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Full Length Research Paper

Ricci tensors for elliptical shaped galaxies and celestial objects

Bijan Nikouravan

[1]Department of Physics, Islamic Azad University (IAU)-Varamin-Pishva Branch, Iran.
[2]Department of Physics, University of Malaya (UM), 50603 Kuala Lumpur, Malaysia. E-mail: nikou@um.edu.my, bijan_nikou@yahoo.com. Tel: (+60) 0123-851513.

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Einstein's field equations are based on Riemannian geometry. One of the great important in Riemannian geometry is the curves that minimize the distance between two given points. The Ricci curvature tensors are also broadly applicable to modern Riemannian geometry and general theory of relativity (GTR). The solution of Einstein's field equations, need Ricci tensors. The line elements based on the ellipsoidal coordinates systems usually are difficult to work but are more perfect than spherical coordinate systems. Here we are trying to get all non-zero *Ricci* tensors coefficients of a line element in an oblate spheroidal coordinate system for an elliptical shaped object. These objects can be as galaxies, stars or planets and other celestial objects that we know today.

Key words: General relativity, gravitational files, oblate spheroidal coordinates, Ricci tensors.

INTRODUCTION

The galaxies and stars are in different types of shapes and forms (Michael and Dana, 2010). These objects are in spherical and/or in elliptical forms usually (Orlov, 2011); most of studies assuming elliptical objects such as spherical and round object for simplification, but really these objects (galaxies and stars or planets) are not perfectly in spherical forms (Roger, 2006). Our galaxy is made up of a mega collection of stars and planets. The percentages of observed galaxies in Universe are with 77% in circular and spirals galaxies, 20% in elliptical or oval egg shaped and finally 3% which have none of these shapes and known as irregulars; the stars within a galaxy remain together due to gravitational forces (NASA).

The masses of elliptical galaxies usually are in the range of about 10^7 up to 10^{13} solar masses (Tim and Marijn, 1991). The smallest of the elliptical galaxies, which are called dwarf elliptical, may be only a little larger than globular clusters, while the giant elliptical galaxies like M87 are among the largest galaxies in the Universe (NOAO); this is a much larger range in size than is seen for the spiral galaxies. Elliptical galaxies are smooth and elastic in appearance; M32 is another type of elliptical dwarf approximately with 3 billion solar masses and a linear diameter of some 8,000 light years (NOAO). It is very small compared to its giant spiral-shaped neighbor.

In 1915, Einstein published a set of differential equations known as the Einstein field equations. Einstein's general theory of relativity (GTR) showed the universe as a geometric system of three dimensions in space and one time dimension (Einstein, 1916). Otherwise general relativity is based on the concept of *space-time* (Stephani et al., 2003). The physical meaning of a space-time does not depend on the particular coordinate system employed in its representation. In general, a transformation of coordinates changes the form of a metric, but not its interpretation (Jerry and Jiri, 2003).

The presence of mass, energy, and momentum, related to mass-energy density or stress-energy in a bending of space-time coordinate system. Therefore, the gravity is movement along the "simplest" or least-energetic route along the curved space-time. Einstein found a relationship between the curvature of space-time and mass-energy density, otherwise,

(Curvature of Space-time) $\propto$ (Mass-energy density)
(Curvature of Space-time) = (Mass-energy density) = $8\pi G/c^4$

The equation shows a direct, constant proportion between the curvature of space-time and mass-energy density; here G comes from Newton's law of gravity and c is the speed of light and is expected from the theory of special relativity. In the case of zero mass energy density (that is, empty space), the space-time is flat. The classical gravitation is a special case of gravity's manifestation in a relatively weak gravitational field, where the c^4 term (a very big denominator) and G (a very small numerator) make the curvature correction small.

