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Full Length Research Paper

Is constructivist learning environment really effective on learning and long-term knowledge retention in mathematics? Example of the infinity concept

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This study investigates the long-term effects of instructing Cantor set theory using constructivist learning approach on student knowledge retention. The participants included 60 first-year secondary mathematics pre-service teachers. Students were divided into two classes one of which was taught via traditional lecture (n = 30) and the other was taught using active learning approach (n = 30). A pre-test named "Minimum Requirements Identification Test" developed by the researcher was used in the determination of the groups. This test involves the concepts such as "set, relation, and function" which were required to be able to learn Cantor set theory. Student retention of Cantor set theory was measured by using a questionnaire which consists of open-ended questions about the topic. The test was administered to all of the students approximately 14 months after the first instruction. In addition, five students from each group were interviewed. Analyses of the data revealed that the students in the constructivist learning environment showed better retention of almost all of the concepts related to Cantor set theory than the students in the traditional class.

Key words: Cantorian set theory, teacher education, infinity, retention, active learning.

INTRODUCTION

There has been an increasing emphasis on active learning during the last decade. It has been regarded as a radical shift from traditional instruction. Active learning method has gained supporters among instructors who seek for alternative means of imroving instruction. However, there are faculty who regarded active learning as no more effective than traditional instruction (Prince, 2004).

Although it is not possible to provide a widely accepted definition for the terms used within active learning instruction, it may be possible to provide commonly accepted definitions in the literature and show how active learning terms are used by different researchers.

Most common defition used for active learning is one that involves students in the learning process. In other words, active learning needs students carry out learning activities on their own and think about their learning (Bonwell and Eison, 1991). While traditional homework activities could be included in this definition, actually the activities that are carried out in the classroom are the ones which referred to. The fundamental elements of active learning are students involved in the activities and in the learning process. Researchers often compares active larning with the traditional instruction where students are though as passive information receivers.

Constructivists believe that people who come to classes with the goals and curiosity given by the nature are information producers and effective inquirers (information seekers) (Brooks and Brooks, 1993, Fosnet 1989, Piaget 1954). Constructivist theorists believe the discovery and transformation of complex knowledge, thus they do not approve teacher-centered instruction, skills, and content (Brooks and Brooks 1993, Fosnet 1993). Constructivists believe that conditions and social activities shape student understanding. These become important when the traditional-type teacher does not provide students with tools required for learning and force students to settle for artificial and shallow resources of learning without an opportunity to use or apply knowledge (Chi, Feltovich, and Gloser 1981). Some educational psychologists say that students from different parts of the world learn better when they gain experience together and tackle with problems which require authenticity and simulation (Bednar, Cunnigham, Duffy and Perry, 1992, Coollins, Brown and Newman 1988, Resnick 1987, Wilson 1993).

Since instructional methods often consists of several elements, they also involve several learning outcomes (Norman and Schmidt, 2000). If one asks about whether active learning works, it should be considered in terms of learning outcomes such as students' content knowledge, skills and abilities, attitudes, and student retention in schools. However, it is not easy to assess all of the outcomes together, thus researchers provide no concrete comprehensive data on the effects of instructional methods on these learning outcomes. On accasional data on multiple learning outcomes researchers report inconclusive results. For instance, in some studies on problem-based learning with medical students (Vernon and Blake, 1993, Albanese, Mitchell, 1993), it was reported that while students' performance on standardized exams declined slightly their clinical performance was slightly enhanced. In such studies, whether this approach works perhaps depends on how one interpret results and also on the procedure used in the study.

A significant issue in assessment is that many learning outcomes are difficult to measure especially for some of the complex learning outcomes intended to be gained by active learning (Prince, 2004).

A determining factor in deciding what works is to understand what an improvement means. It may be possible to have improvements in active learning studies; however, the magnitude of the improvement may be small which may make the results insignificant (Colliver, 2000). The commonly used statistics to measure the effect of an instructional approach is the effect size. The effect size is defined as the difference between the means of an experimental and control groups divided by the pooled standard deviation of the groups. Research indicates that statistics used to measure academic achievement do not particularly depend of instructional approach (Dubin, Taveggia, 1968).

In this case, it may be disputable whether the learning gains provided by constructivist learning environments are due to the method itself or due to a situational increase in students' motivation because of the new instructional method. In addition, the effects of active learning methods on long-term memory are not fully understood. The factors and effects of human memory have been analytically investigated since Ebbinghauss (1885) who tested short-term memory of a subject using nonsense syllables. The beliefs about long-term memory have developed significantly since Craik and Lockhart's (1972) modeling related to long-term memory named as “level-of-processing.” According to this model, familiar and meaningful stimuli are formed by the brain with moving from a less meaningful stimulus to a deeper level. According to previous theories of memory, for instance, Waugh and Norman's (1965) theory of “boxes-in-head,” it was thought that information was transferred to long-term memory especially via rehearsals and practice. Craik and Lockhart on the other hand, argued that long-term memory is the process of information via thinking the recalled items with all the other meanings together rather than how often something is repeated. For example, Craick and Tulving (1975) showed that a deeper processing of a memorized term occurs when the meaning of the term is asked (e.g., is this alive or not?) rather than the structure of it (e.g., does the term include the letter “a”?).

These findings indicate that using instructional methods which include understandable real materials rather than struggling with only mathematical concepts and artificial problems develops long-term memory better (Kvam, 2000). Due to its nature, active learning can provide deeper learning since it enhances long-term memory.

Long-term retention of knowledge

Knowledge retention is a significant goal of education (St. Clair, 2004), as noted in Semb and Ellis (1994: 253) “the very existence of school rests on the assumption that people learn something of what is taught and later remember some part of it”.

