African Journal of Business Management Vol. 5(30), pp. 11925-11945, 30 November, 2011
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM11.412
ISSN 1993-8233 ©2011 Academic Journals

Full Length Research Paper

A risk management method for enhancing patient safety based on interval-valued fuzzy numbers

Ting-Yu Chen[1]* and Hung-Lin Lai[1,2]

[1]Institute of Business and Management, College of Management, Chang Gung University, Taiwan.
[2]Department of Information Management, Lee-Ming Institute of Technology, 259, Wen-Hwa 1st Road, Kwei-Shan, Taiwan.

Accepted 31 May, 2011

As the importance of patient safety increases for hospital management, improvements in patient safety are needed to reduce the high incidence of medical errors. Research on patient safety and medical errors shows that errors and the resulting adverse events are mainly the result of health-care providers, equipment and the quality management system. Most studies focused their research on the risk of the individual patient in health care. However, when facing patient safety problems, a hospital manager must consider the risk to the organization while making decisions about improvements. The risks will be relative to the cost-effectiveness of a health-care organization. Here we used a TOPSIS (technique for order preference by similarity to ideal solution) approach to manage the risk of a health-care organization in linguistic terms in the environment of interval-valued fuzzy numbers (IVFNs). Rather than calculating the distance between the alternatives and the positive/negative ideal solution in a TOPSIS approach, we use the similarity measure between IVFNs of the alternatives' risk and the risk of the positive/negative ideal solution to help hospital managers analyze risks in an uncertain and complex situation and more easily determine the best alternative.

Key words: Risk management, decision making, patient safety, interval-valued fuzzy numbers, TOPSIS, similarity measure.

INTRODUCTION

All people requiring or receiving health care have a right to be safe that is, the right to be kept free of danger or risk of injury while in health care domains (Johnstone and Kanitsaki, 2007; Bianchi, 2009). This right carries with it a correlative duty on the part of health service providers to ensure that people who are receiving care are kept free from danger or risk of injury while receiving that care (Johnstone and Kanitsaki, 2007). The risk of mortality among patients with iatrogenic complications was significantly higher than the risk of mortality among patients without iatrogenic complications (Giraud et al., 1993). Therefore, patient safety and quality health care are primary directives for those in health care (Ross and Ranum, 2009; Kalra, 2004; Teng et al., 2010; Didier et al., 2006; Chao et al., 2007). The term "patient safety" is a relatively recent initiative in health care, which encompasses systems of patient care, reporting of mistakes and the initiation of new systems to reduce the risk of errors in patient care (Vande and France, 2002).

The main aim of patient safety efforts is to eliminate adverse events (Makai et al., 2009). Recent studies denote that nurses are a critical component in the promotion of patient safety. Nurses have an innate capability to intercept near misses and the errors of others on the health care team. Nurses have a pivotal role to play in clinical risk management and promoting patient safety in health care domains (Johnstone and Kanitsaki, 2007; Ross and Ranum, 2009; Borden and Lang, 2001; Hegney et al., 2003; Sasicbay- Akkadechannunt et al., 2003; Dunton et al., 2004; Person et al., 2004; Philbrook, 2004; Sochalski, 2004). Accordingly, nurses need to be prepared educationally to manage clinical risk effectively when delivering patient care (Johnstone and Kanitsaki, 2007). However, global nursing shortages have exacerbated time pressure and burn out among nurses (Teng et

*Corresponding author. E-mail: tychen@mail.cgu.edu.tw. Tel: +886-3-2118800 ext. 5678. Fax: +886-3-2118500.

al., 2010). Therefore, some researchers discussed nursing workload and the work environment (Teng et al., 2010; Hurst, 2005; Carayon and Alvarado, 2007; Lyneham et al., 2008; Gerolamo, 2009).

In addition, most studies focused their research on the risk of individual patients in health care (Ross and Ranum, 2009; Gupta, 2004; Gerson et al., 2004; Welie et al., 2005; Landis and Faries, 2007; Petersen, 2007). However, the risks of a health-care organization should be considered in the decision-making process while improving patient safety. Risk is composed of two factors: the probability of failure and the severity of loss. Many risk management approaches have been based on the use of linguistic assessments instead of numerical values (Nieto-Morote and Ruz-Vila, 2011). The fuzzy set (Yao and Su, 2000) is a mathematical tool for the analysis of data defined in imprecise linguistic terms based on subjective judgments such as low risk, serious impact or high-probability events.

When something is uncertain, such as a measurement, using type-1 fuzzy sets, which represent uncertainty by numbers in the range (0, 1), makes more sense than using conventional sets. However, it may not be reasonable to use an accurate membership function for something that is not only uncertain but also complex (Sepulveda et al., 2007). The concept of type-2 fuzzy sets has thus been proposed by Zadeh (1975), which may better handle linguistic uncertainties in complex situations. A type-2 fuzzy set can be defined by a fuzzy membership function, the grade (fuzzy grade) of which is taken to be a fuzzy set in the unit interval (0, 1) rather than a point in the unit interval (0, 1) (Mizumoto and Tanaka, 1981). The interval-valued fuzzy numbers (IVFNs) were defined from type-2 fuzzy sets by Zadeh (1975) and Sambuc (1975) and have been popularly adopted for handling subjective uncertainties arising from incomplete or imprecise information. Some researchers have applied fuzzy sets in risk management (Schmucker, 1984; García et al., 1992). IVFNs may also be applied to deal with risk management problems. Chen and Chen (2008) and Wei and Chen (2009) proposed methods to deal with risk management problems based on IVFNs. Because uncertainty is an attribute of information (Zadeh, 2005), it appears to be a more applicable method for health-care organizations to handle such variable and uncertain factors in risk management by using IVFNs.

In fuzzy set theory, an expert often find it difficult to identify the opinion as a number in interval (0, 1). Therefore, to represent the degree of certainty of opinions by an interval is more proper for the real world that is the characteristic of IVFNs. According to the aforementioned reason, we analyzed the organization risk to patient safety based on IVFNs in this study. Simultaneously, we used the TOPSIS (technique for order preference by similarity to ideal solution) approach developed by Hwang and Yoon (1981), a widely used multiple-attribute decision-making method. The basic concept of TOPSIS is that the chosen alternative should have the shortest distance from the positive-ideal solution and the farthest distance from the negative-ideal solution. To measure the risk of alternatives, we use TOPSIS to compare it with the smallest risk (positive-ideal solution) and the largest risk (the negative-ideal solution). Rather than measuring the distance between the alternatives and the positive/ negative ideal solution, we measured the similarity between the IVFNs of the risks of alternatives and the least risk (the positive ideal solution) and the greatest risk (the negative ideal solution), which can lead to more intuitive results than measuring the distance.

This paper is organized as follows; In the preliminaries, we briefly review the basic concepts of IVFNs (Chen, 2006; Wang and Li, 1998) and their arithmetic operations (Wei and Chen, 2009; Chen, 1995). We also briefly review some existing similarity measures of fuzzy numbers and IVFNs, followed by, proposing a new risk management method to solve risk-analysis problems between IVFNs using a TOPSIS approach combined with a similarity measure. We then, apply the proposed method to evaluate the patient safety risks when improving problems in health-care organizations and finally conclusions are drawn.

Preliminaries

In the following, we briefly review some basic concepts of IVFNs and their arithmetic operations.

IVFNs and their arithmetic operations

Wang and Li (1998) defined IVFNs and gave them extended operations. From Chen (2006), the IVFN $\tilde{\tilde{A}}$, as shown in Figure 1, can be represented by $\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right]$, where $a_1^L \le a_2^L \le a_3^L \le a_4^L$, $a_1^U \le a_2^U \le a_3^U \le a_4^U$, $\tilde{\tilde{A}}^L$ denotes the lower IVFN, $\tilde{\tilde{A}}^U$ denotes the upper IVFN, and $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$.

Assume that there are two IVFNs $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$, where;

$$\tilde{\tilde{A}} = [\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right], \text{ and } \tilde{\tilde{B}} = \left(\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right) = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U\right)\right], \ 0 \le w_{\tilde{\tilde{A}}}^L \le w_{\tilde{\tilde{A}}}^U \le 1, \ \tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U, \quad 0 \le w_{\tilde{\tilde{B}}}^L \le w_{\tilde{\tilde{B}}}^U \le 1, \text{ and } \tilde{\tilde{B}}^L \subset \tilde{\tilde{B}}^U.$$

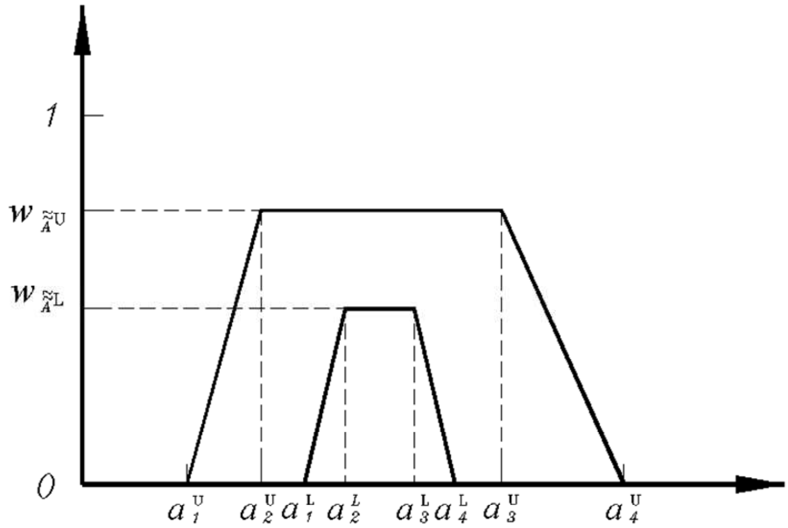

Figure 1. An interval-valued trapezoidal fuzzy number.

The arithmetic operations between IVFNs $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are given in Chen (1995) and Wei and Chen (2009) as follows:

1.IVFNs addition $\oplus$:

$$\begin{aligned}\tilde{\tilde{A}} \oplus \tilde{\tilde{B}} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right] \oplus \\ &\quad \left[(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L), (b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U)\right] \\ &= \left[\left(a_1^L+b_1^L, a_2^L+b_2^L, a_3^L+b_3^L, a_4^L+b_4^L; \min\left(w_{\tilde{\tilde{A}}}^L, w_{\tilde{\tilde{B}}}^L\right)\right),\right. \\ &\quad \left.\left(a_1^U+b_1^U, a_2^U+b_2^U, a_3^U+b_3^U, a_4^U+b_4^U; \min\left(w_{\tilde{\tilde{A}}}^U, w_{\tilde{\tilde{B}}}^U\right)\right)\right]\end{aligned} \tag{1}$$

2. IVFNs subtraction $\ominus$:

$$\begin{aligned}\tilde{\tilde{A}} \ominus \tilde{\tilde{B}} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right] \ominus \\ &\quad \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U\right)\right] \\ &= \left[\left(a_1^L-b_4^L, a_2^L-b_3^L, a_3^L-b_2^L, a_4^L-b_1^L; \min\left(w_{\tilde{\tilde{A}}}^L, w_{\tilde{\tilde{B}}}^L\right)\right),\right. \\ &\quad \left.\left(a_1^U-b_4^U, a_2^U-b_3^U, a_3^U-b_2^U, a_4^U-b_1^U; \min\left(w_{\tilde{\tilde{A}}}^U, w_{\tilde{\tilde{B}}}^U\right)\right)\right]\end{aligned} \quad (2$$

3. IVFNs multiplication $\otimes$:

$$\begin{aligned}\tilde{\tilde{A}} \otimes \tilde{\tilde{B}} &= \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right] \otimes \\ &\quad \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U\right)\right] \\ &= \left[\left(a_1^L \times b_1^L, a_2^L \times b_2^L, a_3^L \times b_3^L, a_4^L \times b_4^L; \min\left(w_{\tilde{\tilde{A}}}^L, w_{\tilde{\tilde{B}}}^L\right)\right),\right. \\ &\quad \left.\left(a_1^U \times b_1^U, a_2^U \times b_2^U, a_3^U \times b_3^U, a_4^U \times b_4^U; \min\left(w_{\tilde{\tilde{A}}}^U, w_{\tilde{\tilde{B}}}^U\right)\right)\right]\end{aligned} \tag{3}$$

4. IVFNs division $\varnothing$:

$$\tilde{\tilde{A}} \varnothing \tilde{\tilde{B}} = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right] \varnothing \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U\right)\right]$$
$$= \left[\left(\min\left(U^L\right), \min\left(U^L\text{-}x^L\right), \max\left(U^L\text{-}y^L\right), \max\left(U^L\right); \min\left(w_{\tilde{\tilde{A}}}^L, w_{\tilde{\tilde{B}}}^L\right)\right),\right.$$
$$\left.\left(\min\left(U^U\right), \min\left(U^U\text{-}x^U\right), \max\left(U^U\text{-}y^U\right), \max\left(U^U\right); \min\left(w_{\tilde{\tilde{A}}}^U, w_{\tilde{\tilde{B}}}^U\right)\right)\right] \quad (4)$$

where $U^L = \left\{\frac{a_1^L}{b_1^L}, \frac{a_2^L}{b_2^L}, \frac{a_3^L}{b_3^L}, \frac{a_4^L}{b_4^L}\right\}$, $U^U = \left\{\frac{a_1^U}{b_1^U}, \frac{a_2^U}{b_2^U}, \frac{a_3^U}{b_3^U}, \frac{a_4^U}{b_4^U}\right\}$, $x^L = \min\left(U^L\right)$,

$x^U = \min\left(U^U\right)$, $y^L = \max\left(U^L\right)$, $y^U = \max\left(U^U\right)$, "$U^L\text{-}x^L$" denotes excluding the element x^L from the set U^L, "$U^U\text{-}x^U$" denotes excluding the element x^U from the set U^U, "$U^L\text{-}y^L$" denotes excluding the element y^L from the set U^L, and "$U^U\text{-}y^U$" denotes excluding the element y^U from the set U^U.