Einstein field equations are based on Riemannian geometry and the major concept of Riemannian geometry is curvature and the resulting space is a 4-dimensional Lorentzian manifold (Landau and Lifshitz, 1987). There are various notions of curvature which are of great interest all of them can be deduced from the so-called *Riemannian curvature tensor*. According to GTR, the curvature created by massive and heavier objects will cause higher gravitation than lighter objects (Chandrasekhar, 1983). For example, the curvature created by the Earth keeps the Moon in orbit, but at the same time, the curvature created by the Moon is enough to affect the tides. Certainly, the curvature of the spherical and elliptical objects in space are different.

All galaxies such as NGC 1407, M32 (low-luminosity elliptical), M96, NGC 3384 or a new discovered object, 2003EL61 which is named, Haumea, by IAU, are in elliptical shaped (Colin et al., 2009; Ragozzine and Brown, 2009). The gravitational fields surrounding these objects are different than spherical celestial objects (Nikouravan, 2009).

The first solution of Einstein field equations was found by Schwarzschild in 1916 (Schwarzschild, 1916). Schwarzschild solution is valid only for spherically symmetric objects. Nevertheless, we are interested in real and for elliptical objects, which is more perfect. In fact, the spherical solution of Einstein's field equations is as an especial case and this solution is not satisfying elliptical shape. Here the Ricci tensors are very essential for the solution of Einstein's field equations (Ramsey.1961). However, the solutions of Einstein's field equations in the oblate spheroidal coordinate system are very tedious and complicated, but it is more accurate (Nikouravan, 2001). This solution not only used for elliptical objects but also it is valid for spherical objects too. The purpose of the present paper is to obtain Ricci tensors directly in an oblate spheroidal coordinate system.

In this regards, we start with some simple definition about ellipsoids in R^3 space and ellipsoids in Minkowski space-time and finally we try for Ricci tensors in a static oblate spheroidal coordinate system.

ELLIPSOIDS IN R^3 SPACE

In the Euclidean geometry of flat space, the notion of an ellipsoid is clear and easily understood. Euclidean space, and Euclidean geometry by extension, is assumed to be

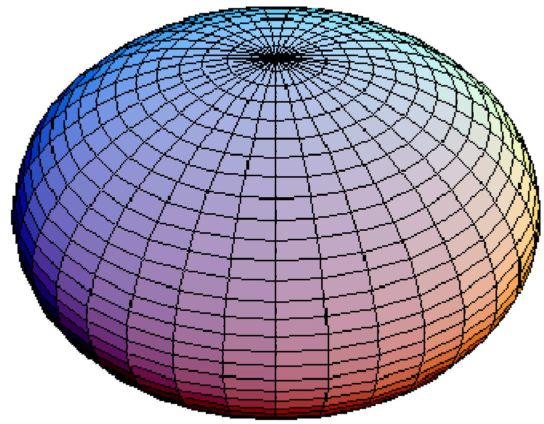

Figure 1. Elliptical object in term of Cartesian coordinates system (Designed with Mathematica 5.1).

flat and non-curved, but according to GTR, the space is curved (Einstein Albert). Here, we consider a review of the properties of ellipsoids in R^3 space. For this, imagine a 3-dimensional Euclidean space, R^3, filled with a congruence of ellipsoids with a common center. We suppose one of the semi-axis of the ellipsoids as the coordinate $r = const$ on a fixed ellipsoid. Let φ be the angle measured around this axis, and let it be a third coordinates defined at will. We define (r, θ, φ) in terms of Cartesian coordinates (x, y, z) as follows:

$$\square = \square_0(\square) \sin \square \cos \square \qquad \square = \square_0(\square) \sin \square \sin \square \qquad \square = (\square)\,\square \tag{1}$$

where $x_0(r)$ and $y_0(r)$ are arbitrary functions whose values are equal the lengths of the other two semiaxes of the ellipsoid (Figure 1) (Mathematica 5.1). The line element $ds_E^2 = dx^2 + dy^2 + dz^2$, is in Euclidean space R^3 (Zsigrai, 2008).

