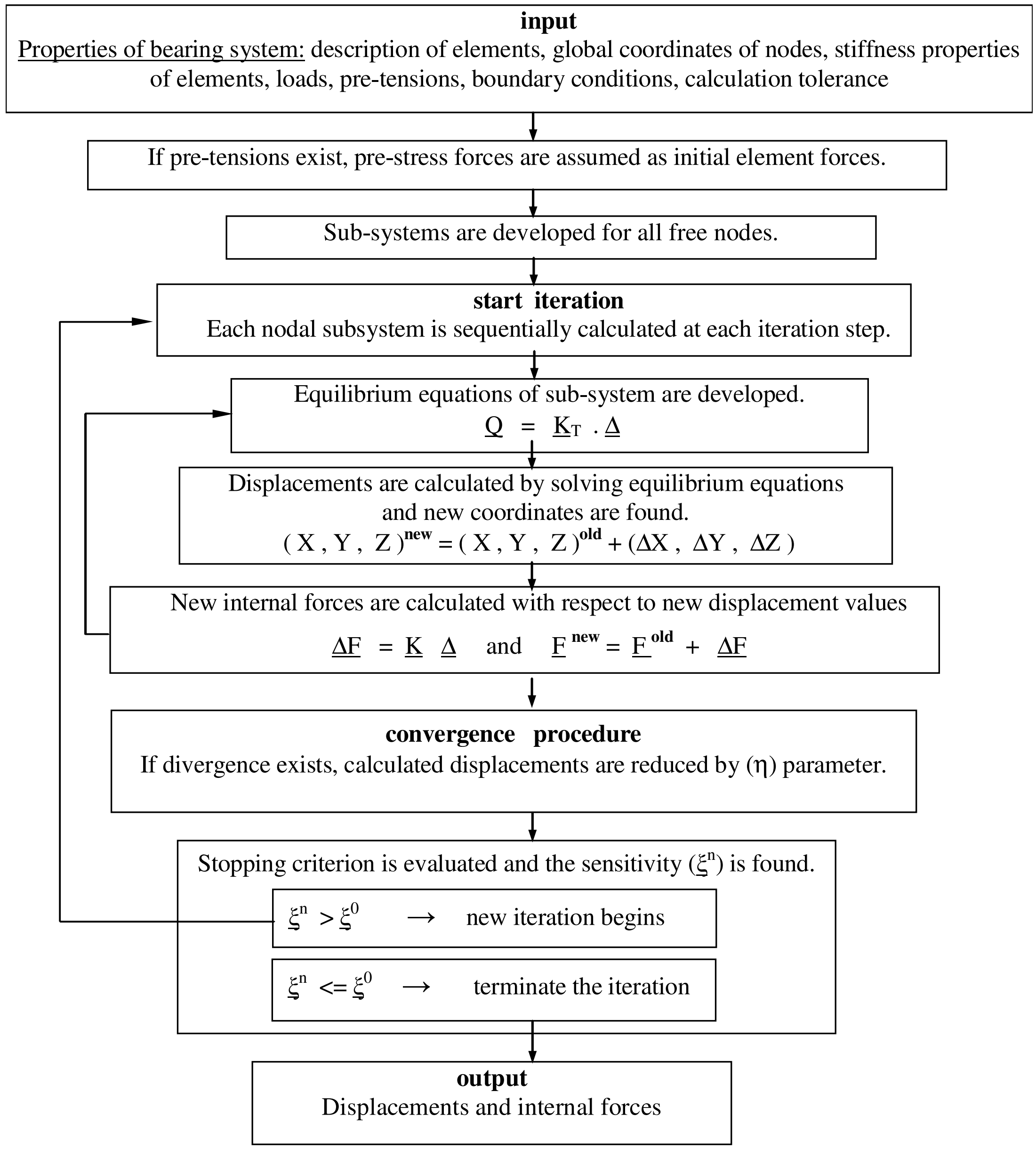
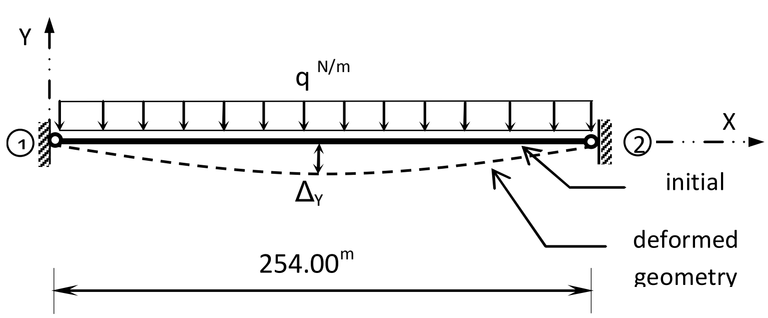


Figure 3. The flow chart of the software developed for point-based iterative procedure.