A review of similarity measures between fuzzy numbers

Several methods of similarity measure between fuzzy numbers have been presented in fuzzy-theory literature. Pappis and Karacapilidis (1995) presented a grade for the similarity of two fuzzy sets and gave its properties. Wu and Mendel (2009) analyzed five existing similarity measures (Bustince, 2000; Gorzalczany, 1987; Mitchell, 2005; Wu and Mendel, 2008; Zeng and Li, 2006) for type-2 fuzzy interval sets and proposed a similarity measure with a reduced computational complexity. Zhang and Zhang (2009) introduced a new definition of an inclusion measure: the hybrid monotonic inclusion measure. Chen and Chen (2008) proposed a similarity measure for calculating the degree of similarity between IVFNs using geometric concepts to calculate the center-of-gravity (COG) points of the lower and upper fuzzy numbers of IVFNs.

Chen and Chen (2004) proposed a method for calculating the degree of similarity between IVFNs based on COG points. Here the COG-based similarity measure was used to calculate the degree of similarity $S(\tilde{\tilde{A}}^L, \tilde{\tilde{B}}^L)$ between the lower trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L$ and $\tilde{\tilde{B}}^L$ and the degree of similarity $S(\tilde{\tilde{A}}^U, \tilde{\tilde{B}}^U)$ between the upper trapezoidal fuzzy numbers $\tilde{\tilde{A}}^U$ and $\tilde{\tilde{B}}^U$ of the two IVFNs $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$. However, the COG-based similarity measure used by Chen and Chen (2004) cannot correctly handle the similarity measure of two different generalized fuzzy numbers that have the same COG points. Therefore, Chen (2006) proposed a fuzzy-number similarity measure to overcome this drawback and to present a new method to calculate the degree of similarity between IVFNs. Wei and Chen (2009) presented a similarity measure between IVFNs that combined the concepts of the geometric distance, theperimeter, the height and the COG points of IVFNs to calculate the degree of similarity between IVFNs. They also provided proofs for three properties of the proposed similarity measure. Wei and Chen's method (2009) can overcome the drawbacks of the existing similarity measures. Here, we briefly describe the similarity measure presented by Wei and Chen (2009).

Let $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ be two IVFNs, where $\tilde{\tilde{A}} = \left[\tilde{\tilde{A}}^L, \tilde{\tilde{A}}^U\right] = \left[\left(a_1^L, a_2^L, a_3^L, a_4^L; w_{\tilde{\tilde{A}}}^L\right), \left(a_1^U, a_2^U, a_3^U, a_4^U; w_{\tilde{\tilde{A}}}^U\right)\right]$ and $\tilde{\tilde{B}} = \left[\tilde{\tilde{B}}^L, \tilde{\tilde{B}}^U\right] = \left[\left(b_1^L, b_2^L, b_3^L, b_4^L; w_{\tilde{\tilde{B}}}^L\right), \left(b_1^U, b_2^U, b_3^U, b_4^U; w_{\tilde{\tilde{B}}}^U\right)\right]$, $0 \le a_1^L \le a_2^L \le a_3^L \le a_4^L \le 1, 0 \le a_1^U \le a_2^U \le a_3^U \le a_4^U \le 1$, $0 \le w_{\tilde{\tilde{A}}}^L \le w_{\tilde{\tilde{A}}}^U \le 1$, $\tilde{\tilde{A}}^L \subset \tilde{\tilde{A}}^U$, $0 \le b_1^L \le b_2^L \le b_3^L \le b_4^L \le 1$, $0 \le b_1^U \le b_2^U \le b_3^U \le b_4^U \le 1$, $0 \le w_{\tilde{\tilde{B}}}^L \le w_{\tilde{\tilde{B}}}^U \le 1$ and $\tilde{\tilde{B}}^L \subset \tilde{\tilde{B}}^U$.

First, the areas $A(\tilde{\tilde{A}}^L)$, $A(\tilde{\tilde{A}}^U)$, $A(\tilde{\tilde{B}}^L)$, and $A(\tilde{\tilde{B}}^U)$ of the lower trapezoidal fuzzy numbers $\tilde{\tilde{A}}^L$ and $\tilde{\tilde{B}}^L$ and the upper trapezoidal fuzzy numbers $\tilde{\tilde{A}}^{\mathrm{U}}$ and $\tilde{\tilde{B}}^{\mathrm{U}}$ are calculated, followed by the COG points $\left(x^*_{\tilde{\tilde{A}}^{\mathrm{L}}}, y^*_{\tilde{\tilde{A}}^{\mathrm{L}}}\right)$, $\left(x^*_{\tilde{\tilde{A}}^{\mathrm{U}}}, y^*_{\tilde{\tilde{A}}^{\mathrm{U}}}\right)$, $\left(x^*_{\tilde{\tilde{B}}^{\mathrm{L}}}, y^*_{\tilde{\tilde{B}}^{\mathrm{L}}}\right)$, $\left(x^*_{\tilde{\tilde{B}}^{\mathrm{U}}}, y^*_{\tilde{\tilde{B}}^{\mathrm{U}}}\right)$ of $\tilde{\tilde{A}}^{\mathrm{L}}$, $\tilde{\tilde{A}}^{\mathrm{U}}$, $\tilde{\tilde{B}}^{\mathrm{L}}$ and $\tilde{\tilde{B}}^{\mathrm{U}}$, respectively.

Next, the COG points ($x^*_{\tilde{\tilde{A}}}, y^*_{\tilde{\tilde{A}}}$; $x^*_{\tilde{\tilde{B}}}, y^*_{\tilde{\tilde{B}}}$) of the IVFNs $\tilde{\tilde{A}}$ and $\tilde{\tilde{B}}$ are calculated, followed by the degrees of similarity, $S(\tilde{\tilde{A}}^{\mathrm{L}}, \tilde{\tilde{B}}^{\mathrm{L}})$ and $S(\tilde{\tilde{A}}^{\mathrm{U}}, \tilde{\tilde{B}}^{\mathrm{U}})$, between the lower trapezoidal fuzzy numbers $\tilde{\tilde{A}}^{\mathrm{L}}$ and $\tilde{\tilde{B}}^{\mathrm{L}}$ and the upper trapezoidal fuzzy numbers $\tilde{\tilde{A}}^{\mathrm{U}}$ and $\tilde{\tilde{B}}^{\mathrm{U}}$, respectively. Finally, the degree of similarity between IVFNs is calculated as follows:

$$S(\tilde{\tilde{A}}, \tilde{\tilde{B}}) = \left[\frac{S(\tilde{\tilde{A}}^{\mathrm{L}}, \tilde{\tilde{B}}^{\mathrm{L}})+S(\tilde{\tilde{A}}^{\mathrm{U}}, \tilde{\tilde{B}}^{\mathrm{U}})}{2}\times(1-\Delta x)\times(1-\Delta y)\right]^{\left(\frac{1}{1+2t}\right)} * \left(1-\left|w^{\mathrm{U}}_{\tilde{\tilde{A}}}-w^{\mathrm{U}}_{\tilde{\tilde{B}}}-w^{\mathrm{L}}_{\tilde{\tilde{A}}}+w^{\mathrm{L}}_{\tilde{\tilde{B}}}\right|\right)^{\frac{U}{2}} \tag{5}$$

Where

$$t = \begin{cases} 1, \text{ if } A(\tilde{\tilde{A}}^{\mathrm{U}})-A(\tilde{\tilde{A}}^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{B}}^{\mathrm{U}})-A(\tilde{\tilde{B}}^{\mathrm{L}}) \neq 0, \\ 0, \text{ otherwise,} \end{cases} \tag{6}$$

$$u = \begin{cases} 1, \text{ if } a^{\mathrm{U}}_1 = a^{\mathrm{U}}_4 \text{ and } b^{\mathrm{U}}_1 = b^{\mathrm{U}}_4, \\ 0, \text{ otherwise} \end{cases} \tag{7}$$

$$S(\tilde{\tilde{A}}^{\mathrm{L}}, \tilde{\tilde{B}}^{\mathrm{L}}) = \begin{cases} [1-\frac{\sum_{i=1}^{4}\left|a^{\mathrm{L}}_i-b^{\mathrm{L}}_i\right|}{4}]\times\frac{\min(L(\tilde{\tilde{A}}^{\mathrm{L}}),L(\tilde{\tilde{B}}^{\mathrm{L}}))+\min(w_{\tilde{\tilde{A}}^{\mathrm{L}}}, w_{\tilde{\tilde{B}}^{\mathrm{L}}})}{\max(L(\tilde{\tilde{A}}^{\mathrm{L}}),L(\tilde{\tilde{B}}^{\mathrm{L}}))+\max(w_{\tilde{\tilde{A}}^{\mathrm{L}}}, w_{\tilde{\tilde{B}}^{\mathrm{L}}})}, \\ \qquad \text{if } \min(w_{\tilde{\tilde{A}}^{\mathrm{L}}}, w_{\tilde{\tilde{B}}^{\mathrm{L}}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{8}$$

$$S(\tilde{\tilde{A}}^{\mathrm{U}}, \tilde{\tilde{B}}^{\mathrm{U}}) = \begin{cases} [1-\frac{\sum_{i=1}^{4}\left|a^{\mathrm{U}}_i-b^{\mathrm{U}}_i\right|}{4}]\times\frac{\min(L(\tilde{\tilde{A}}^{\mathrm{U}}),L(\tilde{\tilde{B}}^{\mathrm{U}}))+\min(w_{\tilde{\tilde{A}}^{\mathrm{U}}}, w_{\tilde{\tilde{B}}^{\mathrm{U}}})}{\max(L(\tilde{\tilde{A}}^{\mathrm{U}}),L(\tilde{\tilde{B}}^{\mathrm{U}}))+\max(w_{\tilde{\tilde{A}}^{\mathrm{U}}}, w_{\tilde{\tilde{B}}^{\mathrm{U}}})}, \\ \qquad \text{if } \min(w_{\tilde{\tilde{A}}^{\mathrm{U}}}, w_{\tilde{\tilde{B}}^{\mathrm{U}}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{9}$$

$$\Delta x = \begin{cases} \left|x^*_{\tilde{\tilde{A}}} - x^*_{\tilde{\tilde{B}}}\right|, \text{ if } A(\tilde{\tilde{A}}^{\mathrm{U}})-A(\tilde{\tilde{A}}^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{B}}^{\mathrm{U}})-A(\tilde{\tilde{B}}^{\mathrm{L}}) \neq 0, \\ 0, \qquad \text{otherwise,} \end{cases} \tag{10}$$

$$\Delta y = \begin{cases} \left|y^*_{\tilde{\tilde{A}}} - y^*_{\tilde{\tilde{B}}}\right|, \text{ if } A(\tilde{\tilde{A}}^{\mathrm{U}})-A(\tilde{\tilde{A}}^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{B}}^{\mathrm{U}})-A(\tilde{\tilde{B}}^{\mathrm{L}}) \neq 0, \\ 0, \qquad \text{otherwise,} \end{cases} \tag{11}$$

Table 1. Nine-member linguistic terms and their corresponding interval-valued fuzzy numbers.

Linguistic terms	interval-valued trapezoidal fuzzy numbers
Absolutely low	[(0.0, 0.0,0.0, 0.0; 1.0), (0.0, 0.0,0.0, 0.0; 1.0)]
Very low	[(0.0075, 0.0075, 0.015, 0.0525; 0.5), (0.0, 0.0, 0.02, 0.07; 1.0)]
Low	[(0.0875, 0.12, 0.16, 0.1825; 0.5), (0.04, 0.10, 0.18, 0.23; 1.0)]
Fairly low	[(0.2325, 0.255, 0.325, 0.3575; 0.5), (0.17, 0.22, 0.36, 0.42; 1.0)]
Medium	[(0.4025, 0.4525, 0.5375, 0.5675; 0.5), (0.32, 0.41, 0.58, 0.65; 1.0)]
Fairly high	[(0.65, 0.6725, 0.7575, 0.79; 0.5), (0.58, 0.63, 0.80, 0.86; 1.0)]
High	[(0.7825, 0.815, 0.885, 0.9075; 0.5), (0.72, 0.78, 0.92, 0.97; 1.0)]
Very high	[(0.9475, 0.985, 0.9925, 0.9925; 0.5), (0.93, 0.98, 1.0, 1.0; 1.0)]
Absolutely high	[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)]

Source: Chen and Chen (2008)

$$L(\tilde{\tilde{A}}^{L}) = \sqrt{(a_1^L-a_2^L)^2+w_{\tilde{\tilde{A}}^L}^2}+\sqrt{(a_3^L-a_4^L)^2+w_{\tilde{\tilde{A}}^L}^2}+(a_3^L-a_2^L)+(a_4^L-a_1^L)\ , \tag{12}$$

$$L(\tilde{\tilde{B}}^{L}) = \sqrt{(b_1^L-b_2^L)^2+w_{\tilde{\tilde{B}}^L}^2}+\sqrt{(b_3^L-b_4^L)^2+w_{\tilde{\tilde{B}}^L}^2}+(b_3^L-b_2^L)+(b_4^L-b_1^L)\ , \tag{13}$$

$$L(\tilde{\tilde{A}}^{U}) = \sqrt{(a_1^U-a_2^U)^2+w_{\tilde{\tilde{A}}^U}^2}+\sqrt{(a_3^U-a_4^U)^2+w_{\tilde{\tilde{A}}^U}^2}+(a_3^U-a_2^U)+(a_4^U-a_1^U)\ , \tag{14}$$

$$L(\tilde{\tilde{B}}^{U}) = \sqrt{(b_1^U-b_2^U)^2+w_{\tilde{\tilde{B}}^U}^2}+\sqrt{(b_3^U-b_4^U)^2+w_{\tilde{\tilde{B}}^U}^2}+(b_3^U-b_2^U)+(b_4^U-b_1^U)\ , \text{ and } S(\tilde{\tilde{A}},\ \tilde{\tilde{B}})\in[0,1]. \tag{15}$$

Wei and Chen (2009) used nine sets of IVFNs to compare their results with the earlier methods presented by Chen (2006) and Chen and Chen (2004). The results from Wei and Chen's similarity measure coincided with human intuition for all of the aforementioned sets, whereas seven results among the nine sets differed from Chen's method (2006) and Chen and Chen's method (2004). This outcome indicates that Wei and Chen's method (2009) can overcome the drawbacks of the earlier methods.

