International Journal of the Physical Sciences Vol. 7(5), pp. 761 - 768, 30 January, 2012
Available online at http://www.academicjournals.org/IJPS
DOI: 10.5897/IJPS11.1342
ISSN 1992 - 1950 © 2012 Academic Journals

Full Length Research Paper

Study on three-dimensional flow of Maxwell fluid over a stretching surface with convective boundary conditions

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Accepted 15 December, 2011

Three-dimensional flow of non-Newtonian fluid induced by a stretching surface has been studied. The constitutive equations of Maxwell fluid are used. The surface possesses convective boundary conditions. Computations have been carried out for the non-linear problem. Convergence of the obtained solutions is discussed. Impact of the influential parameters involved in the heat transfer analysis is emphasized. Comparison with the previous results is shown. It is found that effects of Deborah and Biot parameters on the Nusselt number are opposite. The Prandtl and Biot numbers have qualitative similar impact on the Nusselt number.

Key words: Maxwell fluid, convective boundary condition, stretched surface, three-dimensional flow.

INTRODUCTION

Interest of researchers in the flows of non-Newtonian fluids is increased during the last few years. This is because of their several practical applications in industry and technology. Fluids belonging to this category include drilling muds, shampoo, ketchup, cement, sludge, grease, granular suspension, aqueous foams, slurries, paints, food products, paper pulp, plastics and several others. No doubt, the additional rheological parameters in the constitutive equations of such fluids present more complicated and higher order governing equations than the Navier-Stokes equations. To obtain analytical/ numerical solutions to such equations is not an easy task. Even then several scientists are making considerable efforts just to understand the flow characteristics of non-Newtonian fluids (Sapna; 2009; Yang and Zhu, 2010; Wang and Tan, 2011; Jamil and Fetecau, 2010; Nazar et al., 2010; Ziabakhsh et al., 2010; Hayat and Qasim, 2010; Ahmad and Asghar, 2011; Pakdemirli et al., 2011; Hayat et al., 2011; Rashidi et al., 2011; Iyengar and Vani, 2011) and many references therein.

The study of boundary layer flows generated by a stretching surface has key importance in several engineering processes. An example of stretching surface is a polymer sheet of filament extruding continuous from a die. The cooling of large metallic plate in a bath (which may be an electrolyte) is another example which belongs to this category. More, the occurrence of the stretched flows is obvious in paper production, glass blowing, melt spinning, wire drawing etc. The boundary layer flows of non-Newtonian fluids in the presence of heat transfer have relevance in food engineering, petroleum production, power engineering and in industrial processes including polymer melt and polymer solutions used in the plastic processing industries. Some recent studies on the topic can be seen in Labropulu et al. (2010), Sahoo (2011), Mustafa et al. (2010), Hayat et al. (2011), Rashidi et al. (2011), Hayat et al. (2011), Makinde and Aziz (2011), Sahoo and Poncet (2011), Abbas et al. (2010), Hsiao (2011) and Hayat et al. (2011). It is noted that most studies in the literature discussed the two-dimensional boundary layer flows when heat transfer characteristics are restricted to two boundary conditions of either prescribed temperatures or heat flux at the surface. Very recently, Aziz (2009) examined the Blasius flow subject to convective boundary condition. Few more attempts addressing this issue have been presented by Makinde

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and Aziz (2010), Merkin and Pop (2011), Yao et al. (2011) and Hayat et al. (2011). Having such in mind, the present study addresses the two focal points. First is to consider the three-dimensional flow over a stretching surface. Second generalization is concerned with the boundary condition when the bottom surface of sheet is heated by convection from a hot fluid (at temperature T_f) which provides a heat transfer coefficient h_f. The rheological equations of Maxwell fluid are considered. This model is able to predict the relaxation time effects. This paper is organized as follows: the relevant equations that consist of the solution of expressions by homotopy analysis method (HAM) (Liao, 2003; Vosughi et al., 2011; Hayat et al., 2011; Yao, 2009; Rashidi and Pour, 2010; Abbasbandy and Shirzadi, 2010; Hayat et al., 2010; Nadeem et al., 2010; Iqbal et al., 2011) and many references therein. The convergence of developed solutions and related discussion are given; a comparative study between the present and previous solution is also made.