Data

A: 0.004194 m^2 , (Cross-sectional area)

E: 138 GN/m^2 , (Modulus of elasticity)

σ_0 = 138 MN/m^2 , (Initial stress)

q = 3.50 – 10.50 – 17.50 – 24.50 – 31.50 N/m

Uniform distributed vertical load

Figure 4. Pre-stressed single cable under uniform distributed vertical load.

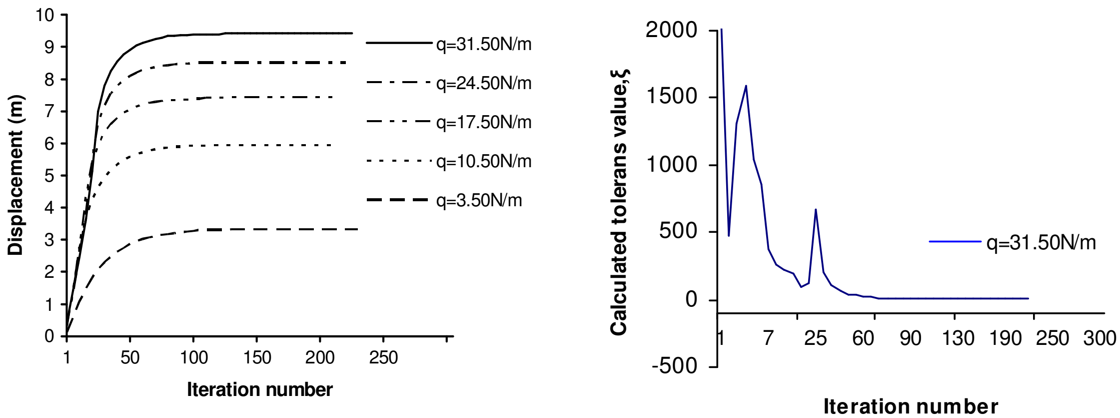


Figure 5. Variation of central vertical displacement and calculated tolerance values (for q=31.50N/m) under increasing iteration number.

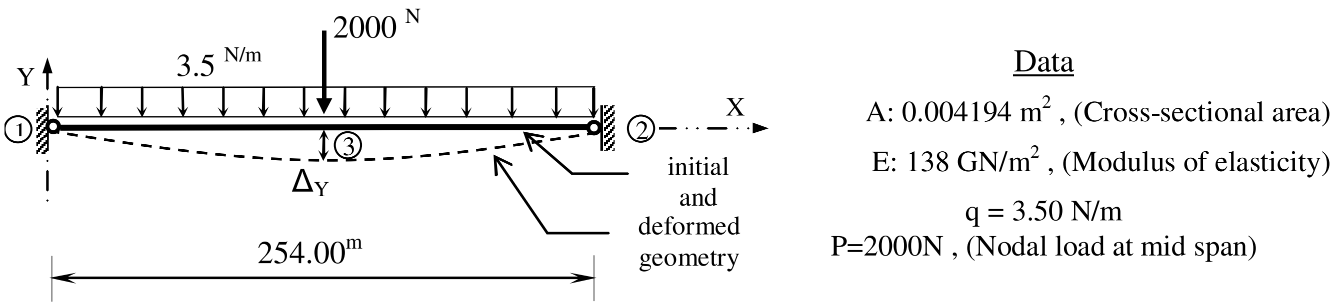


Figure 6. Single cable under uniform distributed and concentrated vertical load.

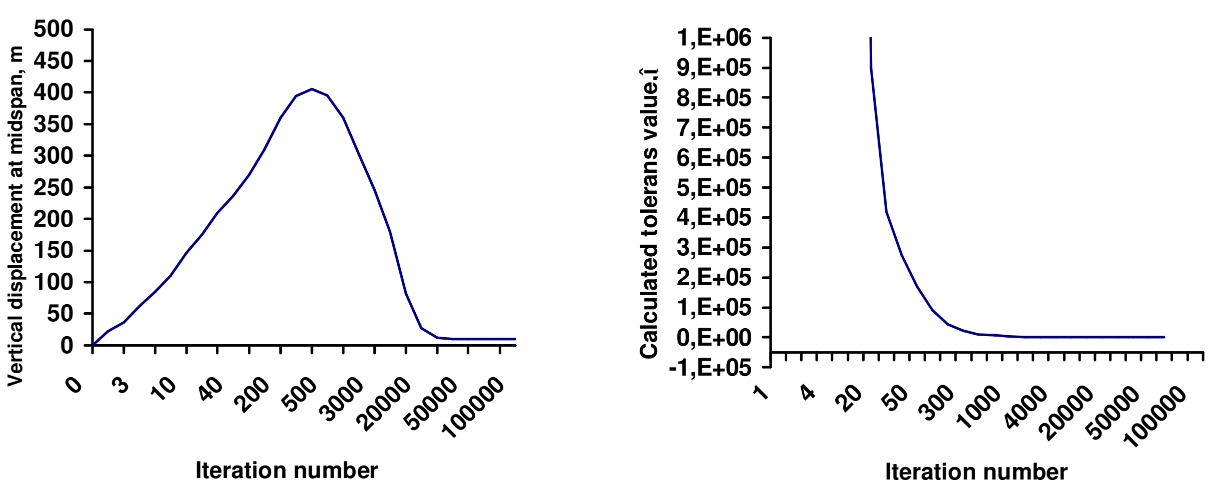


Figure 7. Variation of vertical displacement at mid-span and calculated tolerance values under increasing iteration number, for together under uniform distributed and concentrated vertical load case