Retention of knowledge means recalling or remembering pieces of knowledge, processes, or skills that were learned earlier in time (Semb and Ellis, 1994). However, retention is different from knowledge transfer. Although retention is the ability to remember information as it has been learned, knowledge transfer is both to remember information and apply it to new situations. For knowledge transfer to be possible, there should be retention first.

It is obvious that memory has a significant play in knowledge retention. Activities in classrooms can be used as stimulus for memorization (Engelbrecht et al., 2007). It is suggested that students gain memory systems via which they carry out their classroom experiences by internalizing structures of classroom activities (Nuthall, 2000).

In educational psychology, retention is regarded as one stage in a dynamic model of learning process (Kohen and Kipps 1979). Researchers (Brewer 1987; Derry, 1996) examined the means by which knowledge structures effect the recollection of experience. It was claimed that these knowledge structures grow out of repeated experiences with common characteristics. More general representations are developed from these common properties. It is said that long-term memory involves a stepwise system of these structures, moving from the solid to the more abstract (Neisser 1989).

It may prove to be important to examine factors involving better retention. Sousa (2000) proposes that successful recall depens on efficient encoding that is related to making connections with existing knowledge that can expedite future recall.

Semb and Ellis (1994) argue that unlike the common belief that students forgot much of what they learn in classrooms, long-term knowledge retention is valuable. Results obtained by Anderson et al. (1998) catradict the view of Semb and Ellis. Anderson et al. (1998) examined the long-term knowledge retention by focusing the extent to which certain first year topics are retained and understood. Anderson et al. (1998) stated that

> *... "only about 20% of the responses were substantially correct and almost 50% did not contain anything that could be deemed to be minimally 'credit-worthy'. This suggests that a considerable amount of what is taught to mathematics students in general as 'core material' in the first year is poorly understood or badly remembered. (p. 417)".*

A literature review on retention reveals that there are not much studies on retention that have practical application. In addition, it was shown that when the main source of instruction is lectures, knowledge retention was the lowest. Moreover, different instructional approaches have been effective in improving retention to some extent. Based on these findings, these issues involving retention of mathematical knowledge may prove to be useful (Engelbrecht et al., 2007).

Instructional approaches towards better retention of mathematics

It is suggested that retention can be improved in several ways such as comprehensive learning of concepts and involving different instructional approaches. It was shown that knowledge and skills learned through understanding are retained and transferred better than that which is learned by rote memorization (Katona, 1940). There is more recent evidence that using active learning approaches and involving students in inquiry and discovery processes ehnaces knowledge retention (Handelsman et al., 2004).

Steyn (2003) agrues that knowledge retention is related to the way it is taught. The teachers is seen responsible to guide students in the process of learning and retention. St. Clair (2004) found no significant improvement on long-term retention of knowledge of engineering students over a twenty-five week period after intervention where the effect of the use of technology was investigated. It was concluded that using instructional technology does not hinder long-term knowledge retention and can make instructional process more effective.

In a study with engineering students, Townend (2001) used case studies to contextualize mathematics and concluded that it contributed to students' retention level of mathematical knowledge.

The limited number of studies in the field of knowledge retention indicates that it is generally not good. This may be due to the fact that traditional lecture method is the most popular instructional approach used in higher education (McKeachie, 1999). It should be emphasized that when compared to other types of instructions, lecturing has been shown to result in the lowest level of knowledge retention (Elshorbagy and Schonwetter, 2002). It was found that when learning is measured immediately after instructional intervention, both lecturing and alternative teaching methods had similar effects. However, when learning is measured some time after instruction, in other words when retentin is assessed, students who have received alternative teaching usually outperform students who received only lectures (McKeachie, 1999).

Procedural versus conceptual understanding

Recent studies show interest in conceptual and procedural learning of students in mathematics (Engelbrecht et al., 2005). Allen et al. (2005) investigated students' retention of conceptual and procedural knowledge via analysing their performance by using a post-test after one year of instruction in two differential equations classes. The results of this study suggest that teaching for conceptual understanding can lead to longer retention of mathematical knowledge.

Kwon (2005) compared students exposed to an inquiry-oriented course in differential equations with students in a similar but traditional class. It was reported that inquiry class showed better long-term retention of conceptual knowledge. Garner and Garner (2001) reported the results of a study where a reform and a traditional calculus course were compared with respect to students' long-term retention of basic concepts and skills. They reported no significant difference in the performance of the traditional and reform groups of students, however, the reform class retained conceptual knowledge better and the traditional students retained procedural knowledge better.

Retention and integration of certain topics across different contexts

Integrating knowledge between different subjects and different years of study is related to long-term retention of knowledge. Polanco et al. (2004) did a three-year follow-up study of an experimental integrated curriculum, integrating mathematics, physics and computer science courses, with second-year engineering students. They reported that in probability and statistics and in oral communication, integrated curriculum students performed significantly better than students in a comparison group in the knowledge retention. Finelli and Wicks (2000) measured engineering students understanding and retention of basic concepts in a circuits course. They found that students performed the best immediately after the course is over. They expect to use the results of that study as a feedback when the mathematics and circuits courses' curricula are revised. Cui (2006) investigated students' retention and transfer from calculus to physics. The results yielded that although students seemed to retain their basic calculus well for solving calculus problems, they had difficulties in retaining their calculus knowledge when solving a physics problem.

Cantorian set theory and infinity

Cantor's set theory which involves the concept of infinity brings about some perceptual difficulties due to its nature. It may be worthwhile to investigate whether employing active learning methods in the instruction of this concept would be effective in reducing these difficulties. In this context, the present study investigates the effects of using active learning approach in the instruction of Cantorian set theory on students' long-term retention. A theoretical description of the infinity concept and Cantorian set theory is provided next.