GOVERNING PROBLEMS

We consider the steady three-dimensional flow of an incompressible fluid over a stretched surface at $z = 0$. The flow takes place in the domain $z > 0$. The ambient fluid temperature is taken as T_∞ while the surface temperature is maintained by convective heat transfer at a certain value T_f. The governing boundary layer equations for three-dimensional flow of Maxwell fluid are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}+\frac{\partial w}{\partial z}=0 \quad (1)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-\lambda\begin{pmatrix} u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+w^2\frac{\partial^2 u}{\partial z^2}+2uv\frac{\partial^2 u}{\partial x\partial y} \\ +2vw\frac{\partial^2 u}{\partial y\partial z}+2uw\frac{\partial^2 u}{\partial x\partial z}\end{pmatrix} \quad (2)$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+w\frac{\partial u}{\partial z}=\nu\frac{\partial^2 u}{\partial z^2}-\lambda\begin{pmatrix} u^2\frac{\partial^2 v}{\partial x^2}+v^2\frac{\partial^2 v}{\partial y^2}+w^2\frac{\partial^2 v}{\partial z^2}+2uv\frac{\partial^2 v}{\partial x\partial y}+ \\ 2vw\frac{\partial^2 v}{\partial y\partial z}+2uw\frac{\partial^2 v}{\partial x\partial z}\end{pmatrix} \quad (3)$$

$$\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+w\frac{\partial T}{\partial z}=\sigma\frac{\partial^2 T}{\partial z^2} \quad (4)$$

where the respective velocity components in the $x-$, $y-$ and $z-$ directions are denoted by u, v and w, λ shows the relaxation time, T, the fluid temperature; σ, the thermal diffusivity of the fluid; $\nu = (\mu/\rho)$, the kinematic viscosity; μ, the dynamic viscosity of fluid; ρ, the density of fluid.

The boundary conditions appropriate to the flow under consideration are:

$$u = ax,\ v = by,\ w = 0,\ -k\frac{\partial T}{\partial z} = h(T_f - T) \text{ at } z = 0, \quad (5)$$

$$u \to 0,\ v \to 0,\ T \to T_\infty \text{ as } z \to \infty, \quad (6)$$

where k indicates the thermal conductivity of fluid and a and b have dimension inverse of time.

Using the following new variables:

$$u = axf'(\eta),\ v = byg'(\eta),\ w = -\sqrt{a\nu}(f(\eta)+g(\eta)),\ \theta(\eta) = \frac{T-T_\infty}{T_f-T_\infty},\ \eta = z\sqrt{\frac{a}{\nu}}, \quad (7)$$

Equation 1 is satisfied automatically and Equations 2 to 7 give

$$f''' + (f+g)f'' - f'^2 + \beta^*[2(f+g)f'f'' - (f+g)^2 f'''] = 0, \quad (8)$$

$$g''' + (f+g)g'' - g'^2 + \beta^*[2(f+g)g'g'' - (f+g)^2 g'''] = \quad (9)$$

$$\theta'' + \Pr(f+g)\theta' = \quad (10)$$

$$f = 0,\ g = 0,\ f' = 1,\ g' = \beta, \theta' = -\gamma(1-\theta(0)) \text{ at } \eta = \quad (11)$$

$$f' \to 0,\ g' \to 0,\ \theta \to 0 \text{ as } \eta \to \propto \quad (12)$$

where $\beta^* = \lambda a$ is the Deborah number, $\beta = \frac{b}{a}$ is a parameter, $\Pr = \frac{\nu}{\sigma}$ is the Prandtl number, $\gamma = \frac{h}{k}\sqrt{\frac{\nu}{a}}$ is the Biot number and prime shows the differentiation with respect to η.

The expression for local Nusselt number with heat transfer q_w is

$$Nu_x = \frac{xq_w}{k(T_f - T_\infty)},\ q_w = -k\left(\frac{\partial T}{\partial z}\right)_{z=} \quad (13)$$

In dimensionless form, the above equation can be written as

$$Nu/\mathrm{Re}_x^{1/2} = -\theta'(0), \quad (14)$$

in which $\mathrm{Re}_x = ux/\nu$ is the local Reynolds number.

SERIES SOLUTIONS

The initial approximations and auxiliary linear operators for homotopy analysis solutions are chosen as

$$f_0(\eta)=\left(1-e^{-\eta}\right),\ g_0(\eta)=\beta\left(1-e^{-\eta}\right),\ \theta_0(\eta)=\frac{\gamma\exp(-\eta)}{1+\gamma} \qquad (15)$$

$$?_f=f'''-f',\ ?_g=g'''-g',\ ?_\theta=\theta''-\theta \qquad (16)$$

We note that the auxiliary linear operators in Equation 16 satisfy the following properties

$$?_f(C_1+C_2e^{\eta}+C_3e^{-\eta})=0,\ ?_g(C_4+C_5e^{\eta}+C_6e^{-\eta})=0,\ ?_\theta(C_7e^{\eta}+C_8e^{-\eta})=0 \qquad (17)$$

where $C_i\ \ (i=1-8)$ are the arbitrary constants.