Risk management method for patient safety

In this section, we use the TOPSIS approach and Wei and Chen's (2009) similarity measure method to solve a patient safety risk management problem in a health-care organization. Assume that there are n alternatives: $A_1, A_2, A_3, \ldots, A_n$. Each alternative has sub risks $R_{i1}, R_{i2}, R_{i3}, \ldots, R_{ik}$, and $1 \le i \le n$. Decision makers are concerned not only with the severity of loss but also the probability of failure; thus, the integrated risk of each alternative is composed of these two sub risk factors. We used a nine-member linguistic term set, shown in Table 1, to represent the linguistic terms and their corresponding IVFNs.

The algorithm of the proposed risk management method is presented in the following paragraphs. Based on the IVFN arithmetic operations, we first integrate the linguistic values $\tilde{\tilde{W}}_{ij}$

$$\left(\tilde{\tilde{W}}_{ij}=\left[\left(w_{ij1}^L,\ w_{ij2}^L,\ w_{ij3}^L,\ w_{ij4}^L;\ w_{\tilde{\tilde{W}}_{ij}}^L\right),\ \left(w_{ij1}^U,\ w_{ij2}^U,\ w_{ij3}^U,\ w_{ij4}^U;\ w_{\tilde{\tilde{W}}_{ij}}^U\right)\right]\right)$$

for the severity of loss and the values $\tilde{\tilde{P}}_{ij}$

$$\left(\tilde{\tilde{P}}_{ij}=\left[\left(p_{ij1}^L,\ p_{ij2}^L,\ p_{ij3}^L,\ p_{ij4}^L;\ w_{\tilde{\tilde{P}}_{ij}}^L\right),\ \left(p_{ij1}^U,\ p_{ij2}^U,\ p_{ij3}^U,\ p_{ij4}^U;\ w_{\tilde{\tilde{P}}_{ij}}^U\right)\right]\right)$$

for the probability of failure to obtain the integrated sub risk $\tilde{\tilde{R}}_{ij}$ of each alternative, respectively, which can be calculated as follows and explained in Figure 2:

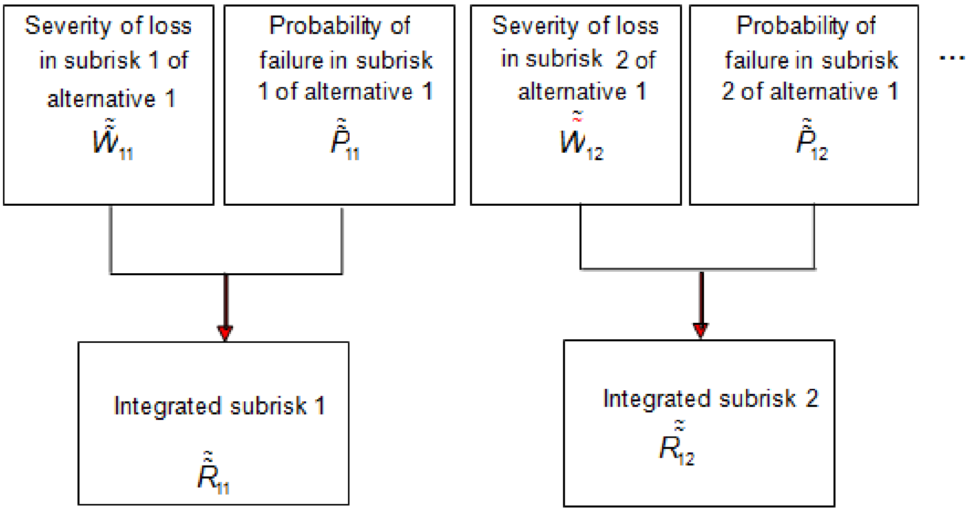

Figure 2. Structure of integration in each sub risk.

$$\begin{aligned}
\tilde{\tilde{R}}_{ij} = \tilde{\tilde{W}}_{ij} \otimes \tilde{\tilde{P}}_{ij} &= \left[\left(w^L_{ij1}, w^L_{ij2}, w^L_{ij3}, w^L_{ij4}; w^L_{\tilde{\tilde{W}}_{ij}}\right), \left(w^U_{ij1}, w^U_{ij2}, w^U_{ij3}, w^U_{ij4}; w^U_{\tilde{\tilde{W}}_{ij}}\right)\right] \otimes \\
&\quad \left[\left(p^L_{ij1}, p^L_{ij2}, p^L_{ij3}, p^L_{ij4}; w^L_{\tilde{\tilde{P}}_{ij}}\right), \left(p^U_{ij1}, p^U_{ij2}, p^U_{ij3}, p^U_{ij4}; w^U_{\tilde{\tilde{P}}_{ij}}\right)\right] \\
&= \left[\left(w^L_{ij1} \times p^L_{ij1}, w^L_{ij2} \times p^L_{ij2}, w^L_{ij3} \times p^L_{ij3}, w^L_{ij4} \times p^L_{ij4}; \min\left(w^L_{\tilde{\tilde{W}}_{ij}}, w^L_{\tilde{\tilde{P}}_{ij}}\right)\right),\right. \\
&\quad \left.\left(w^U_{ij1} \times p^U_{ij1}, w^U_{ij2} \times p^U_{ij2}, w^U_{ij3} \times p^U_{ij3}, w^U_{ij4} \times p^U_{ij4}; \min\left(w^U_{\tilde{\tilde{W}}_{ij}}, w^U_{\tilde{\tilde{P}}_{ij}}\right)\right)\right] \\
&= \left[\left(r^L_{ij1}, r^L_{ij2}, r^L_{ij3}, r^L_{ij4}; w^L_{\tilde{\tilde{R}}_{ij}}\right), \left(r^U_{ij1}, r^U_{ij2}, r^U_{ij3}, r^U_{ij4}; w^U_{\tilde{\tilde{R}}_{ij}}\right)\right]
\end{aligned} \quad (16)$$

Where $1 \le i \le n$ and $1 \le j \le k$

Assume that ϖ_j is the importance of each sub risk R_{ij}.

We modify the linguistic values $\tilde{\tilde{\varpi}}_j$

$$\left(\tilde{\tilde{\varpi}}_j = \left[\left(\varpi^L_{j1}, \varpi^L_{j2}, \varpi^L_{j3}, \varpi^L_{j4}; w^L_{\tilde{\tilde{\varpi}}_j}\right), \left(\varpi^U_{j1}, \varpi^U_{j2}, \varpi^U_{j3}, \varpi^U_{j4}; w^U_{\tilde{\tilde{\varpi}}_j}\right)\right]\right)$$

to $\tilde{\tilde{\omega}}_j$ with the following arithmetic operation:

$$\begin{aligned}
\tilde{\tilde{\omega}}_j &= \frac{\left[\left(\varpi^L_{j1}, \varpi^L_{j2}, \varpi^L_{j3}, \varpi^L_{j4}; w^L_{\tilde{\tilde{\varpi}}_j}\right), \left(\varpi^U_{j1}, \varpi^U_{j2}, \varpi^U_{j3}, \varpi^U_{j4}; w^U_{\tilde{\tilde{\varpi}}_j}\right)\right]}{\sum_{j=1}^{k}\left[\left(\varpi^L_{j1}, \varpi^L_{j2}, \varpi^L_{j3}, \varpi^L_{j4}; w^L_{\tilde{\tilde{\varpi}}_j}\right), \left(\varpi^U_{j1}, \varpi^U_{j2}, \varpi^U_{j3}, \varpi^U_{j4}; w^U_{\tilde{\tilde{\varpi}}_j}\right)\right]} \\
&= \left[\left(\omega^L_{j1}, \omega^L_{j2}, \omega^L_{j3}, \omega^L_{j4}; w^L_{\tilde{\tilde{\omega}}_j}\right), \left(\omega^U_{j1}, \omega^U_{j2}, \omega^U_{j3}, \omega^U_{j4}; w^U_{\tilde{\tilde{\omega}}_j}\right)\right]
\end{aligned} \quad (17)$$

Where $1 \le j \le k$ (Figure 3). The integrated risk $\tilde{\tilde{R}}_i$ of each alternative is calculated as follows:

$$\begin{aligned}
\tilde{\tilde{R}}_i &= \sum_{j=1}^{k}\left(\tilde{\tilde{R}}_{ij} \otimes \tilde{\tilde{\omega}}_j\right) \\
&= \sum_{j=1}^{k}\left\{\begin{array}{l}\left[\left(r^L_{ij1}, r^L_{ij2}, r^L_{ij3}, r^L_{ij4}; w^L_{\tilde{\tilde{R}}_{ij}}\right), \left(r^U_{ij1}, r^U_{ij2}, r^U_{ij3}, r^U_{ij4}; w^U_{\tilde{\tilde{R}}_{ij}}\right)\right] \otimes \\ \left[\left(\omega^L_{j1}, \omega^L_{j2}, \omega^L_{j3}, \omega^L_{j4}; w^L_{\tilde{\tilde{\omega}}_j}\right), \left(\omega^U_{j1}, \omega^U_{j2}, \omega^U_{j3}, \omega^U_{j4}; w^U_{\tilde{\tilde{\omega}}_j}\right)\right]\end{array}\right\} \\
&= \left[\left(r^L_{i1}, r^L_{i2}, r^L_{i3}, r^L_{i4}; w^L_{\tilde{\tilde{R}}_i}\right), \left(r^U_{i1}, r^U_{i2}, r^U_{i3}, r^U_{i4}; w^U_{\tilde{\tilde{R}}_i}\right)\right]
\end{aligned} \quad (18)$$

Where $1 \le i \le n$ and $1 \le j \le k$ (Figure 4).

Next, we rank the alternatives according to the decision-making criteria and give the risk of the ideal

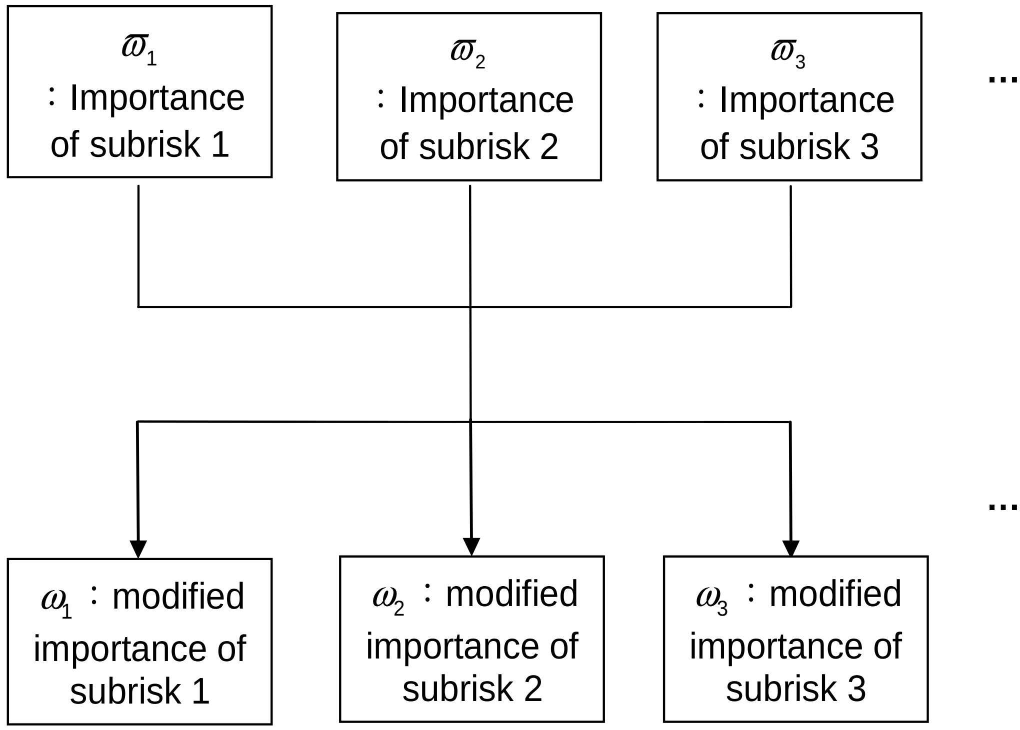

Figure 3. Structure of modifying the importance of each subrisk.

alternative for "excellence":

$$\begin{aligned}\tilde{\tilde{R}}^* &= \left[\left(r_1^{*\mathrm{L}}, r_2^{*\mathrm{L}}, r_3^{*\mathrm{L}}, r_4^{*\mathrm{L}}; w_{\tilde{\tilde{R}}^*}^{\mathrm{L}}\right), \left(r_1^{*\mathrm{U}}, r_2^{*\mathrm{U}}, r_3^{*\mathrm{U}}, r_4^{*\mathrm{U}}; w_{\tilde{\tilde{R}}^*}^{\mathrm{U}}\right)\right] \\ &= \left[(0.0, 0.0, 0.0, 0.0; 1.0), (0.0, 0.0, 0.0, 0.0; 1.0)\right] \end{aligned} \quad (19)$$

