

*Full Length Research Paper*

# Computation of shoreline change: A transient cross-shore sediment transport approach

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**It has been well understood that almost all coasts experience permanent and transient changes due to water waves and other causes in different times. Build-up and erosion of the dunes in the vicinity of the shoreline is a morphodynamic process made by the cycle of windblown fine sediments and wave surges. This phenomenon has been investigated and the beach profile change has been represented by a mathematical morphodynamic model. This model has been constituted based upon the most recent work of Hanson et al. (2010) but with an additional term to consider the steady-state cross-shore sediment transport. A closed form solution to the morphodynamic model has been presented and applied to the northern coastline of the Oman Sea, located in the southern part of Iran.**

**Key words:** Coastal, shoreline, sediment transport, beach, morphodynamic, dune, berm.

## INTRODUCTION

Coastline change is often a long term process which is a result of many factors affecting the beach. Although these factors are numerous, the wind effect and storm surges can be pointed as the most important factors affecting this process (Larson and Kraus, 1989; Short, 1999; Dean and Dalrymple, 2004; Nordstrom, 2008; Neshaei et al., 2009; Hanson et al., 2010). Most of the beaches consist mainly of fine sand and silt and creation of dunes by wind along with destructive effect of wave attacks constitutes a progressive process that may result in landward or seaward build-up of the shoreline, depending the predominant effect of these two constructive and destructive factors (Kriebel and Dean, 1985; Kriebel, 1986; Short, 1999; Dean and Dalrymple, 2004; Larson et al., 2004; Davidson-Arnott et al., 2005; Nordstrom, 2008; Hanson et al., 2010; Sesli, 2010).

Very recently, Hanson et al. (2010) suggested a mathematical model for beach change by simultaneous effect of long-shore sediment transport and cross-shore processes. They assumed that the shoreline change occurs in a cycle of sediment deposition by wind leading to dune build-up and erosion of the dune by storm surges. This cycle is affected by long-shore sediment

transport rate, and, accompanied with the cross-shore process, forms the morphodynamic profile of the coastlines. In Hanson et al. (2010) model, there is no consideration to the cross-shore steady-state sediment transport and the seaward-landward process is assumed to happen in a closed cycle. Several factors are involved in steady-state cross-shore sediment transport, among them wave number and wave period, breaking wave height, water depth in the vicinity of the shoreline, the equilibrium profile and the beach slope can be mentioned (Dean, 1991; Dean and Dalrymple, 2004). In this research, the Hanson et al. (2010) model has been reformed and a cross-shore sediment transport rate is added which seems to increase the ability of this model in prediction of the morphodynamic coastline profile change in particular for beaches prone to sea level rise (Hansen et al., 2010). The main focus is to apply the modified model to the Northern Oman Sea Coastline which is classified as a sandy coast. This model can be considered as a new approach within which, the interrelation between the beach profile changes due to wind transported sediment accumulation or erosion and the shoreface sediment transport regime, seaward or landward, is included. A mathematical model is constituted and an analytical solution has been found to represent the shoreline change. The model is then applied to and verified by a practical case study.

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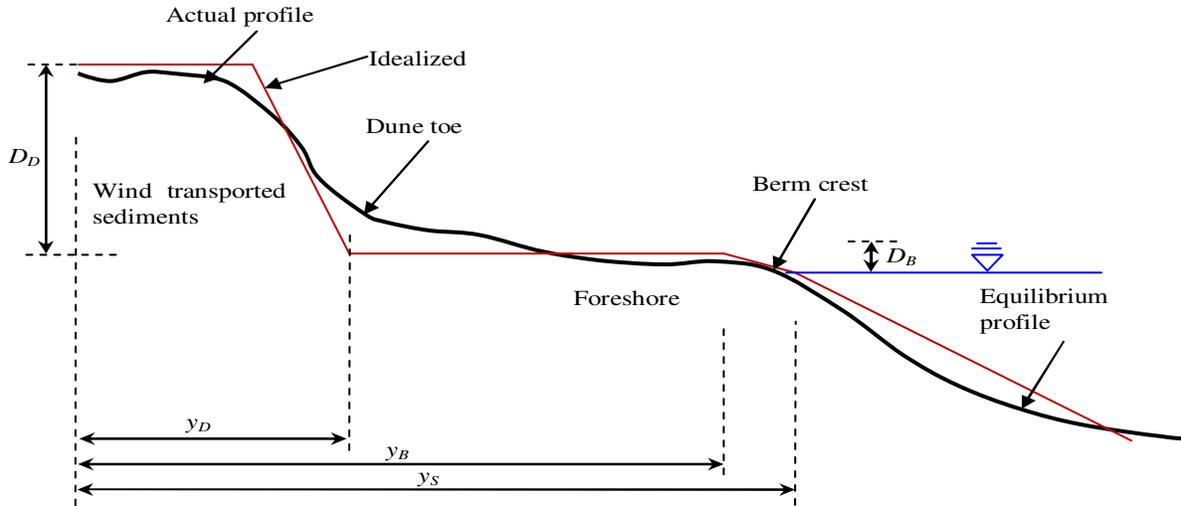


Figure 1. Definition of parameters involved in derivation of the equations.