The associated zeroth order deformation problems can be written as

$$(1-p)?_f\left[\hat f(\eta;p)-f_0(\eta)\right]=ph_f\mathbf{N}_f\left[\hat f(\eta;p),\hat g(\eta;p)\right] \qquad (18)$$

$$(1-p)?_g\left[\hat g(\eta;p)-g_0(\eta)\right]=ph_g\mathbf{N}_g\left[\hat f(\eta;p),\hat g(\eta;p)\right] \qquad (19)$$

$$(1-p)?_\theta\left[\hat\theta(\eta;p)-\theta_0(\eta)\right]=ph_\theta\mathbf{N}_\theta\left[\hat f(\eta;p),\hat g(\eta;p),\hat\theta(\eta,p)\right] \qquad (20)$$

$$\begin{aligned}&\hat f(0;p)=0,\ \hat f'(0;p)=1,\ \hat f'(\infty;p)=0,\ \hat g(0;p)=0,\ \hat g'(0;p)=\beta,\ \hat g'(\infty;p)=0\\&\hat\theta'(0,p)=-\gamma[1-\theta(0,p)],\ \hat\theta(\infty,p)=0,\end{aligned} \qquad (21)$$

$$\begin{aligned}\mathbf{N}_f[\hat f(\eta,p),\hat g(\eta,p)]=&\frac{\partial^3\hat f(\eta,p)}{\partial\eta^3}-\left(\frac{\partial\hat f(\eta,p)}{\partial\eta}\right)^2+(\hat f(\eta,p)+\hat g(\eta,p))\frac{\partial^2\hat f(\eta,p)}{\partial\eta^2}\\&+\beta^*\left[\begin{array}{c}2(\hat f(\eta,p)+\hat g(\eta,p))\frac{\partial\hat f(\eta,p)}{\partial\eta}\frac{\partial^2\hat f(\eta,p)}{\partial\eta^2}\\-(\hat f(\eta,p)+\hat g(\eta,p))^2\frac{\partial^3\hat f(\eta,p)}{\partial\eta^2}\end{array}\right],\end{aligned} \qquad (22)$$

$$\begin{aligned}\mathbf{N}_g[\hat g(\eta,p),\hat f(\eta,p)]=&\frac{\partial^3\hat g(\eta,p)}{\partial\eta^3}-\left(\frac{\partial\hat g(\eta,p)}{\partial\eta}\right)^2+(\hat f(\eta,p)+\hat g(\eta,p))\frac{\partial^2\hat g(\eta,p)}{\partial\eta^2}\\&+\beta^*\left[\begin{array}{c}2(\hat f(\eta,p)+\hat g(\eta,p))\frac{\partial\hat g(\eta,p)}{\partial\eta}\frac{\partial^2\hat g(\eta,p)}{\partial\eta^2}\\-(\hat f(\eta,p)+\hat g(\eta,p))^2\frac{\partial^3\hat g(\eta,p)}{\partial\eta^2}\end{array}\right],\end{aligned} \qquad (23)$$

$$\mathbf{N}_\theta[\hat\theta(\eta,p),\hat f(\eta,p),\hat g(\eta,p)]=\frac{\partial^2\hat\theta(\eta,p)}{\partial\eta^2}+\Pr(\hat f(\eta,p)+\hat g(\eta,p))\frac{\partial\hat\theta(\eta,p)}{\partial\eta} \qquad (24)$$

Here, p is an embedding parameter, h_f, h_g and h_θ are the non-zero auxiliary parameters and $\mathbf{N}_f$, $\mathbf{N}_g$ and $\mathbf{N}_\theta$ indicate the non-linear operators. For $p=0$ and $p=1$, we have

$$\hat f(\eta;0)=f_0(\eta),\ \hat\theta(\eta,0)=\theta_0(\eta)\text{ and }\hat f(\eta;1)=f(\eta),\ \hat\theta(\eta,1)=\theta(\eta). \qquad (25)$$