We use the proposed similarity measure to evaluate the degree of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the ideal alternative, $\tilde{\tilde{R}}^*$

$$\begin{aligned} S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^*) &= \left[\frac{S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{*\mathrm{L}})+S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{*\mathrm{U}})}{2} \times (1-\Delta x)\times(1-\Delta y)\right]^{\left(\frac{1}{1+2t}\right)} \\ &\quad \times \left(1-\left|w_{\tilde{\tilde{R}}_i}^{\mathrm{U}} - w_{\tilde{\tilde{R}}^*}^{\mathrm{U}} - w_{\tilde{\tilde{R}}_i}^{\mathrm{L}} + w_{\tilde{\tilde{R}}^*}^{\mathrm{L}}\right|\right)^{\frac{U}{2}}, \\ &= \left[\frac{S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{*\mathrm{L}})+S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{*\mathrm{U}})}{2}\right] \times \left(1-\left|w_{\tilde{\tilde{R}}_i}^{\mathrm{U}} - w_{\tilde{\tilde{R}}_i}^{\mathrm{L}}\right|\right)^{\frac{U}{2}} \end{aligned} \quad (20)$$

$$t = \begin{cases} 1, \text{ if } A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \neq 0, \\ 0, \text{ otherwise}, \end{cases} \quad (21)$$

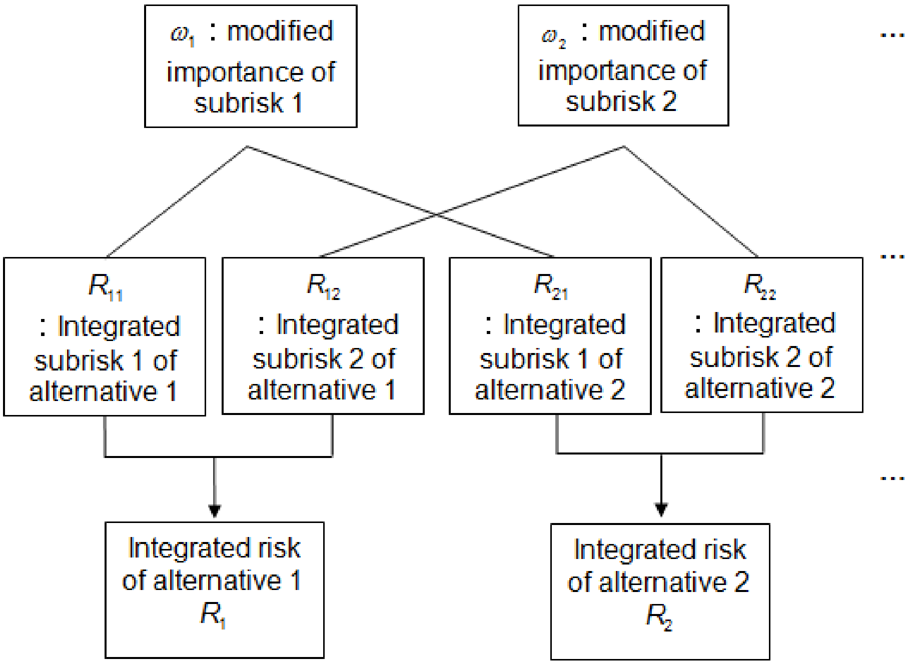

Figure 4. Structure of integrating the risk of each alternative.

$$u = \begin{cases} 1, \text{ if } r_{i1}^{U} = r_{i4}^{U} \text{ and } r_{1}^{*U} = r_{4}^{*U}, \\ 0, \text{ otherwise} \end{cases} \tag{22}$$

The areas $A\left(\tilde{\tilde{R}}_i^{L}\right)$ and $A\left(\tilde{\tilde{R}}_i^{U}\right)$ of the lower trapezoidal fuzzy number $\tilde{\tilde{R}}_i^{L}$ and the upper trapezoidal fuzzy number $\tilde{\tilde{R}}_i^{U}$ and the areas $A\left(\tilde{\tilde{R}}^{*L}\right)$ and $A\left(\tilde{\tilde{R}}^{*U}\right)$ of the lower trapezoidal fuzzy number $\tilde{\tilde{R}}^{*L}$ and the upper trapezoidal fuzzy number $\tilde{\tilde{R}}^{*U}$, respectively, are given in the following equations:

$$A\left(\tilde{\tilde{R}}_i^{L}\right) = \frac{(r_{i3}^{L} - r_{i2}^{L} + r_{i4}^{L} - r_{i1}^{L}) \times w_{\tilde{\tilde{R}}_i^{L}}}{2} \tag{23}$$

$$A\left(\tilde{\tilde{R}}_i^{U}\right) = \frac{(r_{i3}^{U} - r_{i2}^{U} + r_{i4}^{U} - r_{i1}^{U}) \times w_{\tilde{\tilde{R}}_i^{U}}}{2} \tag{24}$$

$$A\left(\tilde{\tilde{R}}^{*L}\right) = \frac{(r_{3}^{*L} - r_{2}^{*L} + r_{4}^{*L} - r_{1}^{*L}) \times w_{\tilde{\tilde{R}}^{*L}}}{2} = 0 \tag{25}$$

$$A\left(\tilde{\tilde{R}}^{*U}\right) = \frac{(r_{3}^{*U} - r_{2}^{*U} + r_{4}^{*U} - r_{1}^{*U}) \times w_{\tilde{\tilde{R}}^{*U}}}{2} = 0 \tag{26}$$

To calculate the degree of similarity $S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^{*})$, we must first calculate the degrees of similarity $S(\tilde{\tilde{R}}_i^{L}, \tilde{\tilde{R}}^{*L})$ and $S(\tilde{\tilde{R}}_i^{U}, \tilde{\tilde{R}}^{*U})$ between the lower trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{L}$ and $\tilde{\tilde{R}}^{*L}$ and the upper trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{U}$ and $\tilde{\tilde{R}}^{*U}$, respectively.

$$S(\tilde{\tilde{R}}_i^{L}, \tilde{\tilde{R}}^{*L}) = \begin{cases} \left[1 - \dfrac{\sum_{k=1}^{4} \left| r_{ik}^{L} - r_{k}^{*L} \right|}{4}\right] \times \dfrac{\min(L(\tilde{\tilde{R}}_i^{L}), L(\tilde{\tilde{R}}^{*L})) + \min(w_{\tilde{\tilde{R}}_i^{L}}, w_{\tilde{\tilde{R}}^{*L}})}{\max(L(\tilde{\tilde{R}}_i^{L}), L(\tilde{\tilde{R}}^{*L})) + \max(w_{\tilde{\tilde{R}}_i^{L}}, w_{\tilde{\tilde{R}}^{*L}})}, \\ \quad \text{if } \min(w_{\tilde{\tilde{R}}_i^{L}}, w_{\tilde{\tilde{R}}^{*L}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{27}$$

$$L(\tilde{\tilde{R}}_i^{\mathrm{L}}) = \sqrt{(r_{i1}^{\mathrm{L}}-r_{i2}^{\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}_i^{\mathrm{L}}}}+\sqrt{(r_{i3}^{\mathrm{L}}-r_{i4}^{\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}_i^{\mathrm{L}}}}+(r_{i3}^{\mathrm{L}}-r_{i2}^{\mathrm{L}})+(r_{i4}^{\mathrm{L}}-r_{i1}^{\mathrm{L}}) \ , \tag{28}$$

$$\begin{aligned} L(\tilde{\tilde{R}}^{*\mathrm{L}}) &= \sqrt{(r_1^{*\mathrm{L}}-r_2^{*\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}^{*\mathrm{L}}}}+\sqrt{(r_3^{*\mathrm{L}}-r_4^{*\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}^{*\mathrm{L}}}}+(r_3^{*\mathrm{L}}-r_2^{*\mathrm{L}})+(r_4^{*\mathrm{L}}-r_1^{*\mathrm{L}}) \\ &= 2 \end{aligned} \tag{29}$$

$$S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{*\mathrm{U}}) = \begin{cases} \left[1-\dfrac{\sum_{k=1}^{4}\left|r_{ik}^{\mathrm{U}}-r_k^{*\mathrm{U}}\right|}{4}\right]\times\dfrac{\min(L(\tilde{\tilde{R}}_i^{\mathrm{U}}),L(\tilde{\tilde{R}}^{*\mathrm{U}}))+\min(w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}, w_{\tilde{\tilde{R}}^{*\mathrm{U}}})}{\max(L(\tilde{\tilde{R}}_i^{\mathrm{U}}),L(\tilde{\tilde{R}}^{*\mathrm{U}}))+\max(w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}, w_{\tilde{\tilde{R}}^{*\mathrm{U}}})}, \\ \qquad \text{if } \min(w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}, w_{\tilde{\tilde{R}}^{*\mathrm{U}}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{30}$$

$$L(\tilde{\tilde{R}}_i^{\mathrm{U}}) = \sqrt{(r_{i1}^{\mathrm{U}}-r_{i2}^{\mathrm{U}})^2+w^2_{\tilde{\tilde{R}}_i^{\mathrm{U}}}}+\sqrt{(r_{i3}^{\mathrm{U}}-r_{i4}^{\mathrm{U}})^2+w^2_{\tilde{\tilde{R}}_i^{\mathrm{U}}}}+(r_{i3}^{\mathrm{U}}-r_{i2}^{\mathrm{U}})+(r_{i4}^{\mathrm{U}}-r_{i1}^{\mathrm{U}}) \ , \tag{31}$$

$$\begin{aligned} L(\tilde{\tilde{R}}^{*\mathrm{U}}) &= \sqrt{(r_1^{*\mathrm{U}}-r_2^{*\mathrm{U}})^2+w^2_{\tilde{\tilde{R}}^{*\mathrm{U}}}}+\sqrt{(r_3^{*\mathrm{U}}-r_4^{*\mathrm{U}})^2+w^2_{\tilde{\tilde{R}}^{*\mathrm{U}}}}+(r_3^{*\mathrm{U}}-r_2^{*\mathrm{U}})+(r_4^{*\mathrm{U}}-r_1^{*\mathrm{U}}) \\ &= 2 \end{aligned} \tag{32}$$

We showed the structure of calculations in the similarity degree between IVFNs of risks in each alternative and the ideal solution in Figure 5.

After calculating the degrees of similarity between the lower trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{\mathrm{L}}$ and $\tilde{\tilde{R}}^{*\mathrm{L}}$ and the upper trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{\mathrm{U}}$ and $\tilde{\tilde{R}}^{*\mathrm{U}}$, respectively, we then calculate the COG points $\left(x^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}}, y^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}}\right)$, $\left(x^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}}, y^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}}\right)$, $\left(x^*_{\tilde{\tilde{R}}^{*\mathrm{L}}}, y^*_{\tilde{\tilde{R}}^{*\mathrm{L}}}\right)$ and $\left(x^*_{\tilde{\tilde{R}}^{*\mathrm{U}}}, y^*_{\tilde{\tilde{R}}^{*\mathrm{U}}}\right)$ of $\tilde{\tilde{R}}_i^{\mathrm{L}}$, $\tilde{\tilde{R}}_i^{\mathrm{U}}$, $\tilde{\tilde{R}}^{*\mathrm{L}}$ and $\tilde{\tilde{R}}^{*\mathrm{U}}$, respectively.

$$x^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}} = \frac{y^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}}(r_{i3}^{\mathrm{L}}+r_{i2}^{\mathrm{L}})+(r_{i4}^{\mathrm{L}}+r_{i1}^{\mathrm{L}})(w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}-y^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}})}{2w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}} \tag{33}$$

$$y^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}} = \begin{cases} \dfrac{w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}\times\left(\dfrac{r_{i3}^{\mathrm{L}}-r_{i2}^{\mathrm{L}}}{r_{i4}^{\mathrm{L}}-r_{i1}^{\mathrm{L}}}+2\right)}{6}, & \text{if } r_{i1}^{\mathrm{L}} \neq r_{i4}^{\mathrm{L}} \text{ and } 0<w_{\tilde{\tilde{R}}_i^{\mathrm{L}}} \leq 1, \\ \dfrac{w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}}{2}, & \text{if } r_{i1}^{\mathrm{L}} = r_{i4}^{\mathrm{L}} \text{ and } 0<w_{\tilde{\tilde{R}}_i^{\mathrm{L}}} \leq 1, \end{cases} \tag{34}$$

$$x^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}} = \frac{y^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}}(r_{i3}^{\mathrm{U}}+r_{i2}^{\mathrm{U}})+(r_{i4}^{\mathrm{U}}+r_{i1}^{\mathrm{U}})(w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}-y^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}})}{2w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}} \tag{35}$$

$$y^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}} = \begin{cases} \dfrac{w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}\times\left(\dfrac{r_{i3}^{\mathrm{U}}-r_{i2}^{\mathrm{U}}}{r_{i4}^{\mathrm{U}}-r_{i1}^{\mathrm{U}}}+2\right)}{6}, & \text{if } r_{i1}^{\mathrm{U}} \neq r_{i4}^{\mathrm{U}} \text{ and } 0<w_{\tilde{\tilde{R}}_i^{\mathrm{U}}} \leq 1, \\ \dfrac{w_{\tilde{\tilde{R}}_i^{\mathrm{U}}}}{2}, & \text{if } r_{i1}^{\mathrm{U}} = r_{i4}^{\mathrm{U}} \text{ and } 0<w_{\tilde{\tilde{R}}_i^{\mathrm{U}}} \leq 1, \end{cases} \tag{36}$$

$$x^*_{\tilde{\tilde{R}}^{*\mathrm{L}}} = \frac{y^*_{\tilde{\tilde{R}}^{*\mathrm{L}}}(r_3^{*\mathrm{L}}+r_2^{*\mathrm{L}})+(r_4^{*\mathrm{L}}+r_1^{*\mathrm{L}})(w_{\tilde{\tilde{R}}^{*\mathrm{L}}}-y^*_{\tilde{\tilde{R}}^{*\mathrm{L}}})}{2w_{\tilde{\tilde{R}}^{*\mathrm{L}}}} = 0 \tag{37}$$

$$y^*_{\tilde{\tilde{R}}^{*\mathrm{L}}} = \left\{\begin{array}{ll} \dfrac{w_{\tilde{\tilde{R}}^{*\mathrm{L}}}\times\left(\dfrac{r_3^{*\mathrm{L}}-r_2^{*\mathrm{L}}}{r_4^{*\mathrm{L}}-r_1^{*\mathrm{L}}}+2\right)}{6}, & \text{if } r_1^{*\mathrm{L}} \neq r_4^{*\mathrm{L}} \text{ and } 0<w_{\tilde{\tilde{R}}^{*\mathrm{L}}} \leq 1, \\ \dfrac{w_{\tilde{\tilde{R}}^{*\mathrm{L}}}}{2}, & \text{if } r_1^{*\mathrm{L}} = r_4^{*\mathrm{L}} \text{ and } 0<w_{\tilde{\tilde{R}}^{*\mathrm{L}}} \leq 1, \end{array}\right\} = 0.5 \tag{38}$$