## REPRESENTATION OF THE MATHEMATICAL MODEL

As stated previously, the formation of a beach is a long-term process consisting of dune build-up and erosion cycles. This phenomenon can be related to constructive and destructive factors which are combined together, along with the cross-shore steady state sediment transport rate, to form a shoreline change model. Wind-blown sand and silts are the constituents of the dunes which are transported landward from the berm and the foreshore (Short, 1999; Dean and Dalrymple, 2004; Hanson et al., 2010). The berm, as defined by Hanson et al. (2010) refers to the mildly sloped ground, fairly horizontal, that forms a portion of sub-aerial beach seaward. The foreshore, starting from the berm seaward from the dune, is the portion of the beach exposed to waves and is prone to continuous erosion and accretion. The subaqueous profile is assumed to obey the equilibrium profile suggested by Dean (1977). Figure 1 show different parameters involved in the equations presented in this work which are consistent with the terminology of Hanson et al. (2010).

### Formation of the Dune

Wind, as the main factor, is responsible for formation of dunes at almost all beaches consisting of fine sand and silt. This formation can be expressed by mass conservation differential equation as follows:

$$\frac{dV_{DW}}{dt} = q_w \quad (1)$$

In this equation,  $V_{DW}$  is the dune volume built up by wind transported sediments,  $t$  is time and  $q_w$  is the wind

transported volume of sand and silt. There are several equations that relate the rate of sand and silt transport due to the wind velocity which mainly depend on the width of the berm (Davidson-Arnott et al., 2005; Hopf and Sherman, 2007; Hanson et al., 2010). For the sake of convenience, the following relationship of Hanson et al. (2010) has been adopted in this research:

$$q_w = q_{w0} \left( 1 - 0.5 \left[ 1 - \tanh \left( \frac{\pi}{q_{grad}} (y_B - y_D - y_{50}) \right) \right] \right) \quad (2)$$

In this equation,  $q_{w0}$  is the maximum rate of transported sand by the wind for an infinite width which depends on the sand and water properties,  $y_B$  and  $y_D$  are the distances to the end of the berm measured in seaward direction,  $y_{50}$  is the distance corresponding to 50% of the maximum sediment deposition and  $q_{grad}$  is the transport gradient (Hanson et al., 2010). They suggested a simplified approximation of this equation as follows:

$$q_w = q_{w0} (1 - \exp[-\alpha(y_B - y_D)]) \quad (3)$$

The coefficient  $\alpha$  is a scale factor.

This equation exhibits similar properties to the preceding equation, that is, a slow and gradual increase in sand transport with the width of the beach, an intensified increase for wider beaches and a saturated state (upper bound) corresponding to very little or no transport for small beaches or beaches with an approach towards being shortened in width when windblown sediments decrease the width of the beach. Further assumption is made, that is, the shape and the height ( $D_0$ ) of the dune remains constant during the deposition/erosion processes. This yield the mass conservation differential equation to be as follows:

$$\frac{d(y_{DW})}{dt} = q_w \Rightarrow \frac{dV_{DW}}{dt} = \frac{q_w}{D_0} \quad (4)$$

### Erosion of the dune

Wave impact during storm surges is responsible for the erosion of the dune. It can be expressed by the following equation:

$$\frac{d(V_{D0})}{dt} = -q_0 \quad (5)$$

In this equation,  $V_{D0}$  is the dune loss (volume eroded by waves) during the waves attack. The negative sign is used to express the loss of sand not accumulated (which is positive in sign). Larson et al. (2004) suggested the following equation to evaluate this quantity:

$$q_0 = 4C_s \frac{(R + \Delta h - z_D)^2}{T} \quad R > z_D - \Delta h \quad (6)$$

In this equation,  $C_s$  is a coefficient that is determined empirically,  $z_D$  is the elevation of the dune toe, with respect to the maximum sea level,  $T$  is taken to be the wave period,  $\Delta h$  is the surge level and  $R$  is the height of the run-up which can be estimated by the following equation:

$$R = a\sqrt{H_0 L_0} \quad (7)$$

where  $a$  is a constant equal to 0.15 (Larson et al., 2004) for a typical foreshore slope, but can be estimated for other slopes occurring in different beaches,  $L_0$  is the deepwater wavelength and  $H_0$  is the deepwater root mean square of wave heights.

