Full Length Research Paper

A novel electrical model to an antenna array

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In this paper, an electrical model is developed to represent the input admittance of an antenna array with a finite number of elements. This model consists of an RLC bock component to represent the input admittance of each elementary antenna element and a capacity component to represent different degrees of antenna coupling effects. The equations based on cavity model are developed to represent physical meaning of each model. Numerical results show that good accuracy for the simulation results can be obtained by using this electrical model to the results obtained by using HFSS. As the array is large and sparse, a very small amount of computation can yield good accuracy. This model is shown not only to be numerically efficient compared to the full wave analysis using the moment method, but also to give physical insight into the antenna array mutual coupling mechanism. Furthermore, this model has no limitation on antenna array geometry and excitation.

Key words: Antenna array, electrical model, coupling capacity.

INTRODUCTION

Recently, there has been a growing interest in creating low-cost, high-performance and high network capacity cellular systems by using an antenna array in the place of a single antenna (Vasquez et al., 2009).

Patch array is useful in microwave application to obtain radiation pattern which present a high directivity. Usually, this kind of antenna is used in application where we search to increasing directivity without having a limitation in the dimension offered to the antenna.

Generally, the array factor is calculated with elements independent for each other. In real life, the coupling between elements can lower the array's performances (Krusevac et al., 2006; Fu et al., 2006; Jarchi et al., 2007; Farkasvolgyi and Nagy, 2007; Fallahi and Roshandel, 2007). It is necessary to take account the coupling between elements to design accurately the array. Electrical coupling is difficult to evaluate, for this reason previous works are concentrated to analyze the array by using the numerical methods (Yuan et al., 2007; Liu et al., 2006; Qiu et al., 2006; Uduwawala et al., 2005; Losito, 2007). From the electromagnetic results given by numerical methods they extract the electrical model in Persson et al. (2003), use the theory of diffraction combining with numerical algorithm in order to calculate the mutual coupling between circular apertures on a doubly curved surface. In (Ert"urk and Rojas (2003), a novel method is developed by the author's by combining the method of moments and the green's function to analyze antenna array.

In our research we propose a technique to calculate the RLC parameters of the array without necessary need of the numerical results. In the proposed model, we introduce the mutual coupling in the case of a planar rectangular array. In order to build our model, we began by analyzing the elementary rectangular patch, then we used two elements, finally we generalized the technique in order to build an electrical model of a 2*2 array. The introduced model is very important because in one hand it helps reducing the time of simulation. In the other hand, the proposed model can help us to display the design of an array taking in consideration all kind of coupling between different elements.

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Figure 1. Electrical model of a planar antenna (a) Parallel configuration (b) Serie parallel.



Figure 2. Rectangular patch.

In order to evaluate our model, we compared the results given by the electrical model and those obtained by using HFSS simulator which is based on FEM method (ANSYS, 2009).

This paper is organized as follow: A brief introduction on the importance of using electrical model. A development of the advantages of using electrical model. An electrical model of a rectangular patch is given. A new approach in building an electrical model for a two planar array is developed. Finally a brief conclusion and a future work consists the fifth section.

ELECTRICAL MODEL

In order to design antenna structures, many techniques are developed. Those techniques use numerical methods based on resolving electromagnetic equations in different form. For this reason, much software was used like ADS simulator, HFSS simulator, and IE3D simulator (Monti et al., 2009). Those entire Simulators give their results by resolving electromagnetic equations in their integral or differential forms (Balanis, 1997). But those techniques have some limitation. First, we cannot take the calculation of all kinds of losses. Besides after simulating the structures, we have not the possibility to control antenna parameters like return loss, input impedance when modifying geometry of antenna, nature of substrate. That is why, replacing an antenna by an equivalent circuit will be very important in parametric analyzing of the proposed structure (Anguera et al., 2004). In this way, the model techniques have a great interest. One of those solutions is modeling an antenna by an electrical model.

Building an electrical model means replacing it by an RLC circuit, Figure 1. There were two configurations: a parallel or a series resonant model. The RLC parameters are calculating the equation developed in (Nasimuddin and Verma, 2004).