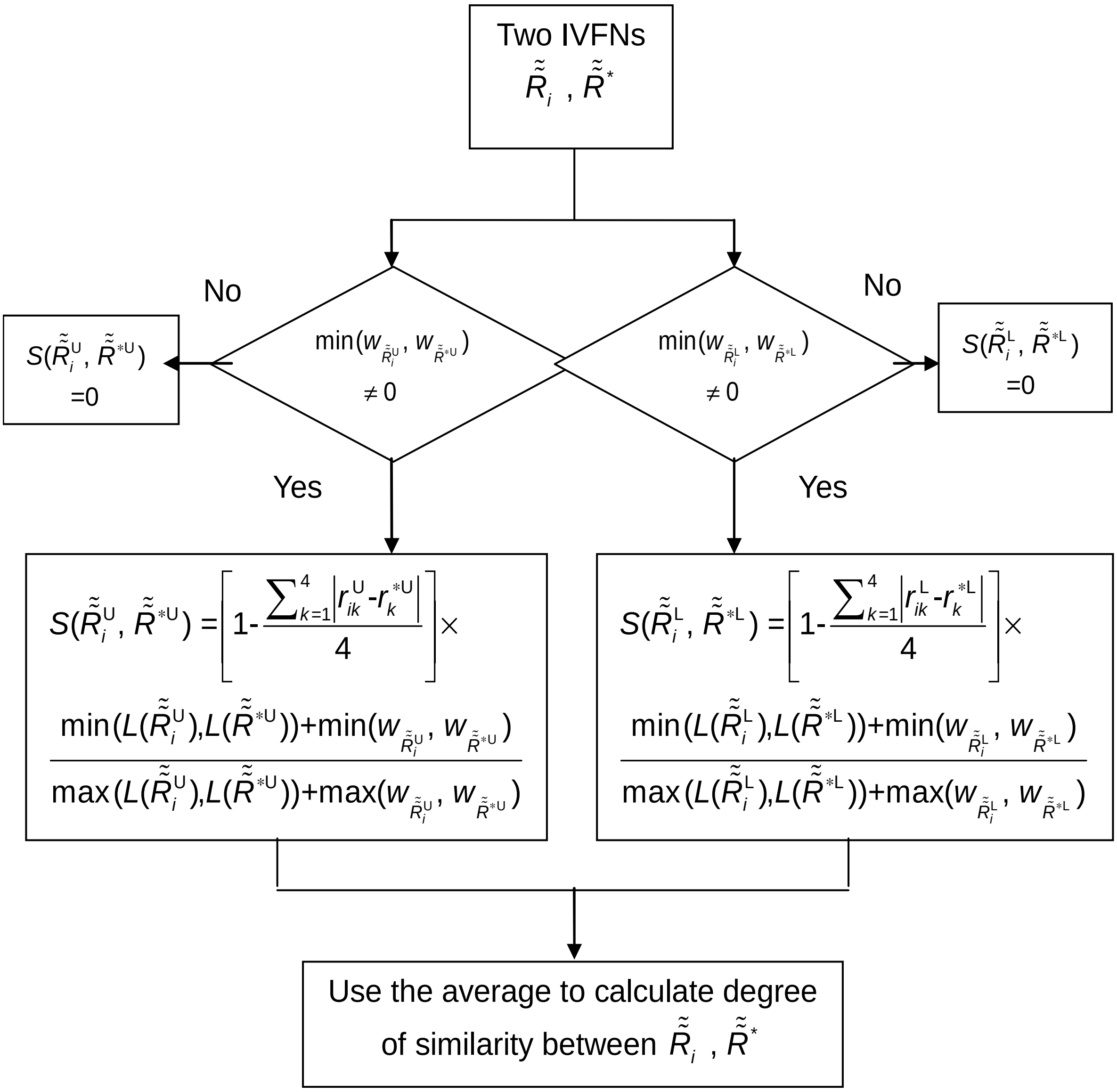

Figure 5. Structure of similarity degree between each alternative and the ideal solution.

$$x^*_{\tilde{\tilde{R}}^{*U}} = \frac{y^*_{\tilde{\tilde{R}}^{*U}}(r_3^{*U}+r_2^{*U})+(r_4^{*U}+r_1^{*U})(w_{\tilde{\tilde{R}}^{*U}}-y^*_{\tilde{\tilde{R}}^{*U}})}{2w_{\tilde{\tilde{R}}^{*U}}} = 0 \tag{39}$$

$$y^*_{\tilde{\tilde{R}}^{*U}} = \left\{ \begin{array}{ll} \dfrac{w_{\tilde{\tilde{R}}^{*U}} \times (\frac{r_3^{*U}-r_2^{*U}}{r_4^{*U}-r_1^{*U}}+2)}{6}, & \text{if } r_1^{*U} \neq r_4^{*U} \text{ and } 0<w_{\tilde{\tilde{R}}^{*U}} \leq 1, \\ \dfrac{w_{\tilde{\tilde{R}}^{*U}}}{2}, & \text{if } r_1^{*U} = r_4^{*U} \text{ and } 0<w_{\tilde{\tilde{R}}^{*U}} \leq 1, \end{array} \right\} = 0.5 \tag{40}$$

Now, we can calculate the COG points $\left(x^*_{\tilde{\tilde{R}}_i}, y^*_{\tilde{\tilde{R}}_i}\right)$ and $\left(x^*_{\tilde{\tilde{R}}^*}, y^*_{\tilde{\tilde{R}}^*}\right)$ of the IVFNs $\tilde{\tilde{R}}_i$ and $\tilde{\tilde{R}}^*$, respectively.

$$x^*_{\tilde{\tilde{R}}_i} = \begin{cases} \dfrac{A(\tilde{\tilde{R}}_i^{\mathrm{U}}) \times x^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}} - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \times x^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}}}{A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}})}, & \text{if } A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0, \\ 0, & \text{Otherwise,} \end{cases} \tag{41}$$

$$y^*_{\tilde{\tilde{R}}_i} = \begin{cases} \dfrac{A(\tilde{\tilde{R}}_i^{\mathrm{U}}) \times y^*_{\tilde{\tilde{R}}_i^{\mathrm{U}}} - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \times y^*_{\tilde{\tilde{R}}_i^{\mathrm{L}}}}{A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}})}, & \text{if } A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0, \\ 0, & \text{Otherwise,} \end{cases} \tag{42}$$

$$x^*_{\tilde{\tilde{R}}^*} = \left.\begin{cases} \dfrac{A(\tilde{\tilde{R}}^{*\mathrm{U}}) \times x^*_{\tilde{\tilde{R}}^{*\mathrm{U}}} - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \times x^*_{\tilde{\tilde{R}}^{*\mathrm{L}}}}{A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}})}, & \text{if } A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \neq 0, \\ 0, & \text{Otherwise,} \end{cases}\right\} = 0 \tag{43}$$

$$y^*_{\tilde{\tilde{R}}^*} = \left.\begin{cases} \dfrac{A(\tilde{\tilde{R}}^{*\mathrm{U}}) \times y^*_{\tilde{\tilde{R}}^{*\mathrm{U}}} - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \times y^*_{\tilde{\tilde{R}}^{*\mathrm{L}}}}{A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}})}, & \text{if } A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \neq 0, \\ 0, & \text{Otherwise,} \end{cases}\right\} = 0 \tag{44}$$

Finally, we calculated the difference Δx on the *x*-axis and the difference Δy on the *y*-axis of the COG points of the IVFNs $\tilde{\tilde{R}}_i$ and $\tilde{\tilde{R}}^*$:

$$\Delta x = \left.\begin{cases} \left| x^*_{\tilde{\tilde{R}}_i} - x^*_{\tilde{\tilde{R}}^*} \right|, & \text{if } A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \neq 0, \\ 0, & \text{otherwise,} \end{cases}\right\} = 0 \tag{45}$$

$$\Delta y = \left.\begin{cases} \left| y^*_{\tilde{\tilde{R}}_i} - y^*_{\tilde{\tilde{R}}^*} \right|, & \text{if } A(\tilde{\tilde{R}}_i^{\mathrm{U}}) - A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{*\mathrm{U}}) - A(\tilde{\tilde{R}}^{*\mathrm{L}}) \neq 0, \\ 0, & \text{otherwise,} \end{cases}\right\} = 0 \tag{46}$$

To rank the alternatives according to the decision-making criteria, we take the risk of negative ideal alternatives for "harmful":

$$\begin{aligned} \tilde{\tilde{R}}^- = & \left[\left(r_1^{-\mathrm{L}}, r_2^{-\mathrm{L}}, r_3^{-\mathrm{L}}, r_4^{-\mathrm{L}}; w^{\mathrm{L}}_{\tilde{\tilde{R}}^-}\right), \left(r_1^{-\mathrm{U}}, r_2^{-\mathrm{U}}, r_3^{-\mathrm{U}}, r_4^{-\mathrm{U}}; w^{\mathrm{U}}_{\tilde{\tilde{R}}^-}\right)\right] \\ & \left[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)\right] \end{aligned} \tag{47}$$

We then use the proposed similarity measure to evaluate the degree of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the negative ideal alternative, $\tilde{\tilde{R}}^{-}$.

$$S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^{-}) = \left[\frac{S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{-\mathrm{L}})+S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{-\mathrm{U}})}{2}\times(1\text{-}\Delta x)\times(1\text{-}\Delta y)\right]^{\left(\frac{1}{1+2t}\right)} \times \left(1\text{-}\left|w^{\mathrm{U}}_{\tilde{\tilde{R}}_i}\text{-}w^{\mathrm{U}}_{\tilde{\tilde{R}}^{-}}\text{-}w^{\mathrm{L}}_{\tilde{\tilde{R}}_i}+w^{\mathrm{L}}_{\tilde{\tilde{R}}^{-}}\right|\right)^{\frac{U}{2}},$$

$$= \left[\frac{S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{-\mathrm{L}})+S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{-\mathrm{U}})}{2}\right]\times\left(1\text{-}\left|w^{\mathrm{U}}_{\tilde{\tilde{R}}_i}\text{-}w^{\mathrm{L}}_{\tilde{\tilde{R}}_i}\right|\right)^{\frac{U}{2}} \tag{48}$$

$$t = \begin{cases} 1, \text{ if } A(\tilde{\tilde{R}}_i^{\mathrm{U}})\text{-}A(\tilde{\tilde{R}}_i^{\mathrm{L}}) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{-\mathrm{U}})\text{-}A(\tilde{\tilde{R}}^{-\mathrm{L}}) \neq 0, \\ 0, \text{ otherwise,} \end{cases} = 0 \tag{49}$$

$$u = \begin{cases} 1, \text{ if } r^{\mathrm{U}}_{i1} = r^{\mathrm{U}}_{i4} \text{ and } r^{-\mathrm{U}}_{1} = r^{-\mathrm{U}}_{4}, \\ 0, \text{ otherwise} \end{cases} \tag{50}$$

The areas $A\left(\tilde{\tilde{R}}_i^{\mathrm{L}}\right)$ and $A\left(\tilde{\tilde{R}}_i^{\mathrm{U}}\right)$ of the lower trapezoidal fuzzy number $\tilde{\tilde{R}}_i^{\mathrm{L}}$ and the upper trapezoidal fuzzy number $\tilde{\tilde{R}}_i^{\mathrm{U}}$ are calculated from Equations (23) and (24). The areas $A\left(\tilde{\tilde{R}}^{-\mathrm{L}}\right)$ and $A\left(\tilde{\tilde{R}}^{-\mathrm{U}}\right)$ of the lower trapezoidal fuzzy number $\tilde{\tilde{R}}^{-\mathrm{L}}$ and the upper trapezoidal fuzzy number $\tilde{\tilde{R}}^{-\mathrm{U}}$, respectively, are calculated as follows:

$$A\left(\tilde{\tilde{R}}^{-\mathrm{L}}\right)=\frac{(r_3^{-\mathrm{L}}\text{-}r_2^{-\mathrm{L}}+r_4^{-\mathrm{L}}\text{-}r_1^{-\mathrm{L}})\times w_{\tilde{\tilde{R}}^{-\mathrm{L}}}}{2} = 0 \tag{51}$$

$$A\left(\tilde{\tilde{R}}^{-\mathrm{U}}\right)=\frac{(r_3^{-\mathrm{U}}\text{-}r_2^{-\mathrm{U}}+r_4^{-\mathrm{U}}\text{-}r_1^{-\mathrm{U}})\times w_{\tilde{\tilde{R}}^{-\mathrm{U}}}}{2} = 0 \tag{52}$$

To calculate the degree of similarity $S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^{-})$, we must first calculate the degrees of similarity $S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{-\mathrm{L}})$ and $S(\tilde{\tilde{R}}_i^{\mathrm{U}}, \tilde{\tilde{R}}^{-\mathrm{U}})$ between the lower trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{\mathrm{L}}$ and $\tilde{\tilde{R}}^{-\mathrm{L}}$ and the upper trapezoidal fuzzy numbers $\tilde{\tilde{R}}_i^{\mathrm{U}}$ and $\tilde{\tilde{R}}^{-\mathrm{U}}$, respectively.