The concept of infinity is an abstract notion that is difficult to conceive for human mind. Infinite sets have not been accepted as mathematical objects since the time of Aristotle who believed that infinity can exist only potentially not in reality (Tirosh, 1991). This view has been a dominant conception in mathematics for many years. One can detect the concept of infinity in the studies of pioneering mathematicians, either implicitly or explicitly. Even among mathematicians the concept has led to conflicting ideas resulting from the nature of the notion itself. For instance, Galileo and Gauss concluded that actual infinity could not be incorporated into logical and coherent thinking. In 1831, Gauss argued that it is not possible to use an infinite magnitude as a complete quantity. Kant introduced the idea that the notions of spatial-temporal finiteness or infiniteness could not be comprehended by the human mind. Consequently, he concluded that our mind tries to understand and organize the outside world and for that it requires some mentally constructed space and time. Therefore, space and time actually do not exist in reality but only in the human mind (Fischbein, 2001).

The notion of infinity has been differentiated as actual and potential by philosophers and mathematicians. Aristotle, for instance, regarded mathematical infinity as potential infinity (Bagni, 1997). The type of infinity that is really challenging for human mind to comprehend is actual infinity, as in the following examples; "an infinite world" and "an infinite number of points in a line segment." Actually our mind is accustomed to finite realities that we acquire via our actions in space and time. Mind can deal with the notions if they are expressed in finite realities. Thus, as soon as we start dealing with actual infinity, contradictions begin to arise.

Cantor formulated a theory for the infinite sets towards the end of 19th century in which he defined infinite set as a set having one-to-one correspondence with one of its proper subsets. Cantor with this formulation disbanded the historical barriers against using numbers as an infinite magnitude. However, Kronecker, Poincaré and their colleagues fiercely criticized this theory where "a set could be at the same length as its subsets, a line could contain the same number of points with a line of half of its length, and infinite processes could be seen as complete things" (Rucker, 1982). Despite these criticism, however, there were significant others, Bertrand Russell and David Hilbert, who accepted and welcomed the theory as an important discovery. Cantor's definition of infinite set includes a significant cognitive obstacle due to the notion of "the equivalence of a set to one of its proper subsets." Approving this theory requires cognitive effort since one who accepts this idea needs to acknowledge the idea that "whole is greater than its parts." Thus, we may not expect students, who had never studied Cantorian Set Theory, to use this definition of infinite sets on their own (Tirosh, 1999).

Since Cantor's definition, the notion of infinity has become a field of study. The work of Piaget and Inhelder (1956: 125-149) has been regarded (Fiscbein et al., 1979: 4-5) as the beginning of studies about children's understanding of infinity (Monaghan, 2001). Fischbein (1987) identified two categories of student conceptions of infinity: Students develop the idea of infinity via personal experiences and formal education. Research also determined that students have some misunderstandings about the notion of infinity (Tall, 1990; Tsamir andTirosh, 1994; Tsamir, 2002; Singer and Voica, 2003). Efforts have been made to find out different empirical methods to study teaching and learning of infinity by mathematics educators (Fishbein et al., 1979). Tall (1980) investigated the intuition of infinity in regard to infinity of real numbers. In addition, with Vinner (1981), he introduced the terms of "concept images" and "concept definition" to explain the difficulties in learning limit and continuity concepts. Duval (1983) studied students' concerns of infinite sets in relation to the difficulty in assigning different roles to mathematical objects (For example, 4 as a whole number, 4 as square of 2, and 4 as an even number). Falk and his colleagues (1986) discussed student reactions to non-existence of very big natural numbers. In 1987, Sierpinska analyzed the types of problems in regard to conceptualizing limit. Furthermore, Moreno and Waldegg (1991) showed the similarities of students' response schema when confronted with the contradictions while studying actual infinity by using intuitions of finite sets and historical developments.

Tall (1992) suggested a transition from primitive mathematical thinking to sophisticated mathematical thinking through discussions of limit, function, mathematical proof, and infinity concepts. Tsamir and Tirosh (1992) presented a paper on students' comprehension of contradictory ideas at the 16th Psychology of Mathematics Education Conference (PME). Several other studies about infinity (Nunez, 1993), comparison of infinite sets with people who gained intuition based on their previous experiences rather than their formal conceptions (Waldegg, 1993b), cognitive challenge of comprehending infinity (Falk, 1994), and infinity intuitions of the Hispanic students in the USA (Gonzalez, 1995) were conducted. From a teacher's perspective, both in-service and pre-service, the concept of infinity was first studied by Mura and Louce (1997). Arrigo and D'Amore (1999) analyzed the answers of students with respect to equivalency of a square and the points on one edge of the same square. Garbin (2000) tried to identify the contradictions of high school students through their conceptual schema about actual infinity. A special issue (Vol. 48) of the Journal of Educational studies in Mathematics was published about infinity. Studies by Monoghan (2001) and Tsamir (2001) investigate opinions of young people about actual infinity. Waldegg (1988) elaborated on Cantor's studies about the collocation of actual infinity from a historical aspect, discussed Aristotle's definition of infinity in Aryan Greek culture (Waldegg, 1993a), and analyzed Bolzano's existential definitions (Waldegg, 2001). Horng (1995) examined the connections between Greek and Chinese mathematics with respect to infinity. There is a recent interest in paradoxical infinities, which include potential equivalence of infinite sets (Waldegg, 2005; Mamolo and Rina, 2008; Dubinsky et al., 2005).