As demonstrated in previous work Ferchichi et al. (2010), the electrical model is used to know what occurred to the input impedance, the return losses when modifying the geometry of the antenna, the nature of substrate and the excitation (Celal and Kerim, 1998). In this way, analyzing an array of antenna will be very interesting.

The main idea is to replace each antenna elements by its equivalent RLC circuits, and then we must introduce the mutual coupling between different elements.

ELECTRICAL MODEL OF SINGLE RECTANGULAR PATCH

Geometry of patch

Figure 2 shows the geometry of a rectangular patch on a dielectric substrate with a ground plane. The patch is characterized by the resonant length L and the width W.

The antenna is placed on an h= 3.2 mm of NeltecNY9260 (IM) substrate material, which has a dielectric constant $\mathcal{E}_r = 2.6$, and loss tangent (tang δ = 0.002). The structure is excited by a cable coaxial.

Electrical model

To build the electrical model of the rectangular patch we use the same method developed in our previous work (Ferchichi et al., 2009). The model is based on the input impedance Z_{in} of a rectangular patch excited by a coax. Z_{in} is given in Nasimuddin and Verma (2004) by the equation (1):

$$Z_{in} = \mathbf{R} + \mathbf{j} \mathbf{X}$$

$$Z_{in} (\mathbf{f}) = \frac{\mathbf{R}}{1 + \mathbf{Q}_{\mathrm{T}}^{2} \left[\frac{f}{f_{r}} - \frac{f_{r}}{f}\right]^{2}} + \mathbf{j} \left[\mathbf{X}_{\mathrm{L}} - \frac{\mathbf{R}\mathbf{Q}_{\mathrm{T}} \left[\frac{f}{f_{r}} - \frac{f_{r}}{f}\right]}{1 + \mathbf{Q}_{\mathrm{T}}^{2} \left[\frac{f}{f_{r}} - \frac{f_{r}}{f}\right]^{2}}\right]$$
(1)



Figure 3. An electrical model of rectangular patch.



Figure 4. Return loss of rectangular patch.

Then we determine the resonant resistance R, the dynamic capacity Cdyn, the inductance L, the quality factor QT and the reactance X_L , (Nasimuddin and Verma, 2004). The antenna will be modeled by the electrical model presented in Figure 3.

Return loss results

The model proposed with the values of R, L, C and X_{L} calculated is simulated in order to evaluate S11 parameter. The results obtained by the model are in good agreement with those obtained by using HFSS which is based on a numerical method. Besides, the model is simulated immediately which take a short time comparing to the time simulation of the HFSS simulator which is about few minutes, Figure 4.

ARRAY OF TWO ANTENNA

Geometry of patch array

At the beginning, we analyzed a planar array that consisted of two elements. The elements of patches are a rectangular patch with the



Metal ground plane

Figure 5. A two element Array of patch.



Figure 6. Electrical model of the proposed array.

same dimension and substrate characteristic of the single patch simulated in the first Figure 5.

Electrical model

The proposed electrical model is built by using two RLC block that represents the equivalent of each antenna element, the two blocks are connected by a single capacitance due to the coupling between the two patches, Figure 6.

Resonant resistance

The resonant resistance R is related essentially to the losses in the conductor, substrate and the radiation quality factor. In the case of an array of patch we use the equation as follows:

$$R = \frac{Q_{\rm T} H}{\pi f_r \ \varepsilon_{\rm dyn} \varepsilon_0 \ \rm WL} \ \cos^2\left(\frac{\pi \ x_0}{\rm L}\right) \tag{2}$$

 Q_T is the quality factor, it can be calculated by using the formulas developed in Nasimuddin and Verma, (2004), f_r is the resonant frequency, H is the dielectric thickness, W and L are the geometry dimensions of the single patch and x_0 is the distance between the excitation and the side edge.

As for \mathcal{E}_{dyn} , it represents the dynamic permittivity of the antenna array, it can be obtained from the equation as follows:



Figure 7. Dynamic capacitance of the two patch.