Further, when p increases from 0 to 1, then $f(\eta,p)$, $g(\eta,p)$ and $\theta(\eta,p)$ vary from $f_0(\eta)$, $g_0(\eta)$, $\theta_0(\eta)$ to $f(\eta)$, $g(\eta)$ and $\theta(\eta)$. Using Taylor's expansion one can write

$$f(\eta,p)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta)p^m,\ f_m(\eta)=\frac{1}{m!}\frac{\partial^m f(\eta;p)}{\partial\eta^m}\bigg|_{p=0}, \qquad (26)$$

$$g(\eta,p)=g_0(\eta)+\sum_{m=1}^{\infty}g_m(\eta)p^m,\ g_m(\eta)=\frac{1}{m!}\frac{\partial^m g(\eta;p)}{\partial\eta^m}\bigg|_{p=0} \qquad (27)$$

$$\theta(\eta,p)=\theta_0(\eta)\sum_{m=1}^{\infty}\theta_m(\eta)p^m,\ \theta_m(\eta)=\frac{1}{m!}\frac{\partial^m\theta(\eta;p)}{\partial\eta^m}\bigg|_{p=0} \qquad (28)$$

where the convergence of above series strongly depends upon h_f, h_g and h_θ. Considering that h_f, h_g and h_θ are selected properly so that Equations 26 to 28 converge at $p=1$ therefore,

$$f(\eta)=f_0(\eta)+\sum_{m=1}^{\infty}f_m(\eta) \qquad (29)$$

$$g(\eta)=g_0(\eta)+\sum_{m=1}^{\infty}g_m(\eta), \qquad (30)$$

$$\theta(\eta)=\theta_0(\eta)+\sum_{m=1}^{\infty}\theta_m(\eta) \qquad (31)$$

The mth order deformation problems are

$$?_f[f_m(\eta)-\chi_m f_{m-1}(\eta)]=h_f\mathbf{R}_f^m(\eta) \qquad (32)$$

$$?_g[g_m(\eta)-\chi_m g_{m-1}(\eta)]=h_g\mathbf{R}_g^m(\eta), \qquad (33)$$

$$?_\theta[\theta_m(\eta)-\chi_m\theta_{m-1}(\eta)]=h_\theta\mathbf{R}_\theta^m(\eta), \qquad (34)$$

$$f_m(0) = f_m'(0) = f_m'(\infty) = 0,\ g_m(0) = g_m'(0) = g_m'(\infty) = 0,\ \theta_m'(0) - \gamma\theta_m(0) = \theta_m(\infty) = 0 \quad (35)$$

$$\begin{aligned}\mathbf{R}_f^m(\eta) = {} & f_{m-1}'''(\eta) - \sum_{k=0}^{m-1} f_{m-1-k}' f_k' + \sum_{k=0}^{m-1}(f_{m-1-k} f_k'' + g_{m-1-k} f_k'') \\ & + \beta \sum_{k=0}^{m-1}\sum_{l=0}^{k}[2(f_{m-1-k} + g_{m-1-k}) f_{k-l}' f_l'' \\ & -(f_{m-1-k} f_{k-l} + g_{m-1-k} g_{k-l} + 2 f_{m-1-k} g_{k-l}) f_l'''],\end{aligned} \quad (36)$$

$$\begin{aligned}\mathbf{R}_g^m(\eta) = {} & g_{m-1}'''(\eta) - \sum_{k=0}^{m-1} g_{m-1-k}' g_k' + \sum_{k=0}^{m-1}(f_{m-1-k} g_k'' + g_{m-1-k} g_k'') \\ & + \beta \sum_{k=0}^{m-1}\sum_{l=0}^{k}[2(f_{m-1-k} + g_{m-1-k}) g_{k-l}' g_l'' \\ & -(f_{m-1-k} f_{k-l} + g_{m-1-k} g_{k-l} + 2 f_{m-1-k} g_{k-l}) g_l'''],\end{aligned} \quad (37)$$

$$\mathbf{R}_\theta^m(\eta) = \theta_{m-1}'' + \Pr \sum_{k=0}^{m-1}(\theta_{m-1-k}' f_k + \theta_{m-1-k}' g_k), \quad (38)$$

$$\chi_m = \begin{cases} 0, & m \le 1 \end{cases} \quad (39)$$

Solving the corresponding mth order deformation problems we have,

$$f_m(\eta) = f_m^*(\eta) + C_1 + C_2 e^{\eta} + C_3 e^{-\eta} \quad (40)$$

$$g_m(\eta) = g_m^*(\eta) + C_4 + C_5 e^{\eta} + C_6 e^{-\eta} \quad (41)$$

$$\theta_m(\eta) = \theta_m^*(\eta) + C_7 e^{\eta} + C_8 e^{-\eta} \quad (42)$$

in which the f_m^*, g_m^* and θ_m^* indicate the special solutions.