Research suggested that 8-year old children think that natural numbers series have no end. Later, by the age of 11–12, children realize the dimensionless feature of points and then argue that line segments can be divided into infinitely many pieces. In these studies, students were asked whether some processes would end or not. Researchers have assumed that students who thought that some processes would end were regarded to be understood that the appearing set was infinite (Tirosh, 1999). Studies involving older students indicated that students had difficulties in understanding Cantorian Set Theory (Tsamir, 1999, 2001, 2002, Narli et al, 2008). The concept of Cantorian Set Theory is very important for the prospective mathematics teachers since the concept of infinity is a prerequisite in courses like Topology, Algebra etc. In addition, prospective teachers may need to associate the notion of infinity with their professional life when teaching infinite sets like natural and real numbers or when talking about infinite rational numbers in a finite interval.

There is a lack of research in the literature about the effects of using active learning methods in teaching the concept of infinity on students' retention of knowledge. Therefore, this study investigates the efficiency of constructivist learning environments on students' retention of knowledge of infinity concept.

METHODS

Subjects

The study involved two groups of freshmen pre-service mathematics teachers in the department of Secondary Science and Mathematics Education of a state university in Turkey.

One of the groups is called the experimental group with 30 students and the other is the control group with 30 students.

A Minimum Requirements Identification Test (MRIT) was used in the formation of the groups. This test includes the prerequisite concepts required to understand Cantorian Set Theory such as "sets, relations, and functions." All students' (n = 60) MRIT scores were listed from the highest to the lowest. Then the first student was assigned to first group, the second student to second group, the third student to first group, the forth student to second group and so on, and the two groups were formed in this way. After forming the two groups, they were randomly assigned to one of the control and experimental groups. The control group was instructed via traditional, formal instructional methods with time-to-time question-and-answer and whole class discussions. The constructive learning approach used for the instruction in the experimental group was designed as follows:

Active learning based course in cantorian set theory

In order to determine the active learning methods to be used in teaching Cantorian Set Theory experts have been consulted. In the light of expert opinions, necessary teaching conditions and learning environments have been prepared. In the preparation of these methods, the subject was divided into four sections, which were (a) basic concepts and definitions about equivalence, (b) special equivalence theorems and proofs, (c) countability, and (d) cardinal numbers.

Finally, in the introduction part of the subject brainstorming method; in the second section, question-and-answer and discussion techniques, and computer animations; in the part (c), problem-based learning (PBL); and in the last section, group study techniques were decided to be used.

These techniques were then applied to the experimental group. The applications were as follows:

Application of brainstorming technique

Brainstorming technique was used particularly in learning the notion of equivalence.

Students were asked to define the concept of "equivalence." Students freely expressed their ideas and stated the definitions, which they regarded as true. These definitions were noted.

No critic or comment was made about students' definitions of equivalence. Definitions were noted exactly as stated by students. These definitions then discussed one by one together with students. Definitions, which were wrong, were discussed as to why they were wrong; definitions, which were short or insufficient, were discussed to correct them; and in the end, a common consensus about the definition of equivalence was reached and students constructed a common definition for equivalence.

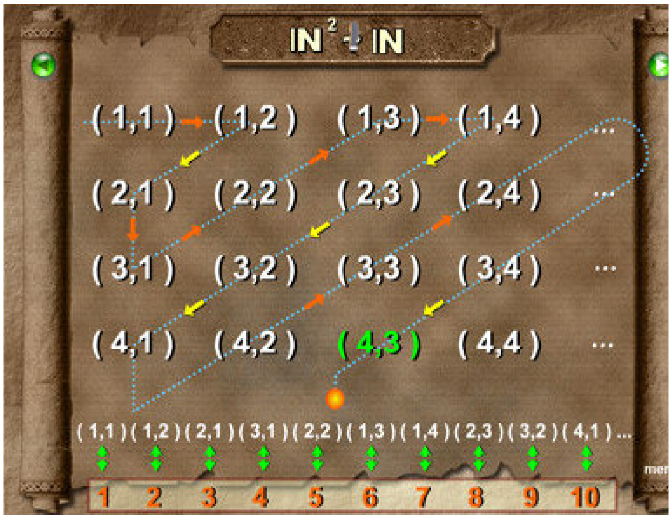

Figure 1. Animation related to N^2 ~ N.

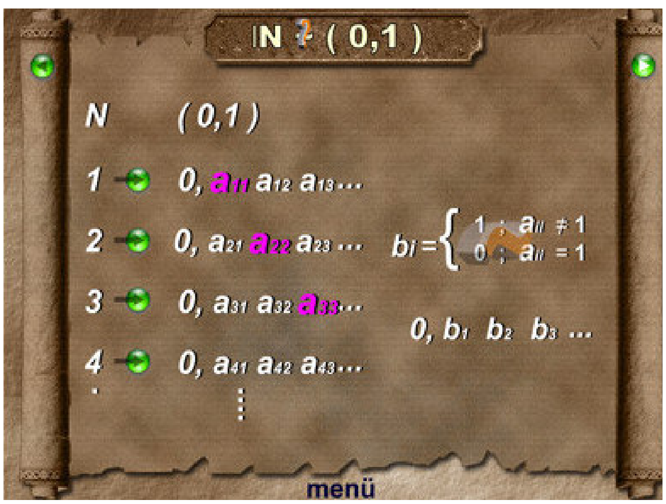

Figure 2. Animation related to N $\not\sim$ (0, 1).

Brainstorming technique was employed from time to time during the later stages of instruction.

The use of question-and-answer, discussion, and animation techniques

The notion of Cantorian Set Theory mainly deals with equivalence of two sets. When two sets are finite, it can be easy to see their equivalence; however, if they are infinite it is hard to show their equivalence to students.