The line element for rotationally invariant oblate ellipsoids is as (Nikouravan, 2011),

$$\square_{\square}^2 = \frac{\square^2 + \square^2\,\square^2\,\square}{\square^2 + \square^2}\,\square^2 + (\square^2 + \square^2\,\square)\,\square^2 + (\square^2 + \square^2)\,\square^2\,\square^2 \tag{2}$$

Here r is the radius of a single ellipsoidal surface and is constant $(r = const)$. It is important to mention that Equation (2) is independent of time.

ELLIPSOIDS IN MINKOWSKI SPACE TIME

The new concept is Minkowski's 4-dimensional space-time or Minkowski space (Hooft, 2009) and the mathematical technique is the '4-vector' calculus in that space-time.

One of the most important results of special relativity is basic physical laws are most simply expressed when they are formulated not in three-dimensional space but in four-dimensional space-time. Hermann (1864–1909) following Poincaré's reasoning, developed the idea of the pseudo-Euclidean geometry with the form,

$$\square\square^2 = \square\square^2 - \square\square^2 - \square\square^2 - \square\square^2 \qquad (3)$$

The Minkowski space-time reflects the dynamical properties common for all types of matter. We can employ many sums and shall follow the summation convention that an index which is repeated represents a sum. This enables us to write (3) in the form:

$$\square\square^2 = \square_{\square\square}\,\square\square^{\square}\square\square^{\square} \qquad (4)$$

Here the elements g_{ij} are called metric tensor or gravitational potential of matter (Wolfgang, 2003). It is convenient that the line element (4) in the Minkowski space-time for static is in the following form.

$$\square\square^2_{\square\square\square} = \square^2\square\square^2 - \frac{\square^2+\square^2\square\square\square^2\square}{\square^2+\square^2}\square\square^2 - (\square^2 + \square\square\square^2\square)\square\square^2 + (\square^2+\square^2)\square\square\square^2\square\square\square^2 \qquad (5)$$

Comparing (5) and (2), shows time dependent and independent, respectively.

RICCI TENSORS FOR A STATIC OBLATE SPHEROIDAL

Galaxies or star-clusters, which rotating with a period of a billion years or more, can be considered as a static ellipsoid, and apparently their rotation may not play an important role for a general purpose. As Schwarzschild solution, which is valid only for spherical shaped objects, we apply Schwarzschild-like solution but exactly in the oblate spheroidal coordinate system (Nikouravan, 2011). It is assumed that the object is in a static and non-rotating or its rotation is very slowly and is neglectable. The line element (4) in the Minkowski space-time is,

$$\square\square^2 = \square^2\square\square^2 - \square\square^2 - \square\square^2 - \square\square^2 \qquad (6)$$

The relation between Cartesian coordinate system to ellipsoidal coordinate system (u, v, φ), (Parry and Domina, 1961) are as,

$$\square = \square(\square\square\square\square\,\square)(\square\square\square\square)\square\square\square\square,\quad \square = \square(\square\square\square\square\,\square)(\square\square\square\square)\square\square\square\square,\quad \square = \square(\square\square\square\square\,\square)(\square\square\square\square),\quad \square = \square \qquad (7)$$

By using (7), the line element (6) in oblate spheroidal coordinate system is,

$$\square\square^2 = \square^2\square\square^2 - \square^2(\square\square\square\square^2\square + \square\square\square^2\square)\square\square^2 - \square^2(\square\square\square\square^2\square + \square\square\square^2\square)\square\square^2 - (\square^2\square\square\square\square^2\square\square\square\square^2\square)\square\square^2 \qquad (8)$$

The line element (8) can be change to the following form,

$$\square\square^2 = \square^2\square\square^2 - \square^2\left(\square^2 + \frac{\square^2(1-\square^2)}{(1+\square^2)}\right)\square\square^2 - \square^2\left(\square^2 + \frac{\square^2(1+\square^2)}{(1-\square^2)}\right)\square\square^2 - \square^2(1+\square^2)(1-\square^2)\square\square^2 \qquad (9)$$

Here for more simplification we supposed $\xi = \sinh u$, $\eta = \sin v$, $(1+\xi^2)^{1/2} = \cosh u$ and $(1-\eta^2)^{1/2} = \cos v$. Another form of this line element had been obtained by some transformation from spherical to elliptical co-ordinate system (Nikouravan, 2011).