parameter and decreased. As a consequence of these operations, the number of iterations is increased and becomes 61405. Maximum tolerance value is taken into consideration (0.001) and analysis duration is approximately 3 min when a 3.0 GHz computer is used. The relationships between iteration number-vertical displacement at node 3 on mid span and iteration number - calculated tolerance value are shown in Figure 7.

Secondly, 3D pre-stressed cable structure, as shown in Figure 8, which consist 12 elements and 12 nodes, has

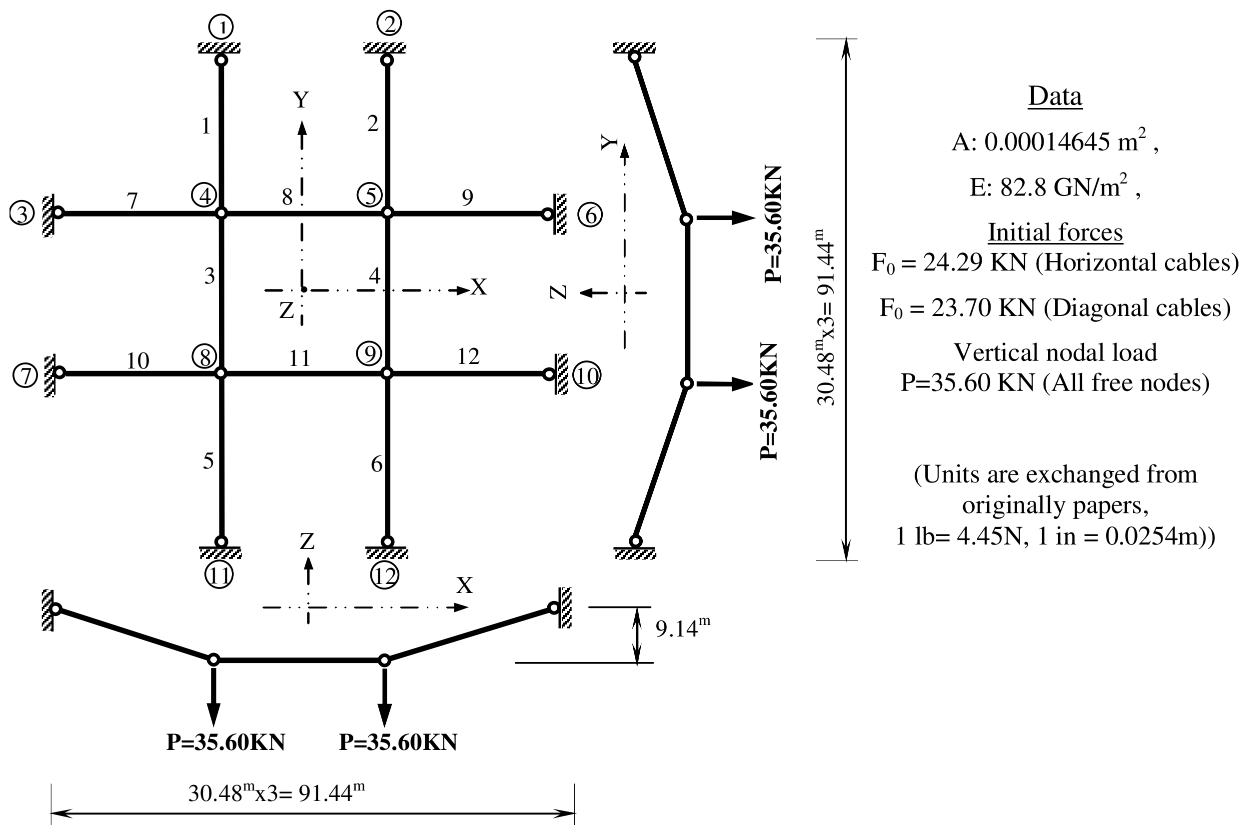


Figure 8. Plan and profile views of pre-stressed cable roof.

Table 2. Displacements at node 4 of pre-tensioned cable roof.

Displacements at node 4 (m)						
Node	**Direction**	**Desai et al. (1988)**	**Jayaraman and Knudson (1981)**	**Saafan (1970)**	**West and Kar (1977)**	**Point based iterative procedure**
	X	-0.0401	-0.0396	-	-0.0404	-0.0402
4	Y	0.0401	0.0402	-	0.0404	0.0402
	Z	-0.4460	-0.4463	-0.4483	-0.4480	-0.4464

been calculated for symmetrically vertical nodal loads. Cross-sectional areas and modulus of elasticity are 0.00014645 m^2 and 82.8 GN/m^2 respectively, to all wire ropes. External vertical load 35.60 KN (self weigh is neglected) is applied simultaneously to the nodes 4, 5, 8 and 9. The pre-stressing axial forces in horizontal (3, 4, 8 and 11 numbered elements) and diagonal cables are 24.29 and 23.70 KN, sequentially.