Figure 8. Coupling capacitance of the two patch.

$$\varepsilon_{\rm dyn} = \frac{C_{\rm dyn}(\varepsilon)}{C_{\rm dyn}(\varepsilon_0)}$$
(3)

The dynamic capacitance

As can be seen in Figure 7, the dynamic capacitance can be divided into three capacitances:

$$C_{dyn} = C_p + C_f + C_{f'} \tag{4}$$

 C_p denotes the parallel capacitance between the element patch and the ground plane; C_f is the fringe capacitance at the outer edge of the patch and C_f the fringe capacitance of a side due to the presence of the other Bahl (2003), Figure 7.

Parallel capacitance

$$C_{p} = \frac{e_{0} e_{r} W L}{H g_{n} g_{m}}$$
(5)

Fringe edge capacitance of each side C_{f}

The fringe capacitance C_f in the rectangular patch can be divided in two parts. Each part is related to a side:

$$C_{f} = C_{f_{1}} + C_{f_{2}}$$
(6)

Where:

 C_{f1} represents the edge capacitance on one side of a patch length L, it can be calculated using the equation as follows:

$$C_{f1}(\varepsilon) = \frac{1}{2\gamma_n} \left(\frac{\varepsilon_{nff}(\varepsilon_r, H, W)}{c_0 Z(\varepsilon_r = 1, H, W)} - \frac{\varepsilon_0 \varepsilon_r W}{H} \right) L$$
(7)

 C_{f2} represents the edge capacitance on one side of a patch length W, it can be calculated using the equation as follows:

$$C_{f2}(\varepsilon) = \frac{1}{2\gamma_m} \left(\frac{\varepsilon_{reff}(\varepsilon_r, H, L)}{c_0 Z(\varepsilon_r = 1, H, L)} - \frac{\varepsilon_0 \varepsilon_r L}{H} \right) W$$
(8)

Fringe edge capacitance of side due to the presence of other $C_{f^{^{\prime}}}$

In the case of two elements patch, there is two sides where $C_{f^{\prime}}$ exists. Its expression can be

calculated via the expression of C_f as can be seen in equation as follows:

(9)

$$C_{f'=\frac{C_{f'}}{1+A\left(\frac{H}{A}\right)\tanh\left(\frac{10a}{H}\right)}\left(\frac{\varepsilon_r}{\varepsilon_{reff}}\right)^{1/4}}$$

Where

$$A = \exp\left[-0.1 \exp\left(2.33 - 1.5 \frac{W}{H}\right)\right]$$
(10)

Coupling capacitance

The coupling capacitance is derived from the odd mode capacitance Bahl (2003), Figure 8. It can be calculated using the equation as follows:

$$C_c = C_{ga} + C_{gd} \tag{11}$$

The gap capacitance in air

It describes the gap capacitance in the air. Its value can be calculated using the equation as follows Bahl (2003):

$$C_{ga} = \varepsilon_0 \frac{K(k')}{K(k)}$$
(12)

Where K(k) and K(k') are the elliptic function and its complement,

$$k = \frac{S}{S + 2W} \tag{13}$$

While S is the distance between the two elements of patches

and
$$k' = \sqrt{1-k^2}$$
 (14)

Finally, we can write the gap capacitance as follows:

$$C_{ga} = \begin{cases} \frac{\varepsilon_0}{\pi} \ln \left\{ 2 \frac{1 + \sqrt{k'}}{1 - \sqrt{k'}} \right\} & \text{for } 0 \le k^2 \le 0.5 \\ \\ \frac{\pi \varepsilon_0}{\ln \left\{ 2 \frac{1 + \sqrt{k}}{1 - \sqrt{k}} \right\}} & \text{for } 0.5 \le k^2 \le 1 \end{cases}$$

$$(15)$$

The gap capacitance in dielectric

 $C_{\rm gd}$ Represents the capacitance value due to the electric flux in the dielectric region, it can be calculated using the equation as follows, Ferchichi et al. (2010):

$$C_{gd} = \frac{\varepsilon_0 \varepsilon_r}{\pi} \ln \coth\left(\frac{\pi S}{4h}\right) + 0.65 C_f \left\{\frac{0.02}{\frac{S}{h}} \sqrt{\varepsilon_r} + \left(1 - \frac{1}{\varepsilon_r^2}\right)\right\}$$
(16)

Return loss

Basing on the equations developed in the previous paragraph, we have calculated the values of: R, L, C and X_L . Then when we compare simulation results obtained by the model and using HFSS, we can say that S11 are similar which prove that our model is accurate. As for S12, there is a little difference between the two results due to variation among mutual coupling calculating by the two methods, Figure 9. Besides, there is a gain in time simulation.