CONVERGENCE ANALYSIS AND DISCUSSION

Obviously, the Equations 29 to 31 consist of the auxiliary parameters $\hbar_f$, $\hbar_g$ and $\hbar_\theta$. These parameters have a key role to adjust and control the convergence of homotopic solutions. The $\hbar-$ curves have been sketched at 18^{th} order of approximations to determine the suitable ranges for $\hbar_f$, $\hbar_g$ and $\hbar_\theta$. From Figures 1 to 3, it is noted that the range of admissible values of $\hbar_f$, $\hbar_g$ and $\hbar_\theta$ are $-1.30 \le \hbar_f \le -0.30,$ $-1.30 \le \hbar_g \le -0.25$ and $-1.40 \le \hbar_\theta \le -0.45.$ We observed that (Table 1) our series solutions converge in the whole region of ζ when $\hbar_f = \hbar_g = -0.90$ and $\hbar_\theta = -1.00.$

Figures 4 to 12 show the behaviors of Deborah number β^*, Prandtl number $\Pr$ and Biot number ξ on temperature $\theta(\eta)$ for different cases when $\beta = 0.0,$ 0.5 and 1.0. Variations of $\Pr$, β^* and ξ are shown in Figures 4 to 6 when $\beta = 0.$ It is seen that an increase in Prandtl number $\Pr$ shows a decrease in the temperature of fluid and the thermal boundary layer thickness (Figure 4). Physically, an increase in Prandtl number leads to an increase in thermal diffusivity due to which the temperature and thermal boundary layer thickness decrease. Figures 5 and 6 show the variations of Deborah and Biot numbers. We conclude from these Figures 5 and 6 that both the temperature profile and thermal boundary layer thickness increase when Deborah and Biot number increase. The Deborah number involves the relaxation time due to which the temperature at the wall increases. The wall temperature increase with the increase of Biot number and it is expected that the convective boundary condition becomes the prescribed wall temperature when Biot number goes to infinity. It is also noted that the fluid temperature is zero when the Biot number is zero. The effects of $\Pr$, β^* and γ on temperature are displayed in the Figures 7 to 9 for $\beta = 0.5.$ The plotted Figures 7 to 9 show that results for $\beta = 0$ and $\beta = 0.5$ are similar in a qualitative sense.

The only change here that we noted is in the variation of β^*. This can be seen by comparing Figures 5 and 8. The variation in temperature for the case $\beta = 0.5$ is a bit smaller than $\beta = 0.$ Similar observations are noted in Figures 10 to 12. Table 1 presents the convergence of homotpic solutions. From this Table 1, it is concluded that we need 20^{th} terms for velocity and 25^{th} order iterations for the temperature for a convergent series solutions. Table 2 is prepared for the comparison between HAM results and previous existing results in a limiting case for various values of $\beta.$ Also Table 2 shows that the skin-friction coefficient in viscous fluid increases when stretching parameter is increased. One can see that our homotopic results have an excellent agreement with the exact and homotopy perturbation (HPM) results in a viscous fluid. Numerical values of local Nusselt number are analyzed in Table 3. The values of $-\theta'(0)$ decreases by increasing Deborah number and it also increases by increasing Prandtl number and Biot number. It is further observed that Nusselt number is an increasing function of the stretching parameter $\beta.$ In summary, the effect of stretching parameter and Prandtl number are qualitatively similar. More, the Deborah number has opposite effect on the Nusselt number when compared with stretching parameter and Prandtl number.

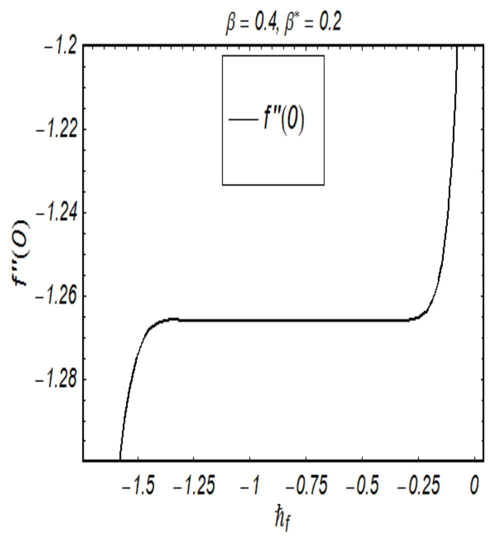

Figure 1. $\hbar$-curve for the function f.