Displacements of free nodes are presented in Table 2, and compared with the results, which were given in the literature (Desai et al., 1988; Kar and Okazaki, 1973; Jayaraman and Knudson, 1981; Saafan, 1970). When using the presented procedure herein, divergence would not occur during the solution, like the first application, as a result of symmetric and homogeneous loading and geometry.

Duration of analysis is approximately 1 s on a 3.0 GHz computer, if the maximum tolerance value used is 0.0001 and the total number of iteration steps becomes 41 in this application for the point based iterative procedure which was presented here.

Then, pre-stressed orthogonal cable net is illustrated in Figure 9, geometrically symmetric about X and Y axes, has been analyzed for self weigh and additional external loads. The idealized bearing system is constituted by 41 nodes and 64 finite cable elements. In the net system, bearing cables and stabilizing cables are in the direction of X and Y axis, respectively. Its surface geometry is of hyperbolic parabolic form and the designed equation is given by:

$$Z = (C_x . X^2) / a^2 - (C_y . Y^2) / b^2 + f \qquad (19)$$

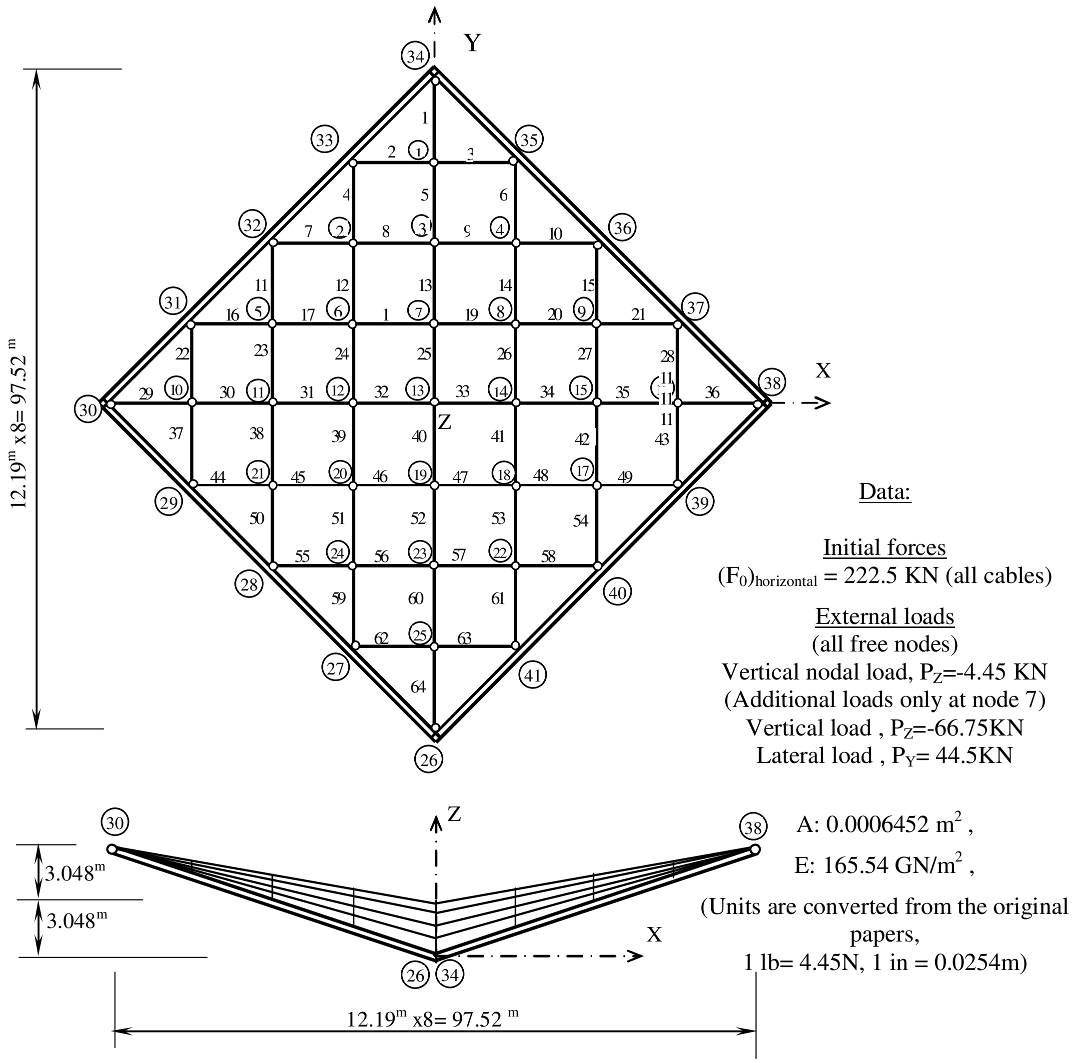


Figure 9. Pre-stressed hyperbolic parabolic cable net under the vertical and lateral loading.