Therefore, the total volume change of the dune in a cycle of deposition and removal of sand by wind and by storm surge induced waves respectively, can be represented as follows:

$$\frac{dV_D}{dt} = \frac{dV_{DW}}{dt} + \frac{dV_{D0}}{dt} = \left( \frac{dy_{DW}}{dt} + \frac{dy_{D0}}{dt} \right) D_0 \quad (8)$$

$$\Rightarrow \frac{dV_D}{dt} = \frac{dy_D}{dt} D_0 = q_w - q_0 \quad (9)$$

### Transient seaward sediment transport

In most of beaches, the deposition/erosion of the dune adjacent to the foreshore is not a closed cycle, instead,

the contribution of the transient sediment transport seaward or landward can influence this cycle as long as the direction and the rate of this transport can be determined. In many beaches, the transient sediment transport is a seasonal process which may change during summer and winter regimes. In order to simplify the problem, an average value of the transient sediment transport is assumed here. Contribution of this cross-shore sediment transport rate,  $q_c$ , is considered in derivation of the governing equations for the build-up/erosion cycle of the dunes and berms which is presented subsequently. This part makes a new form of the equation of Hanson et al. (2010).

There are two important assumptions made to derive the differential equation of the formation/erosion of the dune along with the cross-shore sediment transport:

Dune and beach profile change occur with the same shape (which has been adopted from Hanson et al. (2010) and, simplifies the solution).

Once a volume  $\Delta V_D$  is added to the dune, the same volume should be taken from the profile over its active depth minus the amount of sediment transported seaward, denoted by  $q_c$ . The latter assumption, that is, the seaward (or landward) steady-state sediment transport, is the complementary part which guarantees the coastal erosion that may be permanent in time, or, in a period of time. For example, if this sediment transport rate is positive in one season and negative in another one, prediction of the final location of the shoreline during and at the end of each season would be possible.

By making use of these assumptions, the continuity equation with the contribution of the permanent sediment transport can be expressed as follows:

$$\Delta y_D D_D + \Delta y_B (D_B + D_D) - q_c \Delta t = 0 \quad (10)$$

Therefore, in limits, as  $\Delta y_D$  approaches zero, one may rearrange the last two equations to find the following system of ordinary differential equations for the location of the dune and the berm:

$$\frac{dy_D}{dt} = \frac{q_w - q_0}{D_D} \quad (11a)$$

$$\frac{dy_B}{dt} = -\frac{q_w - q_0 - q_c}{D_B + D_D} \quad (11b)$$

This set of coupled equations can be combined together and solved by introducing a new variable,  $y$ , which defines the relative distance of the dune and the berm as follows:

$$dy = dy_B - dy_D \quad (12)$$

Thus, the system of the ordinary differential equations can

be interrelated and expressed through the following ordinary differential equation:

$$\frac{dy}{dt} = -\frac{q_w - q_0 - q_c}{D_B + D_D} - \frac{q_w - q_0}{D_D} \quad (13)$$

Recalling Equation (3) providing an approximate value of  $q_w$  as a function of  $y_B - y_D$ , this equation would take the following form:

$$\frac{dy}{dt} = -\frac{q_{w0}(1 - \exp(-\alpha y)) - q_0 - q_c}{D_B + D_D} - \frac{q_{w0}(1 - \exp(-\alpha y)) - q_0}{D_D} \quad (14)$$

Which can be further expanded to the following equation:

$$\frac{dy}{dt} = q_{w0} \exp(-\alpha y) \left[ \frac{1}{D_B + D_D} + \frac{1}{D_D} \right] \frac{q_{w0} - q_0 - q_c}{D_B + D_D} - \frac{q_{w0} - q_0}{D_D} \quad (15)$$

Introducing new variables,  $\eta$ ,  $A$  and  $B$ , as:

$$\begin{cases} \eta = \frac{1}{D_B + D_D} + \frac{1}{D_D} \\ A = \eta q_{w0} \\ B = -\left( \frac{q_{w0} - q_0 - q_c}{D_B + D_D} + \frac{q_{w0} - q_0}{D_D} \right) \end{cases} \quad (16)$$