Consequently, the concepts related to the equivalence of infinite sets and their proofs are difficult subjects for students to comprehend. To teach these subjects, question-and-answer and discussion techniques were employed in the experimental group. Moreover, computer animations were created for three proofs that were thought as important and rather abstract. These proofs were; the proofs of "the equivalence of N natural numbers and N^2 set;" "the non-equivalence of N natural numbers set and interval (0, 1);" and "non- equivalence of R real numbers set and F functions set" theorems.

Animations were prepared using Macromedia Flash-mx vector-based Web enhanced animation program, Photoshop-6 image program, and Macromedia Director. The question-and-answer animations were produced so that students could manipulate the program with the help of a button, and necessary information was included where it is needed. Some examples of animation's interfaces are shown Figures 1, 2 and 3.

Application of problem-based learning method in countability

Countability is one of the concepts that constitute an important part in Cantorian Set Theory. Countable and uncountable sets are important concepts for mathematics students. For this reason, special attention is given to the teaching of this subject in this study.

In the experimental group, this concept is taught by using PBL. PBL is applied via a written scenario. PBL scenario is written after consulting experts.

PBL sessions are carried out with groups of 6 to 8 students and with a moderator guiding students (Abacioglu et al., 2002). As an active learning method, the basic principle of PBL is that the information that is assumed necessary and has professional importance is learned by doing research through learning objectives that are developed by students with curious and skeptical approach to problems and is applied to solve a problem (Abacioglu et al., 2002). In accordance with this situation, the story of Hilbert hotel was chosen as the scenario for this study. This story is interesting enough to draw and keep students' attention. The story is as follows:

"You have a hotel. The hotel has infinite number of rooms. Each room has a number: 1, 2, 3, 4, 5, 6… Thereby goes to infinity. There is no "last" room. There is not also a room, which numbered "infinite." Number of each room is finite, only number of rooms is infinite.
First Story: It is your lucky day; a bus full with customers arrives at your hotel. Infinitely many customers…. Customer names numbered as 1, 2, 3, 4, 5, 6….You assign a room to each customer. Room number 1 to customer 1, room number 2 to customer 2, and room number 3 to customer 3, and so on…
Just as you were thinking that everything was going all right, you saw one more customer came to your hotel. How are you going to arrange a room to this customer?
Second Story: Another lucky day, you have a bus full with customers, infinitely many…. They are named as a_1, a_2, a_3, a_4, a_5, a_6 …. You assign a room to each customer. Customer a_1 to room 1, customer a_2 to room 2, and so on…
Just as you were thinking, everything was going all right, all of a sudden….Surprise! Another bus full with customers parks in front of your hotel. There are infinitely many customers in that bus as well. They are named as b_1, b_2, b_3, b_4, b_5, b_6 …. You have no vacancy…. You have infinitely many new customers. How are you going to settle your new customers?
Third Story: This is your luckiest day, you have infinitely many busses every one of which full with infinitely many customers…. The busses are numbered: 1, 2, 3, 4, 5, 6…How can you arrange rooms to your customers?"

The story given above was written as a scenario to be used in PBL sessions to teach the notion of countability. Scenario was revised with corrections and additions in light of expert opinions.

Scenario has three sections, which develops together and taught in three sessions. First session is organized as to be able to teach the notion of infinite and countable set, properties of the union of

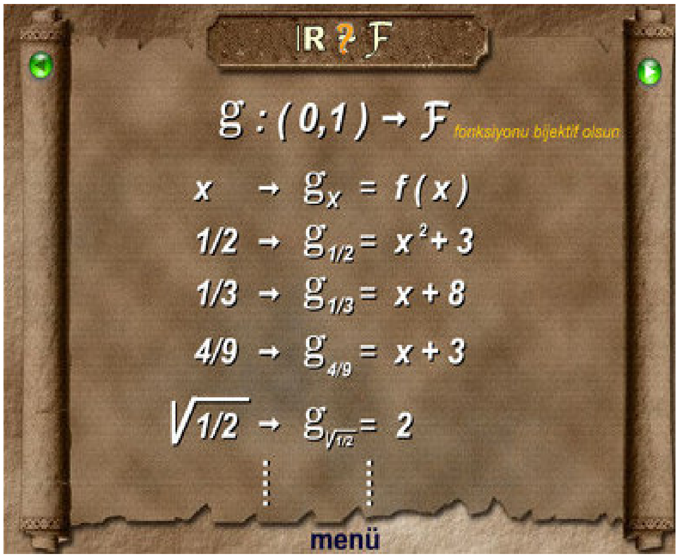

Figure 3. Animation related to IR $\not\sim$ F.

finite sets and countable infinite sets and set properties such as difference and intersection of these sets. In the first session, the questions like "what is countability?, can an infinite set be countable? and is finite set countable?" were determined as learning objectives.

In the second session of the scenario, the union of two countable infinite sets and other set operations, differences between these operations and the operations that are carried out on finite sets, the union and set operations of more than two countable sets has been taught and a connection has been established with uncountable sets. The learning objectives in this session were: how can union of countable sets and the proof of countability of set operations be done. Is there an uncountable set? If so, what does that mean?

In the third session, the students studied the differences and similarities between finite and infinite sets, types of infinite sets, and the proof of countability of union of finite countable or infinite sets.

The sessions were done 3 days apart from each other because of students' time disagreement and the issue of finding moderators. The experimental group was divided into three groups of 10 students each. Each session lasted 90 min.

Employing the group work

In Cantorian Set Theory, the last section that comes after the countability is the notion of cardinal numbers. In this part, group work was employed in the experimental group.

In working groups, students plan to learn a subject, apply the plan, collect information, use that information to solve a complex problem, synthesize the solution, and put together their results (Acikgoz, 2002).