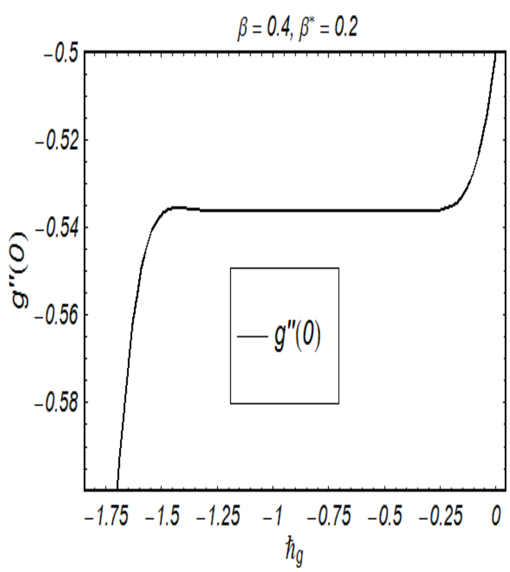

Figure 2. $\hbar$-curve for the function g.

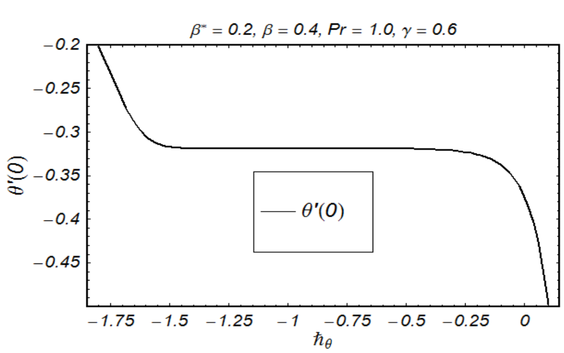

Figure 3. $\hbar$-curve for the function θ.

Table 1. Convergence of series solutions for different order of approximations when $\beta^* = 0.4$, $\beta = 0.5$, $\Pr = 1.0$, $\gamma = 0.6$, $\hbar_f = \hbar_g = -0.9$ and $\hbar_\theta = -1.0$.

Order of approximations	-f''(0)	-g''(0)	-θ'(0)
1	1.232500	0.518750	0.339844
10	1.266203	0.536286	0.318905
15	1.266214	0.536300	0.318769
20	1.22615	0.536301	0.318754
25	1.22615	0.536301	0.318752
30	1.22615	0.536301	0.318752
35	1.22615	0.536301	0.318752

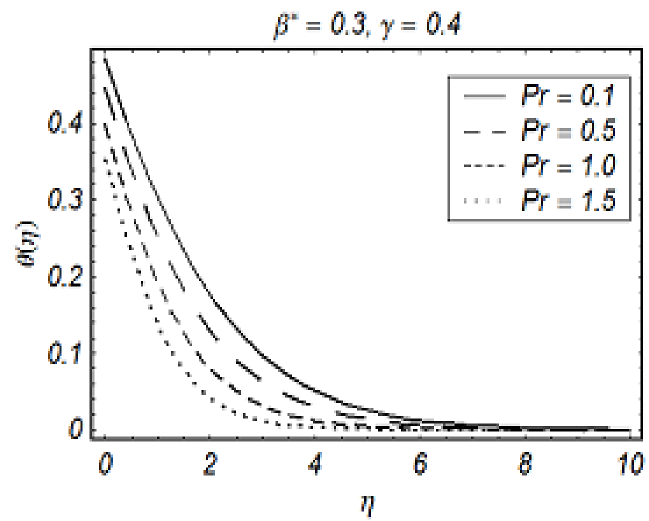

Figure 4. Influence of Pr on θ(η) when β = 0.0.

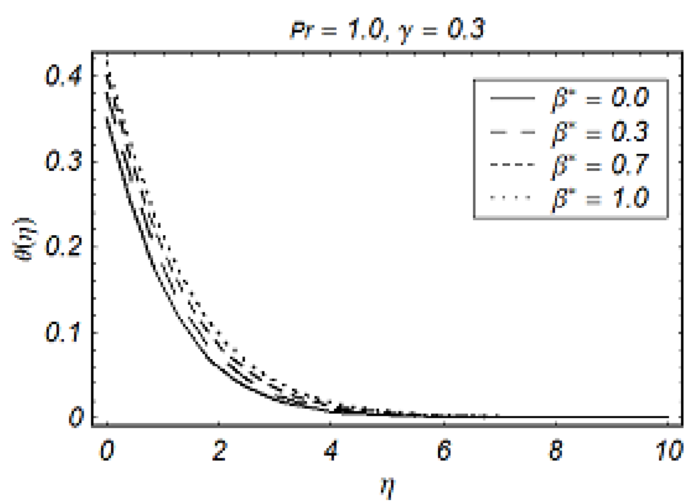

Figure 5. Influence of β^* on θ(η) when β = 0.0.