The last differential equation will be simplified as:

$$\frac{dy}{dt} = A e^{-\alpha y} + B \quad (17)$$

This equation can be further simplified as the following ordinary differential equation with separate variables:

$$\frac{dy}{1 + \frac{A}{B} e^{-\alpha y}} = B dt \quad (18)$$

Integrating both sides to find the solution of the equation yields (note that  $A$  and  $B$  are constant relative to the variables,  $y$  and  $t$ ):

$$\int \frac{1}{1 + \frac{A}{B} e^{-\alpha y}} dy = \int B dt \quad (19)$$

The left side has the following antiderivative:

$$\int \frac{1}{1 + \frac{A}{B} e^{-\alpha y}} dy = y + \frac{A}{\alpha B} \ln\left(1 + \frac{A}{B} e^{-\alpha y}\right) \quad (20a)$$

The right side has the following antiderivative:

$$\int B dt = Bt + C \quad (20b)$$

Equating these two equations and regarding the fact that the expression of  $y$  as a function of  $t$  is rather complicated, it is much easier to find the inversed function as follows:

$$t = \frac{1}{B} \left( y + \frac{A}{\alpha B} \ln\left(1 + \frac{A}{B} e^{-\alpha y}\right) - C \right) \quad (21)$$

In this equation, all constants have been defined which are represented here

$$\begin{cases} A = \eta q_{w0} \\ B = -\left( \frac{q_{w0} - q_0 - q_c}{D_B + D_D} + \frac{q_{w0} - q_0}{D_D} \right) \\ C = \text{Integration constant, determined by the initial condition} \\ \eta = \frac{1}{D_B + D_D} + \frac{1}{D_D} \end{cases} \quad (22)$$

It is remarkable that there are some equations for the cross-shore steady-state sediment transport, among them, the equation of Dean and Dalrymple (2004) defining a *simple cross-shore transport model*, can be addressed:

$$q_c = K(D - D_*) \quad (23)$$

In this equation,  $D$  is the Dean number, a dimensionless parameter defined as the ratio of the breaking wave height to the wave period times the sediment fall velocity,  $D_*$  is the Dean number for equilibrium profile and  $K$  is a dimensional constant (Dean and Dalrymple, 2004). As stated by Dean and Dalrymple (2004), if  $D$  is greater than the equilibrium value,  $D_*$ , it corresponds to a greater *turbulence* in the surfzone relative to the equilibrium condition and hence, the sediments would become unstable due to the destructive forces. As a consequence, a positive sediment transport would occur which means an offshore movement of the sediments. In



**Figure 2.** Location of the Zorabad fishing port in the Southern Iran (Iranian Ministry of Agriculture; Jahad, 2009).

contrast, an onshore sediment transport would occur once this term is smaller than that of an equilibrium condition. The value of  $D$  depends on the bottom slope with a great contribution and the water depth with less influence. It can be expressed by the following equation (Dean and Dalrymple, 2004):

$$D_* = \frac{5}{16} \rho g k^2 \sqrt{gh} \frac{dh}{dy} \quad (24a)$$

$$D = \frac{H_b}{\omega T} \quad (24b)$$

In these equations,  $k$  is the wave number,  $h$  is the water depth,  $H_b$  is the breaking water height,  $\rho$  is the sediments density,  $\omega$  is the sediment fall velocity,  $T$  is the wave period and other parameters are repetitive.

#### VERIFICATION BY A LOCAL CASE STUDY IN IRAN

Here, verification of the model which has been performed by predictions made for a beach profile change in a southern coast in Iran was specified. Unfortunately, there