The line element (9) written exactly in the oblate spheroidal coordinate system had not been considered yet. In the presence of matter and any massive object such as an elliptical galaxy, the line element (9) in static and non-rotating form is,

$$\square\square^2 = \square^{\square}\square\square^2 - \square^{\square}\square^2\left(\square^2 + \frac{\square^2(1-\square^2)}{(1+\square^2)}\right)\square\square^2 - \square^2\left(\square^2 + \frac{\square^2(1+\square^2)}{(1-\square^2)}\right)\square\square^2 - \square^2(1+\square^2)(1-\square^2)\square\square^2 \qquad (10)$$

Here supposed the coefficients $v = v(\xi)$, $\lambda = \lambda(\xi)$ are function of ξ only and $c^2 = 1$ is the velocity of light. By using (10), and non-zero components of the covariant and contravariant metric tensor g_{ij}'s and also covariant and contravariant matrices, we can get the non-vanishing of first and second kind of Christoffel tensors as.

$$\Gamma_{1/11} = -\frac{1}{2}\square^2\square^{\square}\left[\square'\frac{(\square^2+\square^2)}{(1+\square^2)} + \frac{2\square(1-\square^2)}{(1+\square^2)^2}\right]$$

$$\Gamma_{1/12} = \Gamma_{1/21} = -\square^2\square^{\square}\frac{\square}{(1+\square^2)}\ ,\quad \Gamma_{1/22} = \frac{\square^2\square}{(1-\square^2)}$$

$$\Gamma_{1/33} = \square^2\square(1-\square^2)\ ,\quad \Gamma_{1/44} = -\frac{1}{2}\square'\square^{\square}$$

$$\Gamma_{2/11} = \square^2\square^{\square}\frac{\square}{(1+\square^2)}\ ,\quad \Gamma_{2/12} = \Gamma_{2/21} = -\frac{\square^2\square}{(1-\square^2)}$$

$$\Gamma_{2/22} = -\frac{\square^2\square(1+\square^2)}{(1-\square^2)^2}\ ,\quad \Gamma_{2/33} = -\square^2\square(1+\square^2)$$

$$\Gamma_{3/13} = \Gamma_{3/31} = -\square^2\square\square(1-\square^2)$$

$$\Gamma_{3/23} = \Gamma_{3/32} = \square^2\square(1+\square^2)$$

$$\Gamma_{4/14} = \Gamma_{4/41} = \frac{1}{2}\square'\square^{\square}$$

and

$$\Gamma^1_{11} = \frac{\square'}{2} + \frac{\square(1-\square^2)}{(\square^2+\square^2)(1+\square^2)} \;,\; \Gamma^1_{12} = \Gamma^1_{21} = \frac{\square}{(\square^2+\square^2)}$$

$$\Gamma^1_{22} = -\frac{\square^{-\square}\square(1+\square^2)}{(\square^2+\square^2)(1-\square^2)} \;,$$

$$\Gamma^1_{33} = -\frac{\square^{-\square}\square(1+\square^2)(1-\square^2)}{(\square^2+\square^2)} \qquad (11)$$

$$\Gamma^1_{44} = \frac{1}{2}\square^{(\square-\square)}\square^{-2}\square'\left(\frac{1+\square^2}{\square^2+\square^2}\right) \;,\; \Gamma^4_{14} = \Gamma^4_{41} = \frac{1}{2}\square'$$

$$\Gamma^2_{11} = -\frac{\square^{-\square}\square(1-\square^2)}{(\square^2+\square^2)(1+\square^2)} \;,\; \Gamma^2_{12} = \Gamma^2_{21} = \frac{\square}{(\square^2+\square^2)}$$

$$\Gamma^2_{11} = -\frac{\square(1+\square^2)}{(\square^2+\square^2)(1-\square^2)} \;,\; \Gamma^2_{33} = \frac{\square(1+\square^2)(1-\square^2)}{(\square^2+\square^2)}$$

$$\Gamma^3_{13} = \Gamma^3_{31} = \frac{\square}{(1+\square^2)} \;,\quad \Gamma^3_{32} = \Gamma^3_{23} = \frac{-\square}{(1-\square^2)} \qquad (12)$$