If the values are $C_x = C_y = 3.048^m$, $a = b = 148.77^m$, $f = 3.048^m$ taken account in unit in meter, Equation 19 rewritten as:

$$Z = 0.00011906\,(X^2 - Y^2) + 3.048 \qquad (20)$$

The cross-sectional area (0.0006452 m^2), the modulus of elasticity (165540000 KN/m^2) and the horizontal component of pre-stress (222.5 KN) are the same in all members. The vertical point load of 4.45 KN is applied to all nodes, equivalent of the self weigh loads. In this condition, horizontal load of 44.5 KN in the positive Y direction and vertical load of 66.75 KN in the negative Z direction are applied to node 7. Results for displacements and axial forces are presented in Table 3, together with the results taken in literature (Monforton and El-Hakim, 1980; Thornton and Birnstiel, 1967).

If the results of in this study are compared with the data given in the literature, it can be seen that they are in good agreement. The solution of this problem with the point based iterative procedure is obtained by 68 iterative steps for maximum tolerance value which is 0.0001. Computation duration is approximately 15 s in a 3.0G Hz computer, in those conditions. Very appropriate convergence occurred during the analysis, for this application. The relationships between iteration number-vertical displacement at node 7 and iteration number - calculated tolerance value are shown in Figure 10.

Finally, the counter stressed dual cable truss structure is studied for different load cases, analyzed in literature by Thornton and Birnstiel (1967) and Nishino et al. (1989). Firstly, the two dimensional cable system is designed under external pre-stressing forces of 44.50 and 22.25 KN acting at the supports of the parabolic shaped tie-down (stabilizing) and load (bearing) cables, respectively in this study. These values are the horizontal components of axial forces for each straight element at the stabilizing and bearing cables. The geometry is

Table 3. Displacements and axial forces horizontal component of axial forces.

Vertical displacements (m)				Horizontal component of cable axial force (kN)			
Node number	Thornton and Birnstiel (1967)	Monforton and El-Hakim (1980)	Point based iterative procedure	Cable number	Thornton and Birnstiel (1967)	Monforton and El-Hakim(1980)	Point based iterative procedure
34	0.000	0.000	0.000	1	204.46	204.33	204.45
1	0.136	0.134	0.136	5	204.95	204.72	204.92
3	0.417	0.418	0.417	13	205.87	205.77	205.70
7	1.143	1.144	1.144	25	163.12	162.91	163.14
13	0.507	0.509	0.508	40	163.22	162.98	163.24
19	0.293	0.293	0.294	52	163.30	163.00	163.33
23	0.170	0.171	0.170	60	163.36	163.36	163.37
25	0.069	0.068	0.069	64	167.79	167.80	163.63

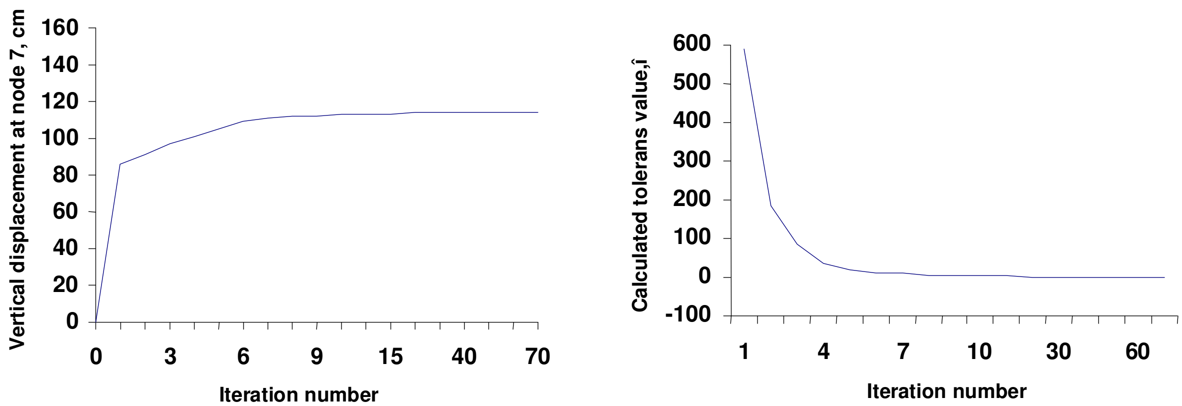
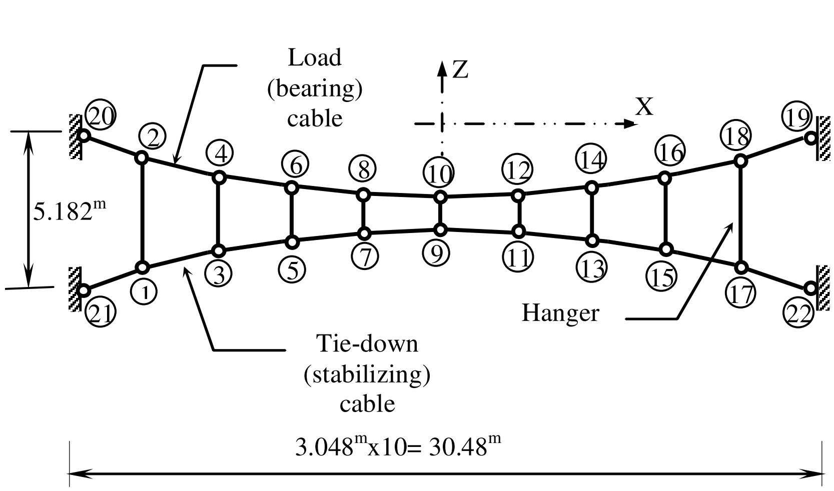