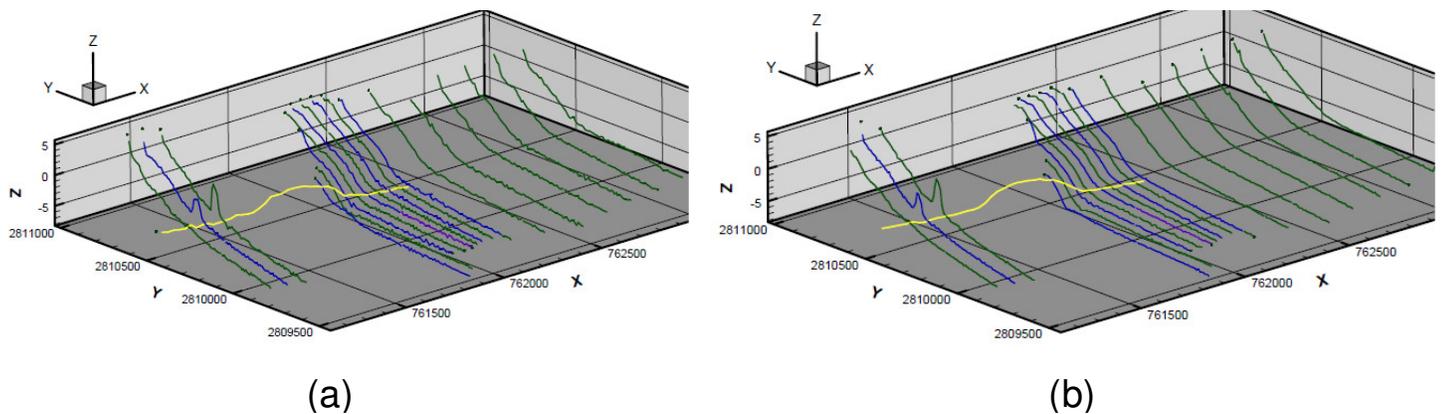
is a lack of data in which both onshore and offshore profiles are included and hence, verifications confronted some difficulties. Nevertheless, the results of a wide surveying on a fishing port in the Southern Iran, called Zorabad fishing port, have been employed for model verification. This port, in Sistan and Baluchestan province, is located in the northern coast of Oman Sea. A satellite image of this area is shown in Figure 2 (Iranian Ministry of Agriculture, Jahad, 2009).

Onshore and offshore surveys during 2006 and 2007 years were performed to investigate the sediment transport and morphological behavior of this coast for future developments plans. Some of important parameters related to the sediment and wave properties have been presented in Table 1. It should be noted that these values are averaged values (Iranian Ministry of Agriculture, Jahad, 2009).

Among a large number of data available for different points in this port, those in which surveying was made in relatively close intervals (intervals of one month) during a winter season were selected. Most of surveys were performed along a line which consists of a 500 m segment, seaward and a 500 m segment, landward, measured from the shoreline. It provides a relatively good and complete overview of the profile change both in the offshore and onshore zones. Zorabad fishing port coast

**Table 1.** Zarabad fishing port wave and sediment characteristics (Iranian Ministry of Agriculture, Jahad, 2009).

Parameter	Quantity	Remark
$T$	6-14 s	Wave period
$\omega$	0.01 m/s	Sediment fall velocity
$H_b$	Up to 3.5 m	Breaking wave height
$D_{50}$	0.2 mm	Mean grain size
$\rho$	1850 kg/m <sup>3</sup>	Sediment density
$e$	0.65	Mean void ratio of the sediments
$\phi$	31°	Sediments mean angle of repose (friction angle)
$(dh/dy)_{win.}$	2%	Average beach slope in winter profile

**Figure 3.** Results of surveying at different locations of Zarabad fishing port: (a) December 2006 and (b) April 2006 (Iranian Ministry of Agriculture, Jahad, 2009).

consists mainly of very fine sand and silt profile which can be easily transported by wind. Figure 3 shows the results of surveying and profile change in onshore and offshore zones of Zarabad fishing port in two different times.

Several parameters are required by the model for prediction of the beach profile change. Seven profiles have been selected and three of them were used for calibration (data1 to data3) whereas the rest of the profiles were employed to verify the model. Evolution of beach profile change is shown in Figure 4 for the three profiles employed in the model calibration phase. This figure shows the average profile evolution prediction by the model by average values of the parameters.

After the calibration had been performed, the model was utilized for prediction in beach profile change in comparison with the rest of data kept for verification phase. To do this, first the average parameters and then, two extrema of the model parameters (the maxima and the minima) have been used. Investigations show that almost all data points are located within a relatively narrow band resulted from any of these two assumptions. Also, average parameters show a rather acceptable

prediction for the beach evolutions in time regarding very complex nature of the beach profile. Figure 5 shows the predictions made by the model with the average, minimum and maximum values of the model parameters. Figure 6 show model predictions in a comparative manner applied to a quadruple set of data (data4 to data7) with the average values of model parameters. It is evident that using the average values provides a relatively good prediction of the beach evolution.

Winter storm surges and seaward sediment transport along with the seaward build-up of the dune cause a decrease in the distance between the berm crest and the toe of the dune. In other words, the general beach profile seems to show a seaward shift in the winter season.