RESULTS

Using Christoffel tensors of first (13) and second (14) kinds, in the oblate spheroidal coordinate system, different and the non-zero values of Ricci tensors R_{ij}'s are as,

$$\square_{11} = \frac{\square''}{2} + \frac{\square'^2}{4} - \frac{\square'\square'}{4} - \frac{\square\square'}{2}\left[\frac{(2\square^2+\square^2+1)}{(\square^2+\square^2)(1+\square^2)}\right] - \frac{\square\square'}{2}\left[\frac{(1-\square^2)}{(\square^2+\square^2)(1+\square^2)}\right] + \square^{\square}\left[\frac{\square^2-\square^2-\square^2-3\square^2\square^2}{(\square^2+\square^2)(1+\square^2)}\right] + \left[\frac{[\square^2\square^2(\square^2+3\square^2+4)-(\square^4-\square^4)-(\square^2-\square^2)]}{(\square^2+\square^2)^2\;(1+\square^2)^2}\right] \qquad (13)$$

$$\square_{22} = \square^{-\square}\left[\frac{\square(1+\square^2)(\square'-\square')}{2(\square^2+\square^2)(1-\square^2)} + \frac{\square^2(\square^2-1)+\square^2(1+3\square^2)}{(\square^2+\square^2)^2(1-\square^2)}\right] - \frac{1}{(\square^2+\square^2)^2(1-\square^2)^2}\left[\square^2\square^2(4-3\square^2-\square^2)+(\square^4-\square^4)\right] \qquad (14)$$

$$\square_{33} = \frac{(1+\square^2)(1-\square^2)}{(\square^2+\square^2)}\left[\square^{-\square}\left(\frac{\square'\square}{2} - \frac{\square'\square}{2} + 1\right) - 1\right] \qquad (15)$$

$$\square_{44} = -\square^{(\square-\square)}\frac{(1+\square^2)}{\square^2(\square^2+\square^2)}\left[\frac{\square''}{2} + \frac{\square'^2}{4} + \frac{\square'\square}{(1+\square^2)} - \frac{\square'\square'}{4}\right] \qquad 16)$$

$$\square_{12} = \frac{\square'\square}{2(\square^2+\square^2)} + \frac{2\square\square\square}{(\square^2+\square^2)^2} - \frac{\square\square\square(1+\square^2)}{(\square^2+\square^2)^2(1-\square^2)} + \frac{\square\square\square}{(1+\square^2)(1-\square^2)} - \frac{\square\square\square(1-\square^2)}{(\square^2+\square^2)^2(1+\square^2)} \qquad (17)$$

Equations (13) to (17) are non-zero values of Ricci tensors in the oblate spheroidal coordinate system. Using these equations and some more mathematical methods, we can solve the Einstein's field equations in the oblate spheroidal coordinate system. This solution also gives some more information about the shape of space surrendered by any non-rotating elliptical object. We found that spherical coordinate system is only a special case of the ellipsoidal coordinate system by some approximation.

DISCUSSION

The simple solution of Einstein's field equations is in the spherical coordinate system and is Schwarzschild solution. This line element is based on the spherical coordinate system, and therefore, it is not appropriate for non-spherical objects in shape. For more accurate study of gravitation field of oblate celestial objects (non-spheroidal), such as elliptical galaxies (NGC 1407, M32, M96, NGC 3384) or other objects, (2003EL61-Haumea, named with IAU), the metric and consequently, Ricci tensors needs, express in exact ellipsoidal coordinate systems. Working with this coordinated system, we can find out some small but may be important coefficients. In real, most celestial objects are not exactly in spherical shape but are in ellipsoidal, and therefore we need to work in exact ellipsoidal coordinate systems.

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