$$S(\tilde{\tilde{R}}_i^{\mathrm{L}}, \tilde{\tilde{R}}^{-\mathrm{L}}) = \begin{cases} \left[1\text{-}\frac{\sum_{k=1}^{4}\left|r_{ik}^{\mathrm{L}}\text{-}r_k^{-\mathrm{L}}\right|}{4}\right]\times\frac{\min(L(\tilde{\tilde{R}}_i^{\mathrm{L}}),L(\tilde{\tilde{R}}^{-\mathrm{L}}))+\min(w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}, w_{\tilde{\tilde{R}}^{-\mathrm{L}}})}{\max(L(\tilde{\tilde{R}}_i^{\mathrm{L}}),L(\tilde{\tilde{R}}^{-\mathrm{L}}))+\max(w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}, w_{\tilde{\tilde{R}}^{-\mathrm{L}}})}, \\ \qquad \text{if } \min(w_{\tilde{\tilde{R}}_i^{\mathrm{L}}}, w_{\tilde{\tilde{R}}^{-\mathrm{L}}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{53}$$

$$L(\tilde{\tilde{R}}^{-\mathrm{L}}) = \sqrt{(r_1^{-\mathrm{L}}\text{-}r_2^{-\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}^{-\mathrm{L}}}}+\sqrt{(r_3^{-\mathrm{L}}\text{-}r_4^{-\mathrm{L}})^2+w^2_{\tilde{\tilde{R}}^{-\mathrm{L}}}}+(r_3^{-\mathrm{L}}\text{-}r_2^{-\mathrm{L}})+(r_4^{-\mathrm{L}}\text{-}r_1^{-\mathrm{L}}) = 2 \tag{54}$$

$L(\tilde{\tilde{R}}_i^{\mathrm{L}})$ is calculated by formula (28).

$$S(\tilde{\tilde{R}}_i^{U}, \tilde{\tilde{R}}^{-U}) = \begin{cases} \left[1-\frac{\sum_{k=1}^{4}\left|r_{ik}^{U}-r_k^{-U}\right|}{4}\right]\times\frac{\min(L(\tilde{\tilde{R}}_i^{U}),L(\tilde{\tilde{R}}^{-U}))+\min(w_{\tilde{\tilde{R}}_i^{U}}, w_{\tilde{\tilde{R}}^{-U}})}{\max(L(\tilde{\tilde{R}}_i^{U}),L(\tilde{\tilde{R}}^{-U}))+\max(w_{\tilde{\tilde{R}}_i^{U}}, w_{\tilde{\tilde{R}}^{-U}})}, \\ \quad \text{if } \min(w_{\tilde{\tilde{R}}_i^{U}}, w_{\tilde{\tilde{R}}^{-U}}) \neq 0 \\ 0, \quad \text{otherwise} \end{cases} \tag{55}$$

$$L(\tilde{\tilde{R}}^{-U}) = \sqrt{(r_1^{-U}-r_2^{-U})^2+w_{\tilde{\tilde{R}}^{-U}}^2}+\sqrt{(r_3^{-U}-r_4^{-U})^2+w_{\tilde{\tilde{R}}^{-U}}^2}+(r_3^{-U}-r_2^{-U})+(r_4^{-U}-r_1^{-U}) = 2 \tag{56}$$

$L(\tilde{\tilde{R}}_i^{U})$ is calculated from Equation (31).

We showed the structure of calculations in the similarity degree between IVFNs of risks in each alternative and the negative-ideal solution in Figure 6.

We previously obtained the COG points $\left(x^*_{\tilde{\tilde{R}}_i^{L}}, y^*_{\tilde{\tilde{R}}_i^{L}}\right)$, $\left(x^*_{\tilde{\tilde{R}}_i^{U}}, y^*_{\tilde{\tilde{R}}_i^{U}}\right)$ of $\tilde{\tilde{R}}_i^{L}$ and $\tilde{\tilde{R}}_i^{U}$ from Equations. (33) to (36).

Next, we calculate the COG points $\left(x^*_{\tilde{\tilde{R}}^{-L}}, y^*_{\tilde{\tilde{R}}^{-L}}\right)$ and $\left(x^*_{\tilde{\tilde{R}}^{-U}}, y^*_{\tilde{\tilde{R}}^{-U}}\right)$ of $\tilde{\tilde{R}}^{-L}$ and $\tilde{\tilde{R}}^{-U}$, respectively, as follows:

$$x^*_{\tilde{\tilde{R}}^{-L}} = \frac{y^*_{\tilde{\tilde{R}}^{-L}}(r_3^{-L}+r_2^{-L})+(r_4^{-L}+r_1^{-L})(w_{\tilde{\tilde{R}}^{-L}}-y^*_{\tilde{\tilde{R}}^{-L}})}{2w_{\tilde{\tilde{R}}^{-L}}} = 1 \tag{57}$$

$$y^*_{\tilde{\tilde{R}}^{-L}} = \left\{\begin{matrix} \frac{w_{\tilde{\tilde{R}}^{-L}}\times(\frac{r_3^{-L}-r_2^{-L}}{r_4^{-L}-r_1^{-L}}+2)}{6}, \text{ if } r_1^{-L} \neq r_4^{-L} \text{ and } 0<w_{\tilde{\tilde{R}}^{-L}} \leq 1, \\ \frac{w_{\tilde{\tilde{R}}^{-L}}}{2}, \quad \text{if } r_1^{-L} = r_4^{-L} \text{ and } 0<w_{\tilde{\tilde{R}}^{-L}} \leq 1, \end{matrix}\right\} = 0.5 \tag{58}$$

$$x^*_{\tilde{\tilde{R}}^{-U}} = \frac{y^*_{\tilde{\tilde{R}}^{-U}}(r_3^{-U}+r_2^{-U})+(r_4^{-U}+r_1^{-U})(w_{\tilde{\tilde{R}}^{-U}}-y^*_{\tilde{\tilde{R}}^{-U}})}{2w_{\tilde{\tilde{R}}^{-U}}} = 1 \tag{59}$$

$$y^*_{\tilde{\tilde{R}}^{-U}} = \left\{\begin{matrix} \frac{w_{\tilde{\tilde{R}}^{-U}}\times(\frac{r_3^{-U}-r_2^{-U}}{r_4^{-U}-r_1^{-U}}+2)}{6}, \text{ if } r_1^{-U} \neq r_4^{-U} \text{ and } 0<w_{\tilde{\tilde{R}}^{-U}} \leq 1, \\ \frac{w_{\tilde{\tilde{R}}^{-U}}}{2}, \quad \text{if } r_1^{-U} = r_4^{-U} \text{ and } 0<w_{\tilde{\tilde{R}}^{-U}} \leq 1, \end{matrix}\right\} = 0.5 \tag{60}$$

Now, we can calculate the COG points $\left(x^*_{\tilde{\tilde{R}}_i}, y^*_{\tilde{\tilde{R}}_i}\right)$ and $\left(x^*_{\tilde{\tilde{R}}^-}, y^*_{\tilde{\tilde{R}}^-}\right)$ of the IVFN $\tilde{\tilde{R}}_i$ and $\tilde{\tilde{R}}^-$, respectively. The COG point $\left(x^*_{\tilde{\tilde{R}}_i}, y^*_{\tilde{\tilde{R}}_i}\right)$ of the IVFN $\tilde{\tilde{R}}_i$ was calculated from Equations (41) to (42). The COG point $\left(x^*_{\tilde{\tilde{R}}^-}, y^*_{\tilde{\tilde{R}}^-}\right)$ of the IVFN $\tilde{\tilde{R}}^-$ is calculated as follows:

$$x^*_{\tilde{\tilde{R}}^-} = \left\{\begin{matrix} \frac{A(\tilde{\tilde{R}}^{-U})\times x^*_{\tilde{\tilde{R}}^{-U}} - A(\tilde{\tilde{R}}^{-L})\times x^*_{\tilde{\tilde{R}}^{-L}}}{A(\tilde{\tilde{R}}^{-U})-A(\tilde{\tilde{R}}^{-L})}, \text{ if } A(\tilde{\tilde{R}}^{-U})-A(\tilde{\tilde{R}}^{-L}) \neq 0, \\ 0, \quad \text{Otherwise,} \end{matrix}\right\} = 0 \tag{61}$$

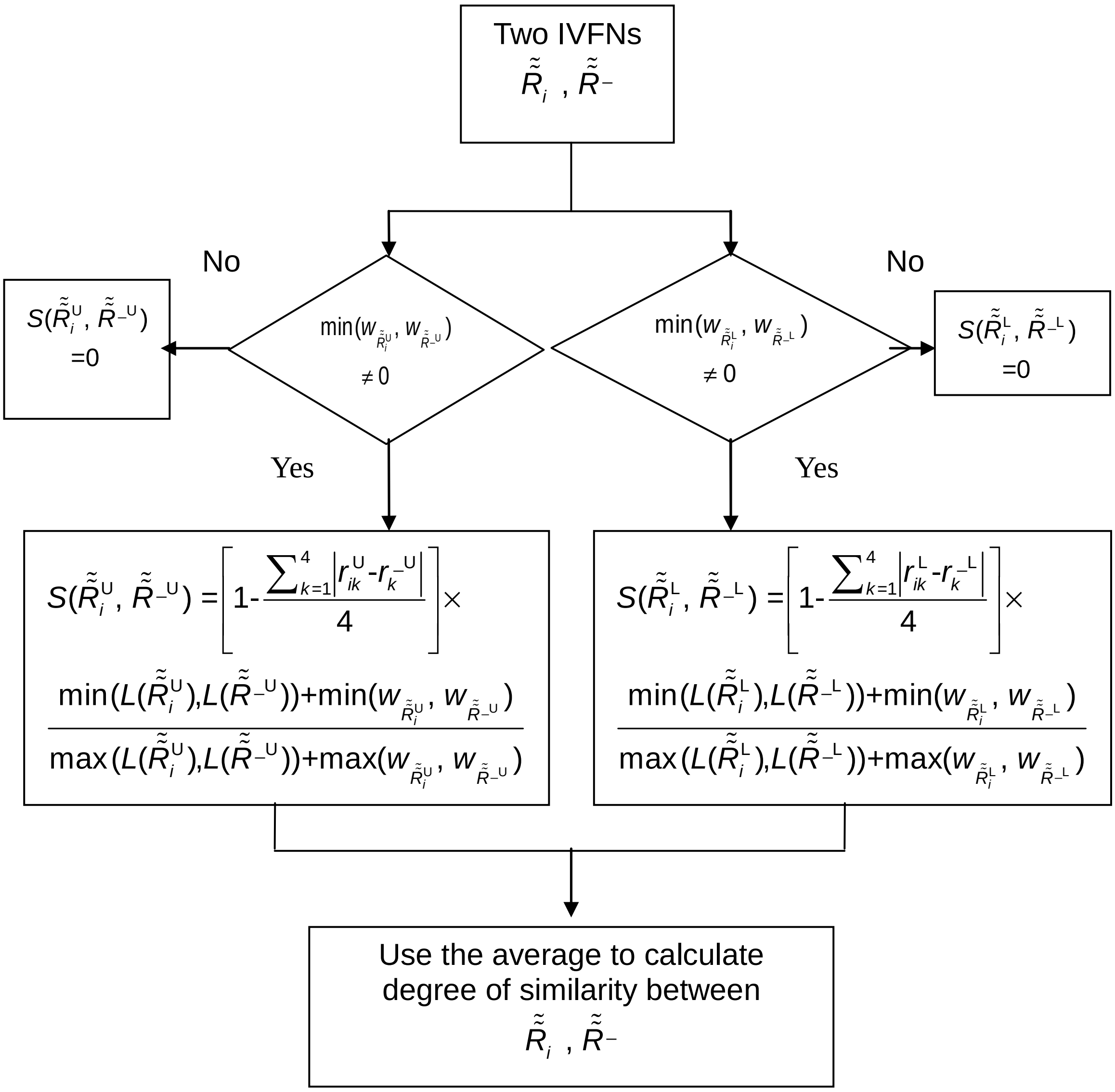

Figure 6. Structure of similarity degree between each alternative and the negative-ideal solution.

$$y^*_{\tilde{\tilde{R}}^-} = \begin{cases} \dfrac{A(\tilde{\tilde{R}}^{-U}) \times y^*_{\tilde{\tilde{R}}^{-U}} - A(\tilde{\tilde{R}}^{-L}) \times y^*_{\tilde{\tilde{R}}^{-L}}}{A(\tilde{\tilde{R}}^{-U}) - A(\tilde{\tilde{R}}^{-L})}, & \text{if } A(\tilde{\tilde{R}}^{-U}) - A(\tilde{\tilde{R}}^{-L}) \neq 0, \\ 0, & \text{Otherwise,} \end{cases} = 0 \tag{62}$$

The difference Δx on the x-axis and the difference Δy on the y-axis of the COG points of the IVFNs $\tilde{\tilde{R}}_i$ and $\tilde{\tilde{R}}^-$ are now calculated as follows:

$$\Delta x = \begin{cases} \left| x^*_{\tilde{\tilde{R}}_i} - x^*_{\tilde{\tilde{R}}^-} \right|, & \text{if } A(\tilde{\tilde{R}}_i^U) - A(\tilde{\tilde{R}}_i^L) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{-U}) - A(\tilde{\tilde{R}}^{-L}) \neq 0, \\ 0, & \text{otherwise,} \end{cases} = 0 \tag{63}$$

$$\Delta y = \begin{cases} \left| y^*_{\tilde{\tilde{R}}_i} - y^*_{\tilde{\tilde{R}}^-} \right|, \text{ if } A(\tilde{\tilde{R}}_i^U)\text{-}A(\tilde{\tilde{R}}_i^L) \neq 0 \text{ and } A(\tilde{\tilde{R}}^{-U})\text{-}A(\tilde{\tilde{R}}^{-L}) \neq 0, \\ 0, \qquad \text{otherwise,} \end{cases} = 0 \tag{64}$$

Finally, we calculate the relative closeness to the positive ideal solution. The relative closeness C_{i^*} of $\tilde{\tilde{R}}_i$ with respect to $\tilde{\tilde{R}}^*$ is defined as follows:

$$C_{i^*} = \frac{S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^*)}{S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^*) + S(\tilde{\tilde{R}}_i, \tilde{\tilde{R}}^-)}, \quad 0< C_{i^*}< 1, \quad i=1, 2, ..., n \tag{65}$$

A set of alternatives can now be preference ranked in descending order of C_{i^*}.