## Conclusion

The main objective of this paper is to develop an analytical model to predict the evolution of the coastline changes for a variety of coasts. This evolution comprises the formation of the dune by wind transported fine sediment and erosion by storm surges accompanied by a

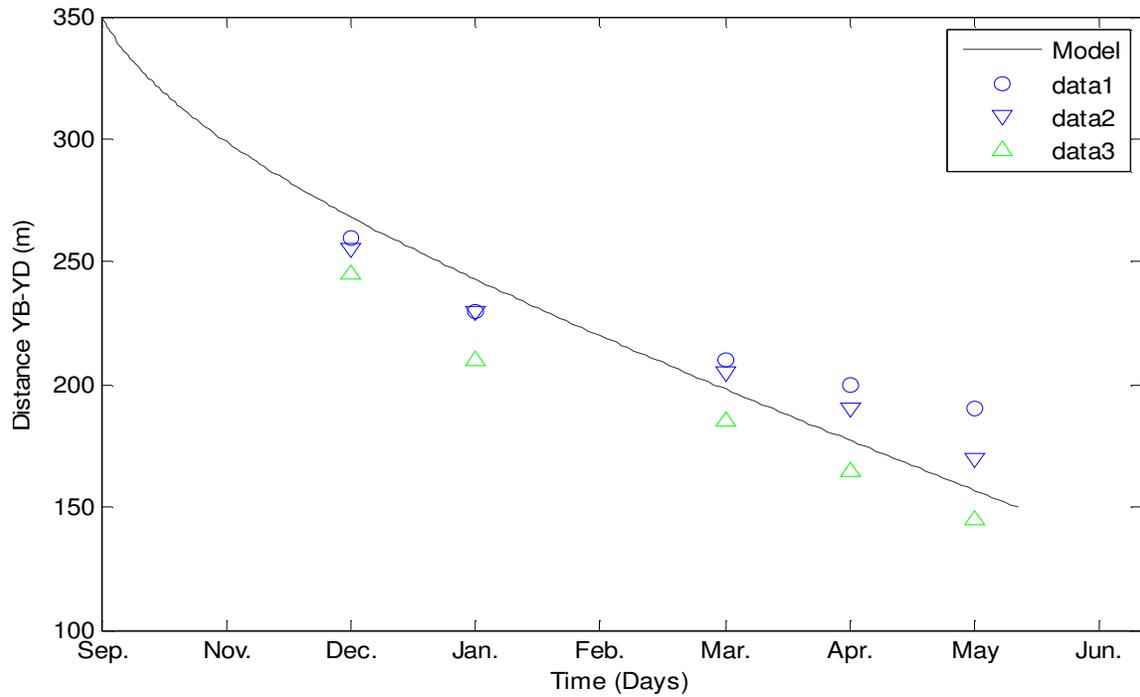


Figure 4. Model calibration with a triple set of data.