Students have enough background from previous mathematics concepts they learned to do research on the cardinal numbers.

The experimental group was divided into 6 groups of 5 students each. Each group researched cardinal numbers and prepared to present their findings in class. During presentations, question-and-answer and discussion techniques were used. Each group submitted their findings as a report after the presentations.

The previous research about the effects of active learning approaches on student achievement and attitudes in Cantorian set theory showed significant improvements in the control group (Narli et al., 2008, 2010). This study investigates whether these instructional approaches are effective in students' retention of knowledge. To determine this, a questionnaire that includes open-ended questions about the Cantorian set theory is administered to the control and experimental groups 14 months after the first study. In addition, five students from each group were interviewed to obtain further information about their knowledge of the topic.

Materials

In this study the MRIT was used in the determination of the experimental and control groups. More information about the MRIT can be found in Narli et al. (2010). In addition, a questionnaire was administered to determine students' knowledge of Cantorian set theory and a semi-structured interview format was planned based on this questionnaire. All students allowed for 100 min for completing the questionnaire. The questionnaire is provided in Appendix 1.

Validity and reliability

The questionnaire was examined by two faculty members from the Mathematics Education Department in terms of the context of the study and its content was found to be valid for this study. Reliability studies were carried out via examination of the qualitative data by two researchers separately (Miles and Huberman, 1984:23). The data were categorized and coded by two different faculty members separately and then the results were compared which yielded 90% consistency between the two codings.

Data analyses

Data were analyzed by using qualitative research methods. χ^2 compatibility tests were used to test the differences among the categories.

RESULTS

Equal sets can be defined in two ways: (1) Two sets that have equal number of elements are said to be equal, or (2) two sets are said to be equal if a 1-1 correspondence can be defined between them (Güney, 1993). Although the former definition can be used to represent the equality of finite sets, it is not sufficient to define the equality of infinite sets. However, 1-1 correspondence can be used to show the equality of both finite and infinite sets. In fact, Cantorian set theory is based on 1-1 correspondence. The first question in the questionnaire was supposed to determine which definition of equality of sets the study groups preferred to choose. A summary of analysis of students' responses to the first question is shown in Table 1.

It can be seen in Table 1 that the experimental group preferred the 1-1 correspondence definition more often than the control group. More than half of the control group did not prefer the 1-1 correspondence definition. Categorized data obtained from two groups can be compared using χ^2 tests (Tekin, 2008). The groups were

Table 1. Frequency of preferred definitions for the equality of sets

Definitions	TC		AC	
	f	%	f	%
Two sets are equal if a 1-1 correspondence can be defined between the sets.	10	33.3	21	70
Two sets which have equal number of elements are said to be equal.	16	53.3	2	6.7
Two sets are said to be equal if their number of elements are equal or if there exist a 1-1 correspondence between them.	4	13.4	7	23.3

TC: traditional class, AC: active clas.

Table 2. Frequency of responses to the second question in the questionnaire.

	2-a		2-b		2-c		2-d		2-e		2-f		2-g		2-h	
	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC
Eq.	30	30	28	29	18	24	23	27	25	24	19	26	20	22	7	3
InEq.	-	-	2	1	11	5	5	2	3	4	9	4	6	4	21	26
UnAnsw.	-	-	-	-	1	1	2	1	2	2	2	-	4	4	2	1

compared using χ^2 test and they were found to differ significantly ($\chi^2_{calculation}$=15,61, sd=2, p<0,05).

The (un)equal judgments and justifications

In the second question, students were asked to compare the number of members of infinite sets. All of these pair of sets are equal except the one given in section (h), "A={1,2,3,4,5,…} and B={points on the segment]AB[}" which are not equal. Students' responses to this question are tabulated and presented in Table 2.

It may be inferred from Table 2 that the experimental group performed better than the control group on the second question.

The difference between the groups gets even clearer when students' explanations are examined. As described in the literature, in general students employ five separate approaches to explain whether two sets are equal or not (Tirosh, 1991; Tsamir, 1990). These are: (1) all infinite sets are 'equal', (2) infinite sets can not be compared, they are incomparable, (3) matching, that is 1-1 correspondence, (4) inclusion, and (5) intervals (i.e., when the elements of two sets have the same range but different intervals, then, the set in which the intervals are larger consists of fewer elements). Tsamir (1999) added two more justifications that are used by students: (1) bounded vs. infinite – bounded sets must have fewer elements than sets that are not bounded, (2) justifying their judgments by referring to the (in)equality of powers of the sets. Students in this study did not use justifications such as "intervals, infinite bounded, and incomparable." Students' justifications are presented in Table 3.

Blank responses and irrelevant ideas are shown as "other ideas" in Table 3. Except for the question 2-h, students who said that "two sets are equal since there is a 1-1 correspondence between the sets" but did not indicate their choice of equality or indicated a wrong choice are coded as W-correspondence. Similarly, students who said that "two sets are equal since they have the same power" but indicated wrong power, categorized as W-power.

The use of 1-1 correspondence and power

According to frameworks of Cantorian set theory described in the literature, two approaches may be mentioned in the justification of (in)equality of infinite sets. These are 1-1 correspondence and power. Table 3 indicates that the correct use of 1-1 correspondence justification is greater in every category for the active group than it is for the control group. The traditional group was found to use 1-1 correspondence justification considerably less especially for the judgments of the equality of whole numbers and natural numbers, equality of even natural numbers and prime numbers, and equality of the sets A={1/n: n, positive integer} and B={$\sqrt[3]{n}$ | n, positive integer}. In the comparison of (in)equal sets in question 2-h, more students in the active

Table 3. Frequencies of judgments and justifications to each problem.