ARRAY OF ANTENNA: 2*2

Geometry of patch

The second application of the developed model is a bidirectional array of antenna. In this case, we use a four rectangular antenna. Each element has the same dimension and substrate as proposed single patch, Figure 10.

Electrical model

In this case the electrical model can be constructed using the same technique. First we must determine the dynamic capacitance of each element in order to calculate the dynamic permittivity and the resonant resistance. Then we calculate the coupling capacitance between the different elements. The proposed model is shown in Figure 11.

The dynamic capacitance

The introduction of two other elements modify the dynamic capacitance.

$$C_{dyn} = C_{p} + C_{f} + C_{f'}$$
 (17)

Where Cp and Cf still unchangeable but Cf' will change because each element have two sides in contact with two other side. Thus the fringe capacitance of side due to the presence of the presence of the other will be divided into two parts.

$$C_{f'} = C_{f_1} + C_{f_2} \tag{18}$$

Where $C_{f_1^{\cdot}} \, \, \operatorname{C}_{f_2^{\cdot}}$ can be calculated using the equation :

$$C_{f_1'} = \frac{C_{f_1'}}{1 + A\left(\frac{H}{A}\right) \tanh\left(\frac{10a}{H}\right)} \left(\frac{\varepsilon_r}{\varepsilon_{reff}}\right)^{1/4}$$
(19)

$$C_{f_2'} = \frac{C_{f_2'}}{1 + A\left(\frac{H}{A}\right) \tanh\left(\frac{10a}{H}\right)} \left(\frac{\varepsilon_r}{\varepsilon_{reff}}\right)^{1/4}$$
(20)

After calculating the dynamic capacitance, we can dedicate the dynamic permittivity $\mathcal{E}_{\rm dyn}$ by applying the Equation 3.

Coupling capacity

In this case, we have four elementary patches so there is a coupling between all those elements. As can be seen in the Figure 11 of model, the coupling capacity can be divided into two kind parts:

The coupling capacity between each two elements $C_{\rm cc}$ The diagonal coupling capacity $C_{\rm ccd}$

Thus, the coupling capacity can be written as follows:

$$C_c = 4C_{cc} + 2C_{ccd} \tag{21}$$

Where:



Figure 9. S-parameters of the two patch (a) S11 and (b) S12.



Figure 10. A four elements array of patch.



Figure 11. Electrical model of the proposed array of 4 elements.

 $C_{cc} = C_{ga} + C_{gd}$, the gap capacitance in the presence of the air and the dielectric C_{ga} and C_{gd} are calculated by the equation 15 and 16.

$$C_{ccd} = C_{gad} + C_{gdd} \tag{22}$$

To calculate the diagonal gap capacitance in the presence of the air and the dielectric C_{gad} and C_{gdd} we use the same equation of C_{ga} and C_{gd} but we change the expression S by $\sqrt{2} S$ which is the distance between the diagonal patch.

RETURN LOSS

After building our electrical model, the simulation results are compared to those given by HFSS. As can be seen, S11, S12 and S13 are in good agreement which means that our model is very accurate, Figure 12. Besides, the time simulation of HFSS simulator increases when adding elementary element but the electrical model keeps the same time simulation when adding element which is instantaneously.

Conclusion

In our paper, we have concentrated on the coupling between elements of an array. The couplings in the arrays are evaluated by modelling the proposed array by an electrical model. Each antenna element is replaced by the RLC equivalent block. We have described the different coupling between array elements in the case of two and four array antenna. The proposed approach can be easily implemented without complicated mathematical programming methods. The technique has shown its ability to generate reasonable results in all checked cases.

Therefore, the usage of electrical model to analyse array antenna shows very interesting and useful results, even though if, considering the approximations for different geometry of array and for different kind of excitation. Moreover, a further development will be in case of mushroom antenna were the array element are related to the GND by using via hole.



Figure 12. S-parameters of the array of 4 elements patch. (a) S11, (b) S12, (c) S13.

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