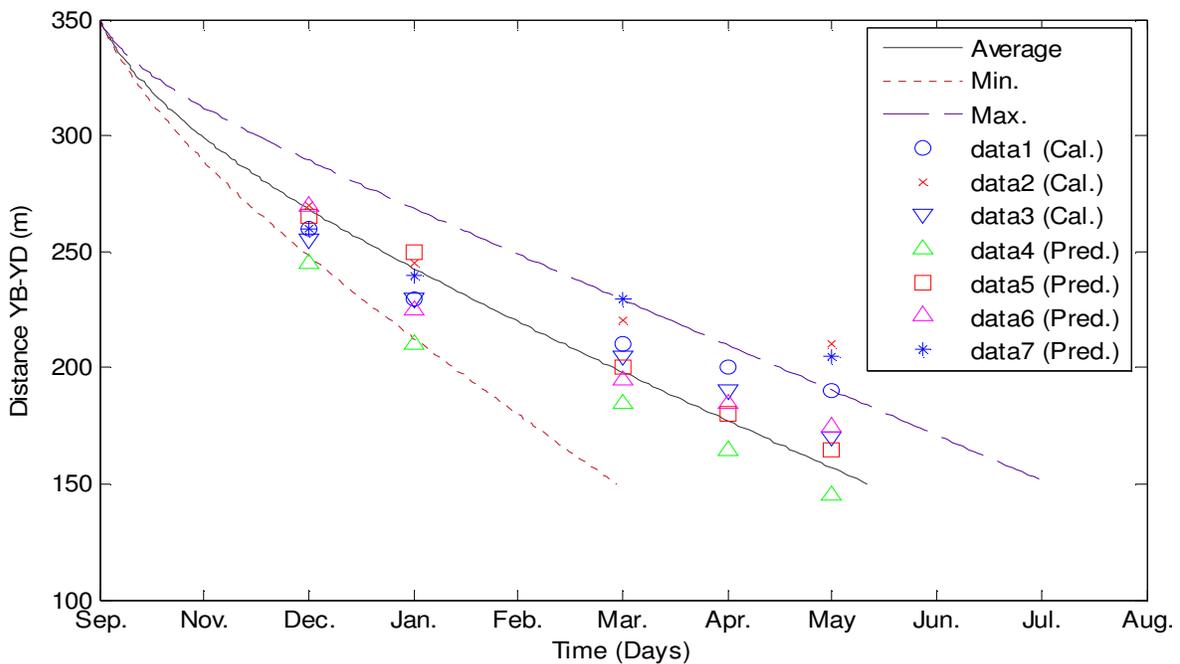
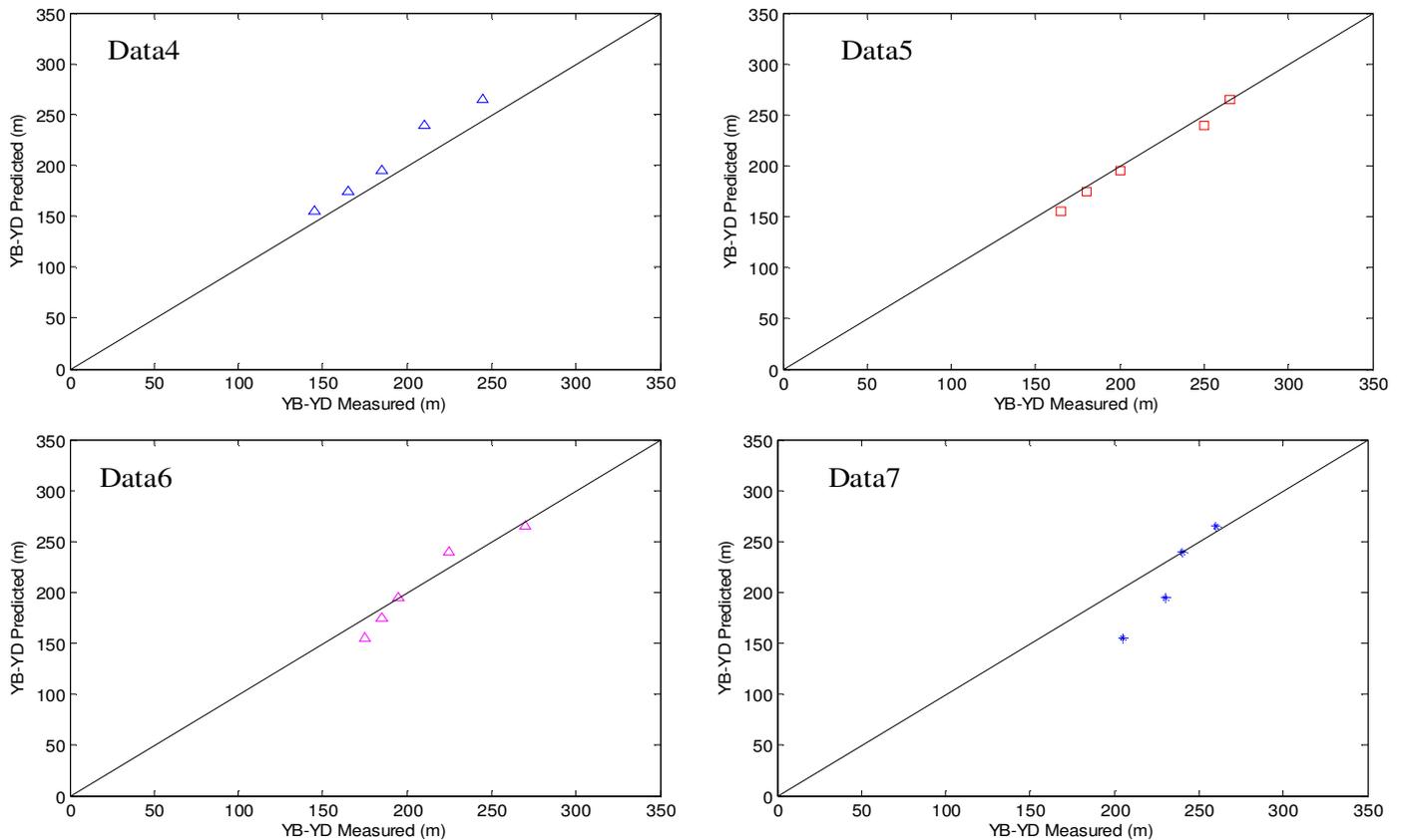


Figure 5. Measured and predicted dune-to-berm distance ( $Y_B - Y_D$ ) by the developed model based on average, minimum and maximum parameters obtained from model calibration.