	Equivalent Sets- $\aleph_o$										Equivalent sets-c				Unequivalent sets	
	{1,2,3,,…} {-1,-2,-3,…}		{{1,2,3,…} {{50,51,…}		{1,2,3,4,…} {…,-1,0,1,…}		{2,4,6,8,…} {prime numbers}		{1/n: n is a Natural number}{$\sqrt[3]{n}$ | n is a Natural number}		(2,7) IR		{points-circle of 1 cm}{points-square of 2 cm }		{1,2,3,…} {points on A-----B}	
	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC	TC	AC
Equal																
1-1 corres.	17	23	18	21	5	19	4	17	8	19	-	2	-	3	-	-
Power	4	2	3	3	1	1	5	2	2	2	6	9	4	9	-	-
W-1-1 corres.	5	2	4	3	8	3	8	5	9	1	4	2	3	2	-	2
W-Power	-	1	-	1	-	1	-	-	1	1	-	1	-	-	2	-
∞ = ∞	1	-	2	-	2	-	2	1	2	-	4	1	6	2	4	-
Other	3	2	1	1	2	-	4	2	3	1	5	11	7	6	1	1
Inequal																
1-1 corres.	-	-	-	-	-	-	-	-	-	-	-	-	-	-	1	4
Power	-	-	-	-	-	-	-	-	-	-	-	-	-	-	9	13
W-1-1 corres.	-	-	-	-	7	2	4		1	1	2	1	1	2	-	1
W-Power	-	-	-	-	-	1	-	1	1	-	1	2	-	1	-	-
Inclusion	-	-	1	1	1	1	1	-	-	-	2	-	-	-	2	-
Other	-	-	1	-	3	1	-	1	1	3	4	1	5	1	9	8
Missed																
	-	-	-	-	1	1	2	1	2	2	2	-	4	4	2	1

group provided valid justification. Students, who said that the sets were not equal, used power justification for their judgments. The number of students who used 1-1 correspondence justification correctly in this question is also greater in the active group than those in the control group. Students in both groups used "power" justification more often than 1-1 correspondence in questions where they compared pairs of sets that are equal to real numbers (2-f, 2-g). This may be due to the difficulty of determining a bijective correspondence between the pairs of sets in these questions. In addition, in the justification of "(2,7) ~ IR" equality, only two students from the active group and none from the control group; and in the justification of "points on a circle of radius of 1 cm ~ points on a square of 2 cm on one side" equality, only three students from the active group and none from the control group were found to use 1-1 correspondence as their justification.

In addition, it was determined that in all questions, all students used 1-1 correspondence and power justifications and the control group used incorrect justifications (coded as W-1-1 correspondence and W-power) more often than the experimental group. Students' justification in the equality of the sets {1, 2, 3, …} and {…, -2, -1, 0, 1, 2, …} includes incorrect use of 1-1 correspondence justification, for instance statements such as

> "*the infinities of these sets are different*", "*since there is a zero, it is confusing, I couldn't form a 1-1 correspondence*", "*there is not 1-1 correspondence between the sets*", "*because of uncountability*", and "*although these sets have infinite number of elements, since the set A contains half of the number of elements of the set B, it is impossible to define a 1-1 correspondence between them*".

In question 2-h, where two inequal infinite sets are compared, 21 students in the traditional group and 26 students in the active group correctly determined the inequality. Those who provided the correct answer used "power" justification more often than 1-1 correspondence. The number of students who used 1-1 correspondence is larger in the active group than they are in the control group. One particular student in the active group, although not with perfect notation, used an evidence for

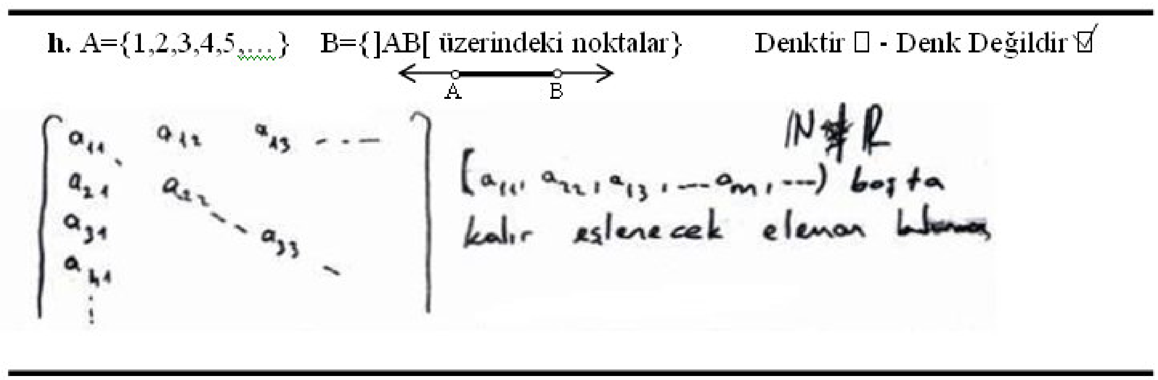

Figure 4. The explanation of a student in the active group for the question 2-h.
([*…the element (a_{11}, a_{22}, a_{33}, …a_{nn},…) will be left alone, there will not be any element to match with*])

Table 4. Students' ideas about the inequality "$A \nsim 2^A$".

Responses	TC		AC	
	f	%	f	%
There is a larger infinite set from every infinite set.	2	6.7	9	30
There is a larger infinite set from every infinite set and therefore there is a bigger cardinal number from every cardinal number.	-	0	2	6.7
It enables the comparison of infinite sets.	5	16.6	2	6.7
Other	23	76.7	17	56.6

his proof of why these sets could not be equal, which shows that he remembered the proof quite well. His proof is provided in Figure 4.