Figure 10. Variation of vertical displacement at node 7 and calculated tolerance values under increasing iteration number.



Data

A= 12.90 10^{-4} m^2 (bearing cable)
A= 6.45 10^{-4} m^2 (stabilizing cable)
A= 6.45 10^{-5} m^2 (hanger)
E= 165.54GN/m^2 ,

Initial forces
F_0 = 22.25 KN (bearing cables)
F_0 = 44.50 KN (stabilizing cables)

(Units are exchanged from originally papers,
1 lb= 4.45N, 1 in = 0.0254m))

Figure 11. Counter stressed dual cable truss structure.

shown in Figure 11, where the system is pre-stressed and fixed at the supports nodes having geometric and boundary conditions. Secondly, the system is loaded by concentrated vertical loads corresponding triangularly to the distributed loading, located at the joints of the stabilizing cable, which vary linearly from 1.335 KN at

Table 4. Nodal coordinates and axial forces for attitude to pre-stressing.

Coordinates (m)						Axial forces (KN)					
Node	X	Z	Node	X	Z	Element	Forces	Element	Force	Element	Forces
1	-12.194	-4.633	7	-3.048	-3.719	20-2	23.65	21 - 1	45.22	1-2	1.78
2	-12.194	-1.951	8	-3.048	-2.926	2-4	23.10	1 - 3	44.94	3-4	1.78
3	-9.144	-4.206	9	0.000	-3.658	4-6	22.69	3 - 5	44.72	5-6	1.78
4	-9.144	-1.951	10	0.000	-3.048	6-8	22.41	5 - 7	44.58	7-8	1.78
5	-6.096	-3.901	20	-15.24	-0.000	8-10	22.27	7 - 9	44.51	9-10	1.78
6	-6.096	-2.560	21	-15.24	-5.182	Not: System is symmetric about to Z axis.					

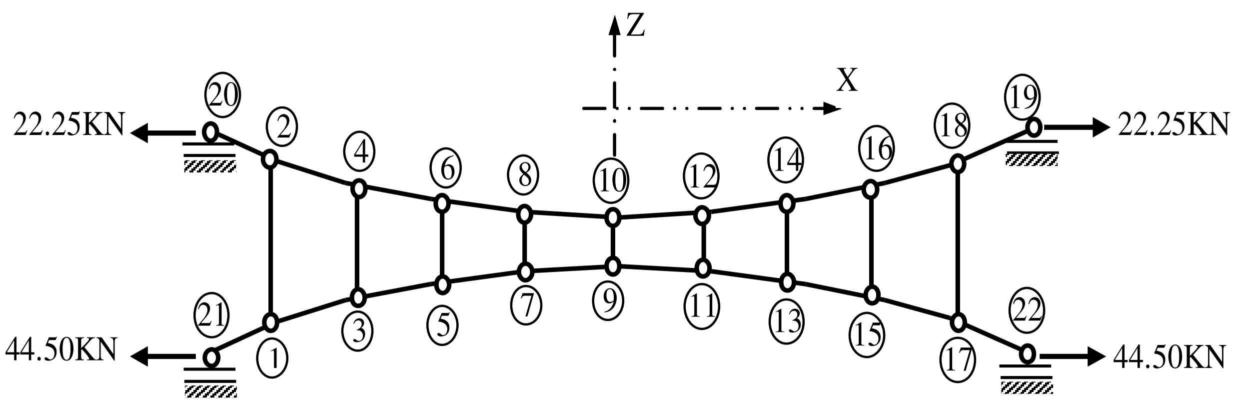


Figure 12. Pre-stress loading application to cable truss.