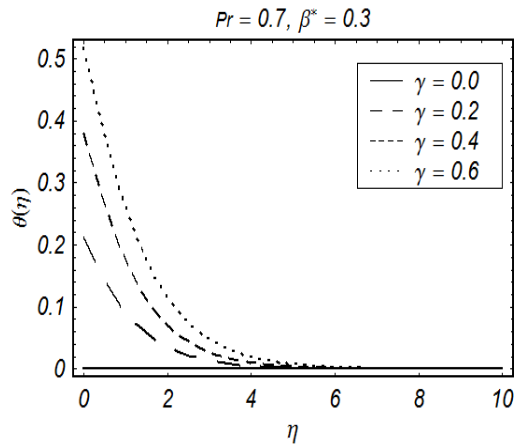

Figure 6. Influence of γ on θ(η) when β = 0.0.

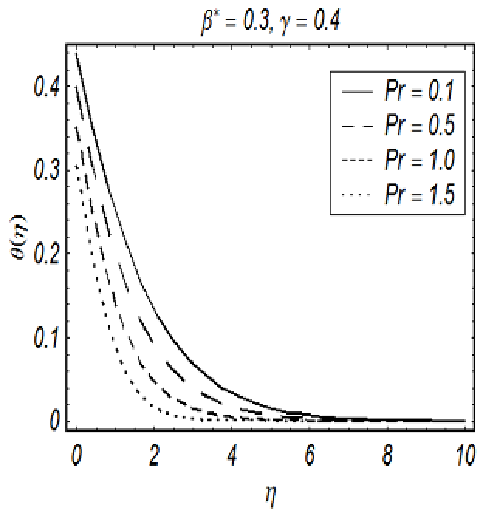

Figure 7. Influence of Pr on θ(η) when β = 0.5.

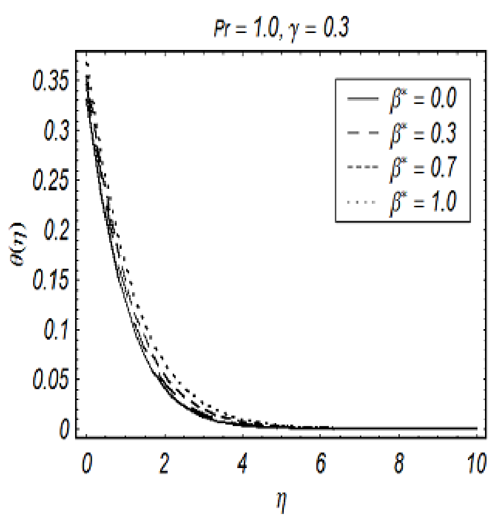

Figure 8. Influence of β^* on θ(η) when β = 0.5.

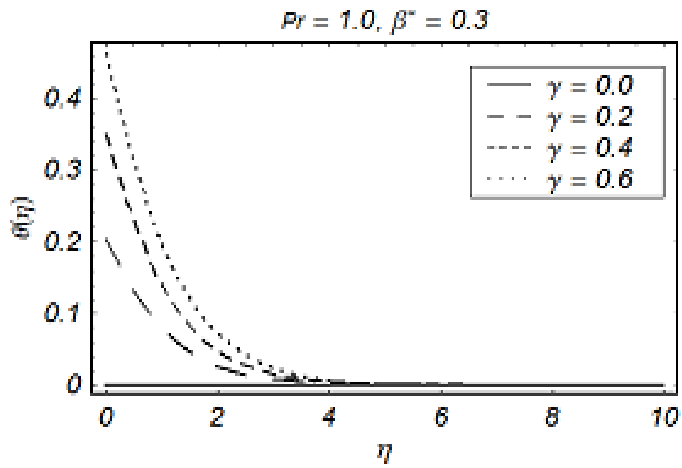

Figure 9. Influence of γ on θ(η) when β = 0.5.

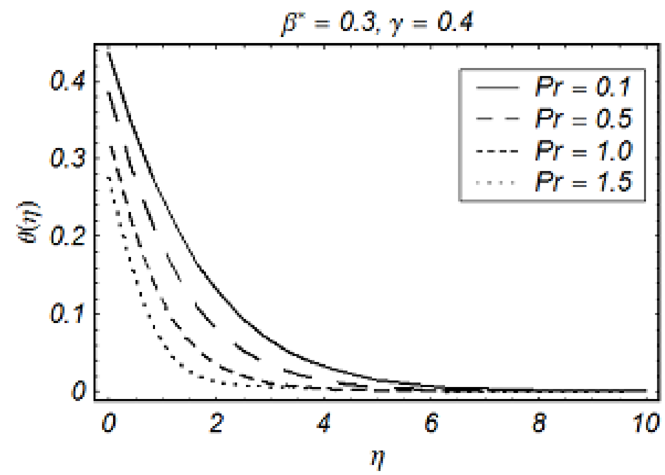

Figure 10. Influence of Pr on θ(η) when β = 1.0.