The proposed fuzzy risk-analysis algorithm steps are as follows:

Step 1: Integrate the linguistic values $\tilde{\tilde{W}}_{ij}$ of the severity of loss and the linguistic values $\tilde{\tilde{P}}_{ij}$ of the probability of failure to obtain the IVFNs of the sub risk $\tilde{\tilde{R}}_{ij}$ of each alternative (formula (16)).

Step 2: Modify the linguistic values $\tilde{\tilde{\varpi}}_j$ of importance in each risk R_{ij} to $\tilde{\tilde{\omega}}_j$ (Equation 17).

Step 3: Integrate the sub risk $\tilde{\tilde{R}}_{ij}$ and importance $\tilde{\tilde{\omega}}_j$ of each alternative (Equation18).

Step 4. Use the proposed similarity measure to evaluate the degrees of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the ideal alternative $\tilde{\tilde{R}}^*$ (Equations 19 to 46).

Step 5: Use the proposed similarity measure to evaluate the degrees of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the negative ideal alternative $\tilde{\tilde{R}}^-$ (Equations 47 to 64).

Step 6: Calculate the relative closeness to the ideal solution (Equation 65).

Step 7: Rank the preference order (Figure 7).

Illustration of the proposed method for improving patient safety

Illustration of the proposed method

Giraud et al. (1993) found that 44% of all iatrogenic complications in their study were associated with either human errors (insufficient surveillance, inadequate experience) or equipment-related problems (equipment failure, inadequate equipment). A human error was defined as a deviation from standard conduct, as well as addition or omission of actions related to standard operational instructions or routines of the unit (Donchin et al., 1995). Verbano and Turra (2010) also mentioned that organizations in the healthcare sector acknowledged the fact that human errors must be managed and controlled. However, several studies identified that nurse/patient ratios have an impact on adverse outcomes, task completion, medication errors, falls and staff retention and costs (Borden and Lang, 2001; Hegney et al., 2003; Sasicbay-Akkadechannunt et al., 2003; Dunton et al., 2004; Person et al., 2004; Philbrook, 2004; Sochalski, 2004). Because the workload per nurse may decrease as overall staffing size increases, a large staff is likely to have a reduced workload and therefore reduced nursing time pressure (Teng et al., 2010).

Patient safety is also affected by the work environment. In the matter of employee well-being, recent studies of the quality of working life have been carried out to measure job stress, job dissatisfaction, and burnout experienced by the professionals working in a health-care organization (Richard et al., 2010). Time was not the only factor when nurses were stressed; it was also important how other coworkers reacted. Nurses were preoccupied with their relationship to other professionals and how this could have a negative effect on their work. The negative effects identified in the study of Berland et al. (2008) were as follows: a lack of concentration, interrupted thought processes, energy not being used constructively, an increase in errors, lack of time for equipment maintenance, insecurity, an inability to act, and verbal abuse. Therefore, a good relationship with other professionals was felt to be important for patient safety. In a demanding work environment, such as in caring for critically ill patients, participation in decisions and support from colleagues can have a very positive effect on patient safety (Berland et al., 2008). Therefore, providing a pleasant work environment may improve the relationships between nurses and other professionals and may positively influence patient safety.

Risk management is primarily concerned with protecting an institution from financial losses from malpractice claims, as well as protecting professionals from the stress and disruption that result from the litigation process (Bower, 2002). To decrease the possibility of litigation, risk management focuses on maintaining minimum standards. Risk managers want to make sure that the standard of care is followed and that health-care

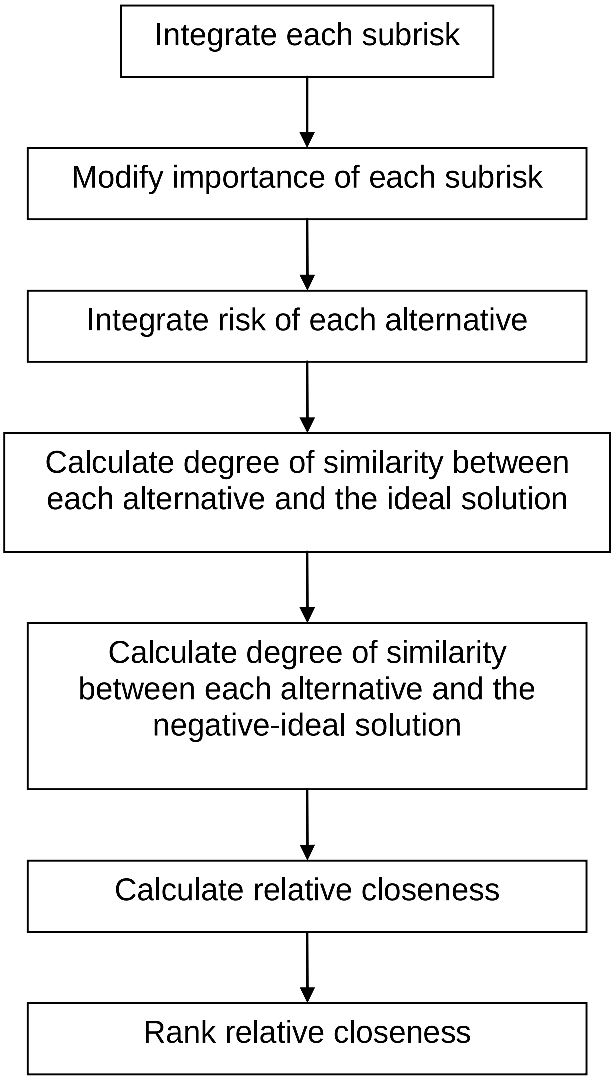

Figure 7. Structure of the proposed risk-analysis steps.

providers obtain informed consent, document thoroughly, communicate clearly and act in compliance with regularity and accreditation (Bower, 2002). Some researchers also found that patient safety should be integrated into the quality management system (Makai et al., 2009; Auerbach et al., 2007; Brennan et al., 2005). Quality management systems are defined as all processes that have been explicitly designed to monitor, assess and improve the quality of care (Makai et al., 2009). Kalra (2004) suggested that adopting intelligent systems approaches to promote efficiency and enhancing team coordination to facilitate optimal outcomes in patient care is a necessity. Here, we provide three alternatives (A_1, A_2 and A_3) to illustrate the risk management process of the proposed method in a health-care organization to improve patient safety. Assume that there is a hospital H whose management found that several medical errors that were harmful to patients were continuously happening in this organization. The management wants to improve the quality and safety of care provided to patients. After considering the causes of the patient safety problem described in the literature hospital H identified several alternatives to improve patient safety. However, the alternatives could not be executed completely because the budget was limited. The management in hospital *H* decided to chose the alternative that had the lowest risk to be the first alternative implemented to improve patient safety. The first alternative (A_1) is to hire more nurses to reduce the workload and to simultaneously provide periodical training. The second alternative (A_2) is to build a lounge for employees and to improve employee well-being. The third alternative (A_3) is to establish a risk management unit overseeing regularity and accreditation

Table 1. Nine-member linguistic terms and their corresponding interval-valued fuzzy numbers.

Linguistic terms	**interval-valued trapezoidal fuzzy numbers**
Absolutely low	[(0.0, 0.0,0.0, 0.0; 1.0), (0.0, 0.0,0.0, 0.0; 1.0)]
Very low	[(0.0075, 0.0075, 0.015, 0.0525; 0.5), (0.0, 0.0, 0.02, 0.07; 1.0)]
Low	[(0.0875, 0.12, 0.16, 0.1825; 0.5), (0.04, 0.10, 0.18, 0.23; 1.0)]
Fairly low	[(0.2325, 0.255, 0.325, 0.3575; 0.5), (0.17, 0.22, 0.36, 0.42; 1.0)]
Medium	[(0.4025, 0.4525, 0.5375, 0.5675; 0.5), (0.32, 0.41, 0.58, 0.65; 1.0)]
Fairly high	[(0.65, 0.6725, 0.7575, 0.79; 0.5), (0.58, 0.63, 0.80, 0.86; 1.0)]
High	[(0.7825, 0.815, 0.885, 0.9075; 0.5), (0.72, 0.78, 0.92, 0.97; 1.0)]
Very high	[(0.9475, 0.985, 0.9925, 0.9925; 0.5), (0.93, 0.98, 1.0, 1.0; 1.0)]
Absolutely high	[(1.0, 1.0, 1.0, 1.0; 1.0), (1.0, 1.0, 1.0, 1.0; 1.0)]

Source: Chen and Chen (2008)

compliance for patient safety.

To analyze the risks, there are two evaluating items $\tilde{\tilde{W}}_{ij}$ and $\tilde{\tilde{P}}_{ij}$ used to derive the probability of failure $\tilde{\tilde{R}}_{ij}$ of the alternative A_i selected by the hospital, where $\tilde{\tilde{W}}_{ij}$ denotes the severity of loss of alternative A_i in sub risk $\tilde{\tilde{R}}_{ij}$, $\tilde{\tilde{P}}_{ij}$ denotes the probability of failure of alternative A_i in sub risk $\tilde{\tilde{R}}_{ij}$, and $\tilde{\tilde{\varpi}}_j$ denotes the importance of sub risk $\tilde{\tilde{R}}_{ij}$, $1 \le i \le 3$, $1 \le j \le 2$. We use the linguistic term set shown in Table 1 to represent the linguistic terms and their corresponding IVFNs. The linguistic values evaluating items $\tilde{\tilde{W}}_{ij}$ and $\tilde{\tilde{P}}_{ij}$ of alternative A_i and the importance of sub risk $\tilde{\tilde{\varpi}}_j$ are shown in Table 2. In the following, we use the proposed algorithm to solve the risk management problem.

Step 1

Based on Equation (15), we integrate the evaluating items $\tilde{\tilde{W}}_{ij}$ and $\tilde{\tilde{P}}_{ij}$ of alternative A_i as follows:

$$\tilde{\tilde{R}}_{11} = \tilde{\tilde{W}}_{11} \otimes \tilde{\tilde{P}}_{11}$$
$$= \left[(0.0685, 0.0978, 0.1416, 0.1656; 0.5),\ (0.0288, 0.078, 0.1656, 0.2231; 1.0)\right]$$
$$\tilde{\tilde{R}}_{12} = \left[(0.162, 0.2048, 0.2889, 0.3221; 0.5),\ (0.1024, 0.1681, 0.3364, 0.4225; 1.0)\right]$$
$$\tilde{\tilde{R}}_{21} = \left[(0.0352, 0.0543, 0.086, 0.1036; 0.5),\ (0.0128, 0.041, 0.1044, 0.1495; 1.0)\right]$$
$$\tilde{\tilde{R}}_{22} = \left[(0.0352, 0.0543, 0.086, 0.1036; 0.5),\ (0.0128, 0.041, 0.1044, 0.1495; 1.0)\right]$$
$$\tilde{\tilde{R}}_{31} = \left[(0.0203, 0.0306, 0.052, 0.0652; 0.5),\ (0.0068, 0.022, 0.0648, 0.0966; 1.0)\right]$$
$$\tilde{\tilde{R}}_{32} = \left[(0.0352, 0.0543, 0.086, 0.1036; 0.5),\ (0.0128, 0.041, 0.1044, 0.1495; 1.0)\right]$$

Step 2

Based on Equations (17) and (4), we modify the importance $\tilde{\tilde{\varpi}}_j$ of each sub risk $\tilde{\tilde{R}}_{ij}$ to $\tilde{\tilde{\omega}}_j$:

$$\tilde{\tilde{\omega}}_1 = \left[(0.5, 0.5, 0.5, 0.5; 0.5),\ (0.5, 0.5, 0.5, 0.5; 0.5)\right]$$

$$\tilde{\tilde{\omega}}_2 = \left[(0.5, 0.5, 0.5, 0.5; 0.5),\ (0.5, 0.5, 0.5, 0.5; 0.5)\right]$$

Step 3

Based on Equation (18), we calculate the risk $\tilde{\tilde{R}}_i$ as follows:

Table 2. Linguistic values for evaluating the risks for the alternatives to improving patient safety.

Items	Financial risk		Operating risk	
	Linguistic values of the severity of loss	Linguistic values of the probability of failure	Linguistic values of the severity of loss	Linguistic values of the probability of failure
A_1	$\tilde{\tilde{W}}_{11}$ = high	$\tilde{\tilde{P}}_{11}$ = low	$\tilde{\tilde{W}}_{12}$ = medium	$\tilde{\tilde{P}}_{12}$ = medium
A_2	$\tilde{\tilde{W}}_{21}$ = low	$\tilde{\tilde{P}}_{21}$ = medium	$\tilde{\tilde{W}}_{22}$ = low	$\tilde{\tilde{P}}_{22}$ = medium
A_3	$\tilde{\tilde{W}}_{31}$ = fairly low	$\tilde{\tilde{P}}_{31}$ = low	$\tilde{\tilde{W}}_{32}$ = medium	$\tilde{\tilde{P}}_{32}$ = low
ϖ	$\tilde{\tilde{\varpi}}_1$ = high		$\tilde{\tilde{\varpi}}_2$ = high	

$$\tilde{\tilde{R}}_1 = \left(\tilde{\tilde{R}}_{11} \otimes \tilde{\tilde{\omega}}_1\right) \oplus \left(\tilde{\tilde{R}}_{12} \otimes \tilde{\tilde{\omega}}_2\right)$$
$$= \left[(0.1153,\ 0.1513,\ 0.2153,\ 0.2439;\ 0.5),\ (0.0656,\ 0.1231,\ 0.251,\ 0.3229;\ 1.0)\right]$$
$$\tilde{\tilde{\mathrm{R}}}_2 = \left[(0.0352, 0.0544, 0.086, 0.1036; 0.5), (0.0128, 0.041, 0.1044, 0.1496; 1.0)\right]$$
$$\tilde{\tilde{\mathrm{R}}}_3 = \left[(0.0278, 0.0425, 0.069, 0.0844; 0.5), (0.0098, 0.0315, 0.0846, 0.1231; 1.0)\right]$$

Step 4

Based on Equations (19) to (46), we use the proposed similarity measure to evaluate the degrees of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the positive ideal alternative $\tilde{\tilde{R}}^*$, respectively. The results are as follows:

$S(\tilde{\tilde{R}}_1, \tilde{\tilde{R}}^*)$ =0.5894 ; $S(\tilde{\tilde{R}}_2, \tilde{\tilde{R}}^*)$ =0.6806 ; $S(\tilde{\tilde{R}}_3, \tilde{\tilde{R}}^*)$ =0.6933

Step 5

Based on Equations (47) to (64), use the proposed similarity measure to evaluate the degrees of similarity between the IVFNs of $\tilde{\tilde{R}}_i$ and the risk of the negative ideal alternative $\tilde{\tilde{R}}^-$. The results are as follows:

$S(\tilde{\tilde{R}}_1, \tilde{\tilde{R}}^-)$ =1356 ; $S(\tilde{\tilde{R}}_2, \tilde{\tilde{R}}^-)$ =0.0547 ; $S(\tilde{\tilde{R}}_3, \tilde{\tilde{R}}^-)$ =0.0442

Step 6

Calculate the relative closeness to the ideal solution.