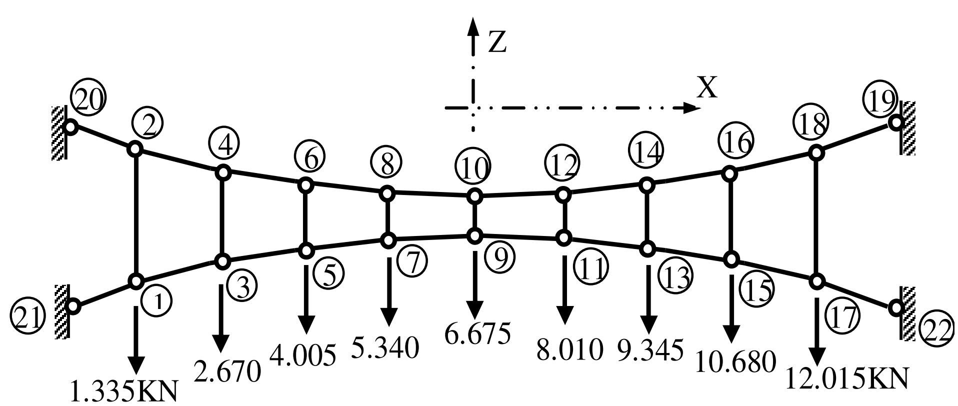


Figure 13. Vertically concentrated loading at the pre-stressed cable system.

joint 1 to 12.015 KN at joint 17. Finally, only node 10 is loaded with 50 KN single load, in the direction to Y axis when the cable truss is pre-stressed. The modulus of elasticity is 165.54 GN/m^2 and the cross-sectional areas are 6.45 10^{-5}, 6.45 10^{-4} and 12.90 10^{-4} m^2 for hanger, stabilizing and bearing cables, respectively.

The axial forces of elements and nodal coordinates given in Table 4, are used in the presented study. First of all, support nodes 19, 20, 21 and 22 are unconstrained in horizontal direction only for applying pre-stress (Figure 12) and these nodes are fixed after pre-stressing is applied. Then, concentrated vertical external loads are placed on nodes in the stabilizing cable and single horizontal load is applied at node 10 in the bearing cable separately when the system is pre-stressed. According to the point based iterative procedure, the element forces and nodal coordinates found by calculation of pre-stress step are the data inputted for the following geometric nonlinear analysis (vertical and horizontal concentrated loading cases).

Table 4 summarizes the starting data for loading cases as shown in Figures 13 and 14.

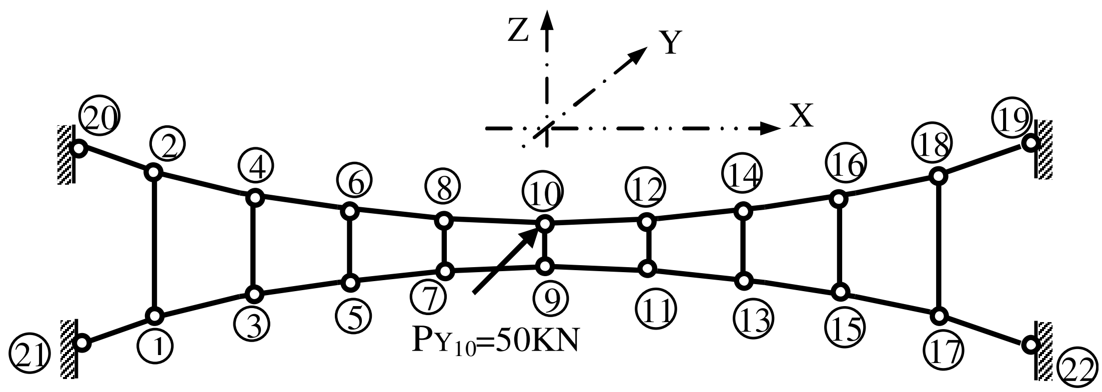


Figure 14. Single load in Y direction at node10 and the truss is pre-stressed.

Table 5. Displacements for vertically concentrated loading, truss is pre-stressed.

Displacements (m)									
	X		Z			X		Z	
Node	Thornton and Birnstiel (1967)	Point based iterative procedure	Thornton and Birnstiel (1967)	Point based iterative procedure	Node	Thornton and Birnstiel (1967)	Point based iterative procedure	Thornton and Birnstiel (1967)	Point based iterative procedure
1	-0.0213	-0.0219	-0.1106	-0.1128	2	0.0393	0.0402	-0.1113	-0.1143
3	-0.0259	-0.0268	-0.1466	-0.1512	4	0.0503	0.0518	-0.1460	-0.1508
5	-0.0232	-0.0237	-0.1253	-0.1301	6	0.0469	0.0484	-0.1234	-0.1286
7	-0.0198	-0.0204	-0.0668	-0.0695	8	0.0405	0.0420	-0.0631	-0.0676
9	-0.0204	-0.0192	0.0088	0.0097	10	0.0378	0.0390	0.0137	0.0122
11	-0.0204	-0.0213	0.0844	0.0884	12	0.0411	0.0423	0.0878	0.0902
13	-0.0238	-0.0247	0.1408	0.1460	14	0.0485	0.0499	0.1423	0.1469
15	-0.0253	-0.0259	0.1582	0.1621	16	0.0530	0.0530	0.1579	0.1621
17	-0.0192	-0.0198	0.1180	0.1207	18	0.0423	0.0433	0.1152	0.1176

Table 6. Element forces for vertically concentrated loading, truss is pre-stressed.