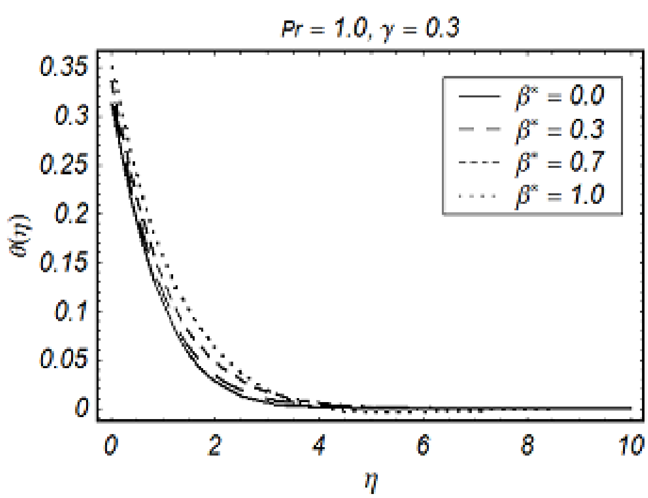

Figure 11. Influence of β^* on θ(η) when β = 1.0.

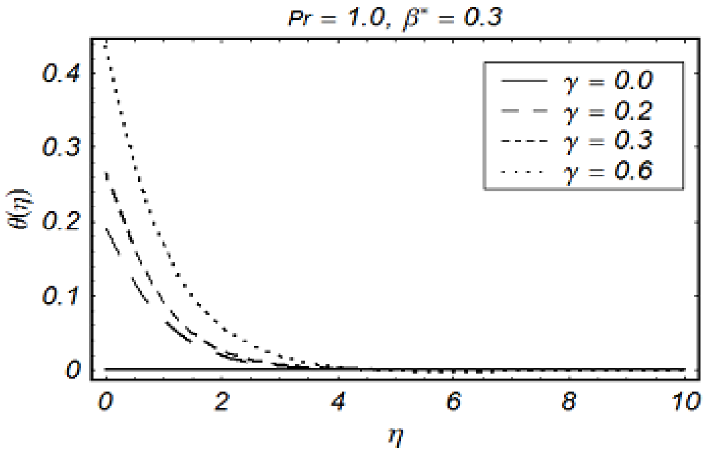

Figure 12. Influence of γ on θ(η) when β = 1.0.

Table 2. Comparison for the different values of β by HAM, HPM and exact solutions with Ariel (2007).

β	Ariel (2007)		Ariel (2007)		HAM	
	-f″(0)	-g″(0)	-f″(0)	-g″(0)	-f″(0)	-g″(0)
0.0	1.0	0.0	1	0	1.0	0.0
0.1	1.02025	0.06684	1.020259	0.066847	1.020260	0.066847
0.2	1.03949	0.14873	1.039495	0.148736	1.039495	0.148737
0.3	1.05795	0.24335	1.057954	0.243359	1.057955	0.243360
0.4	1.07578	0.34920	1.075788	0.349208	1.075788	0.349209
0.5	1.09309	0.46520	1.093095	0.465204	1.093095	0.465205
0.6	1.10994	0.59052	1.109946	0.590528	1.109942	0.590529
0.7	1.12639	0.72453	1.126397	0.724531	1.126398	0.724532
0.8	1.14248	0.86668	1.142488	0.866682	1.142489	0.866683
0.9	1.15825	1.01653	1.158253	1.016538	1.158254	1.016539
1.0	1.17372	1.17372	1.173720	1.173720	1.173721	1.173721

Table 3. Values of local Nusselt number $-\theta'(0)$ for the different values of the parameters β^*, β and Pr when $\gamma = 0.6$.

β*	β	Pr	-θ′(0)
0.0	0.5	1.0	0.33040
0.3			0.32160
0.8			0.30799
1.2			0.29873
0.4	0.0		0.28908
	0.4		0.31664
	0.7		0.33017
	1.0		0.34070
		0.7	0.28279
		1.2	0.34042
		1.6	0.36840
		2.0	0.38887

ACKNOWLEDGEMENT

The research of Dr. Alsaedi was partially supported by Deanship of Scientific Research (DSR), King Abdulaziz University, Jeddah, Saudi Arabia.

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