cross-shore sediment transport which has been included as a new and complementary contributor to the assumptions made by Hanson et al. (2010). There are

several parameters involved in the coastline profile evolution based upon which, the model has been established. A differential equation was first developed to



**Figure 6.** Prediction and measured data for quadruple set of data based on average values of the model parameters.

relate the rate of dune formation by wind transported sediment, erosion of the dune by storm surges and seaward or landward cross-shore sediment transport. The governing differential equation was then solved analytically and an equation which describes the coastline profile change was derived. The equation comprises some terms which can be determined or estimated by some surveys at suitable intervals during a certain season within which, the cross-shore sediment transport is entirely seaward or entirely landward.

An average value of the sediment transport rate should be estimated to be used in the equation. If the sediment transport direction is gradually or suddenly changed, the results will not be valid. Because in a gradual change in direction, some derivatives of this sediment transport rate would be required to be included in the main statement of the differential equation and in the latter, the derivative of this rate with respect to time will approach infinity and the main structure of the differential equation would break. In fact, the equation is valid only for a specific period of time within which, this rate can be assumed to be constant. Although this assumption could be thought of a limitation to this equation, but it works reasonably in short and medium time intervals. Based on the assumptions made in derivation of equations, this model is only applicable to

sandy a beach; that is. beaches with fine sand obeying the governing equations presented in this paper.

This equation was finally utilized and applied to a case study located in southern Iran. After model calibration, predictions were made and showed reasonable results. Predictions were performed based on three types of selected parameters, that is, average values, minimum values and maximum values. A majority of data points measured by surveying are located within a narrow band obtained by the last two set of model parameters. Using the average values of the model parameters showed reasonable prediction regarding the complex nature of the coastline profile change in time. Since this research is limited to the sandy beaches, a more versatile research would be required in the future, to predict the other types of beaches morphodynamic profile.

## REFERENCES

- Davidson-Arnott RGD, Law MN, MacQuarrie K, Agaard T (2005). The Effect of Wind Gusts, Moisture Content and Fetch Length on Sand Transport on a Beach, *Geomorphol.*, 68:115-129.
- Dean RG (1977). *Equilibrium Beach Profiles*, U.S. Atlantic and Gulf Coasts, Ocean Eng. Report, No. 12, Uni. Delaware, Newark, U.S.A.
- Dean RG (1991). *Equilibrium Beach Profiles: Characteristics and Applications*, *J. Coastal Res.*, 7(1):53-84.

- Dean RG, Dalrymple RA (2004). Coastal Process with Engineering Applications, Cambridge Uni. Press.
- Hanson H, Larson M, Kraus NC (2010). Calculation of Beach Change under Interacting Cross-Shore and Long-Shore Processes, Coastal Eng., 57: 610-619.
- Hopf F, Sherman DJ (2007). Aeolian Processes, Coastal Dunes and the Coastal Engineering Manual Part III, Chapter 4: Wind-Blown Sediment Transport, Proc. ASCE Coastal Sediment 07:1241-1254.
- Iranian Ministry of Agriculture Jahad (2009). Sedimentation Analysis of Zarabad Fishing Port, Research Report Provided by Tehran-Berkeley Group of Companies, May 2009.
- Kriebel DL (1986). Verification Study of a Dune Erosion Model, Shore and Beach, 54 (3):13-21.
- Kriebel DL, Dean RG (1985) Numerical Simulation of Time-Dependent Beach and Dune Erosion, Coastal Eng., 9:221-245.
- Larson M, Erikson L, Hanson H (2004). An analytical Model to Predict Dune Erosion due to Wave Impact, Coastal Eng., 51: 675-696.
- Larson M, Kraus NC (1989). SBEACH: Numerical Model to Simulate Storm-Induced Beach Change, Army Corps. Of Eng., Waterway Expt. Station, Tech. Report CERC 89-9.
- Neshaei MAL, Mehrdad MAH, Veiskarami M (2009). The Effect of Beach Reflection on Undertow, Iranian J. Sci. and Tech. (IJST), Shiraz University, Trans B: Eng., 33 (B1):49-60.
- Nordstrom KF (2008). Beach and Dune Restoration, Cambridge Uni. Press.
- Sesli FA (2010). Mapping and monitoring temporal changes for coastline and coastal area by using aerial data images and digital photogrammetry: A case study from Samsun, Turkey, Int. J. Phy. Sci., (IJPS), 5(10):1567-1575.
- Short AD (1999). Handbook of Beach and Shoreface Morphodynamics, John-Wiley and Sons Ltd.