Axial forces (kN)										
Element	20 - 2	2 - 4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 19
Force	118.55	117.04	115.97	114.85	114.63	115.30	116.63	118.90	122.20	126.65
Element	21 - 1	1 - 3	3-5	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 22
Force	68.53	67.64	66.88	66.26	65.55	64.84	64.04	63.59	63.32	63.19
Element	1 - 2	3 - 4	5 - 6	7 - 8	9-10	11-12	13-14	15 - 16	17 - 18	
Force	5.79	6.68	7.57	8.41	9.256	10.10	10.99	11.88	12.82	

During the geometric nonlinear analysis of pre-stress and vertically concentrated load cases, for 10^{-5} and 10^{-6} maximum tolerance values, 331 and 2668 iteration steps are constituted, respectively. The value of correction parameter is taken in to consideration as 1 and any divergence problem do not occur in the iterative calculations. If maximum tolerance value is selected as a large value (10^{-4} and 10^{-5}), although the number of iteration steps are reduced (95 and 1959), the results differ from actual values.

The displacements calculated by the presented procedure are given in Tables 5, 6 and the vertical loading case is shown in Figure 13. The results are compared with that of Thornton and Birnstiel (1967) given the incremental load method also.

Sequentially, the displacements and element axial forces are given in Tables 7 and 8, obtained for single loading case in Y direction at node 10, shown in Figure

Table 7. Displacements for horizontal load in Y direction at node 10 and the truss is pre-stressed.

Displacements (m)							
Node	**X**	**Y**	**Z**	**Node**	**X**	**Y**	**Z**
1	-0.0049	0.1506	0.0463	2	0.0140	0.1914	0.0497
3	-0.0070	0.2999	0.0856	4	0.0201	0.3850	0.0863
5	-0.0064	0.4444	0.1164	6	0.0204	0.5867	0.1097
7	-0.0039	0.5761	0.1448	8	0.0140	0.8105	0.1094
9	-0.0000	0.6770	0.1948	10	0.0000	1.0866	0.0387
11	-0.0039	0.5761	0.1448	12	0.0140	0.8105	0.1094
13	-0.0064	0.4444	0.1164	14	0.0204	0.5867	0.1097
15	-0.0070	0.2999	0.0856	16	0.0201	0.3850	0.0863
17	-0.0049	0.1506	0.0463	18	0.0140	0.1914	0.0497

Table 8. Element forces for horizontally loading to Y direction at node 10, truss is pre-stressed.

Axial forces (kN)										
Element	20 - 2	2-4	4 - 6	6 - 8	8 - 10	10 - 12	12 - 14	14 - 16	16 - 18	18 - 19
Force	177.55	173.98	171.34	169.82	169.42	169.42	169.82	171.34	173.98	177.55
Element	21 - 1	1 - 3	3 - 5	5 - 7	7 - 9	9 - 11	11 - 13	13 - 15	15 - 17	17 - 22
Force	294.79	297.03	295.22	293.85	293.00	293.00	293.85	295.22	297.03	294.79
Element	1 - 2	3 - 4	5 - 6	7 - 8	9 -10	11 - 12	13 - 14	15 - 16	17 - 18	
Force	12.50	12.61	12.09	10.00	28.85	10.00	12.09	12.61	12.50	

14. Due to the exceptional loading at the node 10, the system behavior is highly nonlinear and the divergence problems occur while starting the calculation steps. For this reason, the value of correction parameter is taken into account as the value of the three. Excessive displacements are divided by the correction parameter at the end of each iteration steps.

When the results, obtained for the numerical applications are evaluated together, the geometrical nonlinear analysis procedure presented in this study could be used for all types of cable structures successfully. When divergence of solution occurred in the iterative analysis, a correction parameter value is adapted in to calculation automatically. Consequently, total number of iterations and duration of analysis are increased. But this situation does not create an important problem. Since the computer process speeds have reached high values, in addition, this problem can be minimized by utilizing the professional programming techniques.

Conclusion

The geometrical nonlinear analysis of cable systems is carried out using point-based iterative procedure. Within the analysis performed with finite element direct stiffness principles, pre-stress forces are applied using tangent stiffness matrix. The maximum number of equilibrium equations and unknown displacements are limited with three. Because numerical calculations are carried out only for sub-structures established for nodal points. A simple and effective convergence procedure is adopted in the method to avoid divergence problem. The convergence procedure intervenes with the excessive displacements in order to restore normal iteration steps.

A computer program has been developed and presented in accordance with the point-based iterative method. The outcomes of the analysis of the developed program are consistent with the data given in the literature. Even though the number of iterations is excessive for the presented methodology, the memory required is drastically decreased. In additional solution, divergence problem prevailed and are controlled practically.

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