C_{1*}=0.8129 ; C_{2*}=0.9256 ; C_{3*}=0.9400

Step 7

Rank the preference order. According to the descending order of the C_{i*} values, the preference order is: $A_3 \succ A_2 \succ A_1$.

Without a doubt, hospital H will be a health care organization with a high level of quality in patient safety if all feasible alternatives are implemented by the management. According to the limited budget, in the above risk management method for a health care organization focusing on patient safety, we found that the alternative A_3 resulting in the lowest risk to hospital H from among the alternatives. To improve patient safety and lower management risk, hospital H should first establish a risk management unit overseeing regularity and accreditation compliance for patient safety. If there is remaining money in the budget after executing alternative A_3, the management of hospital H should consider implementing alternative A_2. Using the proposed method, management can easily choose the alternative most appropriate for their organization's situation using their own opinions of the risk in linguistic terms.

A comparison of the relative closeness to the ideal solution

In this section, we use interval valued trapezoidal fuzzy numbers of these three alternatives to compare the calculation results of the proposed method with the TOPSIS method by calculating distance. Table 3 shows

Table 3. Comparison of the closeness calculation results of the proposed method and the existing TOPSIS method by calculating distance.

Methods / Alternatives	TOPSIS method by calculating distance	The proposed method
A_1	0.7595	0.8129
A_2	0.8214	0.9256
A_3	0.8272	0.9400

the comparisons. From Table 3, we can see the relative closeness of A_2 and A_3 resulting from TOPSIS method by calculating distance are too close to be the basis of judgment by the decision maker. Decision makers maybe difficult to identified if A_3 is really better than A_2. However, based on the proposed method, we can see that A_3 is better than A_2 clearly.

As we mentioned in the preliminaries, even with the same distance between IVFNs, the IVFNs may have different shapes or directions, and that also lead the different closeness. In summary, we can see that the proposed method can overcome the drawbacks of the TOPSIS method by calculating distance.

Conclusion

In this study, we used the TOPSIS approach to manage the risk of a health-care organization in the IVFN environment while improving patient safety. The proposed method can overcome the drawbacks of the existing TOPSIS method by calculating distance of IVFNs. Simultaneously, rather than measuring the risk in terms of individual patients, we measured the risks in the health care organizations. Considering the communication patterns, we used linguistic terms to represent their corresponding IVFNs because IVFNs are more appropriate for expert opinions in some complex situations. Generally, the risk of patient safety is affected by several factors, such as health care providers (nurses and others), equipment and the quality management system. These have been discussed in the literature in the past several decades. However, because of cost limitations, it may not be possible to take action to simultaneously improve all of the factors that will affect patient safety. The health care organization should consider the decision making risk while reducing the patient safety risk. The proposed method provides a useful way for decision makers in health care organizations to handle risks in a variable, complex and uncertain environment. Moreover, it may reduce the cost to the organization in determining these risks.

REFERENCES

Auerbach AD, Landefeld CS, Shojana KG (2007). The tension between needing to improve care and knowing how to do it. N. Engl. J. Med., 357: 1405–1409.

Berland A, Gerd KN, Doris G (2008). Patient safety and job-related stress: a focus group study. Intensive Crit. Care Nurs., 24(2): 90–97.

Bianchi N, Carta G, Renzetti T (2009). Clinical risk management at the Bambino Gesu` Children's Hospital. Paediatr. Child Health. 19(Suppl. 2): S176–S181.

Borden S, Lang A (2001). Nurse workloads and distractions lead to medication errors in 1999 – a report from the USP. Nevada Reformation. 10: 22.

Bower JO (2002). Designing and Implementing a patient Safety Program for the OR. AORN J., 76(3): 452–456.

Brennan TA, Gawande A, Thomas E, Studdert D (2005). Accidental deaths, saved lives, and improved quality. N. Engl. J. Med., 353: 608–613.

Bustince H (2000). Indicator of inclusion grade for interval-valued fuzzy sets. Application to approximate reasoning based on interval-valued fuzzy sets. Int. J. Approx. Reason., 23(3): 137–209.

Carayon P, Alvarado CJ (2007). Workload and Patient Safety among Critical Care Nurses. Crit. Care Nurs. Clin. North Am., 19(2): 121–129.

Chao CC, Jen WY, Hung MC, Li YC, Chi YP (2007). An innovative mobile approach for patient safety services: The case of a Taiwan health care provider. Technovation, 27(6-7): 342–351.

Chen SJ (2006). A new method for handling the similarity measure problems of interval-valued fuzzy numbers. In Proceedings of the second international conference on natural computation and the third international conference on fuzzy systems and knowledge discovery. Xi'an China, pp.325–334.

Chen SJ, Chen SM (2004). A new similarity measure between interval-valued fuzzy numbers. In Proceedings of the joint second international conference of soft computing and intelligent systems and fifth international symposium on advanced intelligent systems. Yokohama, Japan.

Chen SJ, Chen SM (2008). Fuzzy risk analysis based on measures of similarity between interval-valued fuzzy numbers. Comput. Math. Appl., 55(8): 1670–1685.

Chen SM (1995). Arithmetic operations between vague sets. In proceeding of international joint conference of CFSA/TFIS/SOFT '95 on fuzzy theory and applications, Taipei Taiwan, Republic of China. pp.206–211.

Didier P, Benedetta A, Julie S, Liam D (2006). 'Clean Care is Safer Care': the global patient safety challenge 2005—2006. Int. J. Infect. Dis., 10(6): 419–424.

Donchin Y, Gopher D et al. (1995). A look into the nature and causes of human errors in the intensive care unit. Crit. Care Med., 23: 294–300.

Dunton N, Gajewski B, Taunton RL, Moore J (2004). Nurse staffing and patient falls on cute care hospital units. Nurs. Outlook, 52(1): 53–59.

García J, Pazos J, Ríos J, Yagüe J (1992). Methodology of linguistics evaluation in risk situations using fuzzy techniques. Fuzzy Sets Syst., 48(2): 185–194.

Gerolamo AM (2009). An exploratory analysis of the relationship between psychiatric nurses' perceptions of workload and unit activity. Arch. Psychiatr. Nurs., 23(3): 243–250.

Gerson LB, Triadafilopoulos G, Gage BF (2004). The management of anticoagulants in the periendoscopic period for patients with atrial fibrillation: a decision analysis. Am. J. Med., 116(7): 451–459.

Giraud T, Dhainaut J, Vaxelaire J (1993). Iatrogenic complications in adult intensive care units: A prospective two-center study. Crit. Care Med., 21(1): 40–51.

Gorzalczany MB (1987). A method of inference in approximate

reasoning based on interval-valued fuzzy sets. Fuzzy Sets Syst., 21(1):1–17.
Gupta A (2004). Strategies for outpatient anaesthesia. Best Pract. Res. Clin. Anaesthesiol. 18(4): 675–692.
Hegney D, Plank A, Parker V (2003). Nursing workloads: the results of a study of Queensland nurses. J. Nurs. Manage., 11: 307–314.
Hurst K (2005). Relationships between patient dependency, nursing workload and quality. Int. J. Nurs. Stud., 42(1): 75–84.
Hwang CL, Yoon K (1981). Multiple Attribute Decision Making: Methods and Applications. Spronger, Berlin, Heidelberg, New York.
Johnstone MJ, Kanitsaki O (2007). Clinical risk management and patient safety education for nurses: A critique. Nurse Educ. Today, 27(3): 185–191.
Kalra J (2004). Medical errors: impact on clinical laboratories and other critical areas. Clin. Biochem., 37(12): 1052-1062.
Landis GS, Faries PL (2007). A critical look at “high-risk” in choosing the proper intervention for patients with carotid bifurcation disease. Semin. in Vasc. Surg., 20(4): 199–204.
Lyneham J, Cloughessy L, Martin V (2008). Workloads in Australian emergency departments a descriptive study. Int. Emerg. Nurs., 16(3): 200–206.
Makai P, Klazingac N, Wagner C, Boncz I, Gulacsi L (2009). Quality management and patient safety: Survey results from 102 Hungarian hospitals. Health Policy, 90(2-3): 175–180.
Mitchell HB (2005). Pattern recognition using type-II fuzzy sets. Inf. Sci., 170(2-4): 409–418.
Mizumoto M, Tanaka K (1981). Fuzzy sets and type 2 under algebraic product and algebraic sum. Fuzzy Sets Syst., 5(3): 277–290.
Nieto-Morote A, Ruz-Vila F (2011). A fuzzy approach to construction project risk assessment. Int. J. Proj. Manage., 29(2): 220–231.
Pappis CP, Karacapilidis NI (1995). Application of a similarity measure of fuzzy sets to fuzzy relational equations. Fuzzy Sets Syst., 75(2): 135–142.
Person SD, Allison JJ, Kiefe CI, Weaver MT, Williams OD, Centor RM, Weissman NW (2004). Nursing staffing and mortality for Medicare patients with acute myocardial infarction. Med. Care, 42: 4–12.
Petersen BT (2007). Reducing risk during endoscopy in patients with implanted electronic devices. Tech. Gastrointest. Endosc., 9(4): 208–212.
Philbrook P (2004). To ratio or not to ratio: that is the question. Maine Nurse. 6(1): 2–3.
Richard JH, Neal RP, Matthew CS, Theresa MS, Judi MA, Ben-Tzion K (2010). Effects of mental demands during dispensing on perceived medication safety and employee well-being: A study of workload in pediatric hospital pharmacies. Res. Social Adm. Pharm., 6(4): 293–306.
Ross J, Ranum D (2009). Improving patient safety by understanding past experiences in day surgery and PACU. J. PeriAnesth. Nurs., 24(3): 144–151.
Sambuc R, Fonctionsψ F (1975). Application A L’Aide Au Diagnostic En Pathologie Thyroidienne. PhD thesis. University of Marseille. France.
Sasicbay-Akkadechannunt T, Scalzi CC, Jawad AF (2003). The relationship between nurse staffing and patient outcome. J. Nurs. Adm., 33: 478–485.
Schmucker KJ (1984). Fuzzy sets, natural language computations, and risk analysis. Computer Science Press. Rockville. 192.
Sepulveda R, Castillo O, Melin P, Rodriguez-Diaz A, Montiel O (2007). Experimental study of intelligent controllers under uncertainty using type-1 and type-2 fuzzy logic. Inf. Sci., 177(10): 2023–2048.
Sochalski J (2004). Is more better? The relationship between nurse staffing and the quality of nursing care in hospitals. Med. Care, 42(Suppl. 2): 67–73.
Teng CI, Shyu YI L, Chiou WK, Fan HC, Lam SM (2010). Interactive effects of nurse-experienced time pressure and burnout on patient safety: A cross-sectional survey. Int. J. Nurs. Stud., 47(11): 1442–1450.
Vande Voorde KM, France AC (2002). Proactive error prevention in the intensive care unit. Crit. Care Nurs. Clin. North Am. 14(4): 347–358.
Verbano C, Turra F (2010). A human factors and reliability approach to clinical risk management: evidence from Italian cases. Saf. Sci., 48(5): 625–639.
Wang G, Li X (1998). The applications of interval-valued fuzzy numbers and interval-distribution numbers. Fuzzy Sets Syst., 98(3): 331–335.
Wei SH, Chen SM (2009). Fuzzy risk analysis based on interval-valued fuzzy numbers. Expert Syst. Appl., 36(2): 2285–2299.
Welie SPK, Dute J, Nys H, Wijmen FCBV (2005). Patient incompetence and substitute decision-making: an analysis of the role of the health care professional in Dutch law. Health Policy, 73(1): 21–40.
Wu D, Mendel JM (2008). A vector similarity measure for linguistic approximation: interval type-2 and type-1 fuzzy sets. Inf. Sci., 178(2): 381–342.
Wu D, Mendel JM (2009). A comparative study of ranking methods, similarity measures and uncertainty measures for interval type-2 fuzzy sets. Inf. Sci., 179(8): 1169–1192.
Yao JS, Su JS (2000). Fuzzy inventory with backorder for fuzzy total demand based on interval-valued fuzzy set. Eur. J. Oper. Res., 124(2): 390–408.
Zadeh LA (1975). The concept of a linguistic variable and its application to approximate reasoning—I. Inf. Sci., 8(3): 199–249.
Zadeh LA (2005). Toward a generalized theory of uncertainty (GTU) – an outline. Inform. Sci., 172(1-2): 1–40.
Zeng W, Li H (2006). Relationship between similarity measure and entropy of interval valued fuzzy sets. Fuzzy Sets Syst., 157(11): 1477–1484.
Zhang HY, Zhang WX (2009). Hybrid monotonic inclusion measure and its use in measuring similarity and distance between fuzzy sets. Fuzzy Sets Syst., 160(1): 107–118.