The global justifications

There are two global methods to compare infinite sets (Tsamir, 1999). These methods are: "All infinite sets are equal" and "infinite sets can not be compared (they are incomparable)." When the Table 3 is examined, it can be seen that the incomparability method is never used in both groups and the justification "all infinite sets are equal" is used quite less frequently. In addition, this justification is used only in some questions by the traditional group. Particularly in question 2-h where two inequal sets were compared, four students from the traditional group used the justification "all infinite sets are equal" and incorrectly claimed that these sets were equal. Another interesting finding was that this justification was used more often in questions where it was difficult to set a 1-1 correspondence.

Results of third question

The last question in the questionnaire was determined as "*in terms of infinite sets what could be the most important consequence of inequality of a set to its power set? Explain*." By this question it was intended to determine students' knowledge of the principle that "there is a larger infinite set from every infinite set" and the "infinity of cardinal numbers." Students' responses to this question are summarized and presented in Table 4.

In Table 4, other ideas represent irrelevant ideas and the choices left blank. More than half of the students in both groups did not respond to this question. However, in the active group, proportion of the non-respondents was less and relatively more students could provide the expected answer as compared to the traditional group.

Interviews

The interview results also support the data given above. Regarding Cantorian set theory, first students were asked what general ideas or thoughts they had or remembered. The students in the active group were observed to provide more clear responses to this question. To illustrate students' ideas, an interview excerpt from each group is provided below:

Researcher (R): Hello. Let me ask you first in which group you were with in the presentation of Cantorian set theory?

Student (S): In active group.

R: OK. Let's start with, what do you remember regarding that presentation?

S: I remember that it belongs to George Cantor. I know that for two sets to be equal, it is sufficient to find a bijective or 1-1 onto function between them. Then we looked at various propositions for two sets to be equal. We have seen proofs of equality or inequality of two sets, I mean, we have seen natural numbers, whole numbers, IN, 2IN, 2IN+1, prime numbers were equal to or not equal, and the proof of these. Rational number and natural numbers are (in)equal, the proof of that. I remember that rational numbers were equal to natural numbers, I remember there was a proof for this. Then, regarding cardinality, I mean, I remember that cardinal 1 was equal to infinity of all the sets which have one element, and that cardinal 2 represented all the subsets having two elements. Um, I remember that there was a fancy F, I remember that there was a "c" and that was I think the infinity of the real numbers. What I remember about cardinal numbers is that they had infinitely infinite varieties, that is, set of cardinal numbers was infinite. I had something in my mind about power set, that is, the infinity of the power set of a set was always greater than the infinity of the set. Then there was a paradox, was it zenno paradox or something like that. No not that. Is there an infinity or not between the infinity of natural numbers and the infinity of real numbers which are equal to power set of natural numbers? Then I remember there was a proof like, it does not make much difference whether that infinity existed or not. It might or might not exist, didn't matter. Apart from that, I remember equality of sets; I mean, set of functions were equal to power set of real numbers, interval (0,1) is equal to IR, we could even find a linear function between them. In fact, there wasn't any curve at a higher level, since if it intersects at two different points, it prevents the bijectivity. That's all I remember for now.

R: Hello, you were in the control group in the presentation of Cantorian set theory, weren't you?
S: Yes.
R: What do you remember about that presentation?
S: I remember lots of photocopies (laughing), shall I talk about those?
R: Sure, you can tell anything you remember.
S: I remember making photocopies, I do not remember much about the subject. I do not remember much about topics, like cardinality. I remember something about the equality of two sets, I mean, if 1-1 correspondence can be defined between two sets, then they are equal.

We defined something as cardinality, cardinality of infinity, I do not remember exactly.

In summary, it can be deduced that active group students were more successful in answering all the questions in the questionnaire than the traditional group students.

Conclusion

One-to-one correspondence forms the bases of Cantor's set theory (Guney, 1993). When asked the meaning of equality of two sets, proportion of the active group students who preferred to respond as "two sets are equal if a 1-1 correspondence can be defined between them" was significantly larger than that of the traditional group. This finding may indicate that the method used in the instruction enabled students to internalize the use of "1-1 correspondence" in defining the equality of sets.

Research indicates that students who did not receive an instruction on Cantorian set theory tended to employ intuitional approaches to compare infinite sets (Fischbein et al., 1979; Martin and Wheeler, 1987; Tirosh and Tsamir, 1996; Tsamir, 1999). These students were reported to use comments such as "all infinite sets are equal" or "infinite sets can not be compared, they are incomparable." In the present study, both groups received an instruction in Cantorian set theory and they hardly used these justifications. None of the students from both groups used "incomparable" justification and "∞ = ∞" justification was used very rarely. In addition, in this study, students' explanations were not as intuitive as those that were reported in the literature. Because students who used this justification did not respond to the other questions as "∞ = ∞". This justification was used in questions where it was difficult to define a 1-1 correspondence. Students wrote statements such as they remembered that *since both sets were infinite, they would be equal*. Notwithstanding, the traditional students preferred to use this justification 23 times and the active group students preferred to use it four times. In most of the questions, active group students never used the justification, "all infinite sets are equal." This finding may also indicate a positive effect of the active learning approach on the retention of knowledge about infinity.

It was also determined that active group students used the "power" justification correctly and also more often than that of the traditional group students. Similar results were reported in the literature (Tsamir, 1999). The use of "power" justification in the active group more often than that in the traditional group in the questions 2-f, 2-g, and 2-h may indicate that computer animations enhance retention of knowledge. Since mostly animations were used to compare these kinds of sets, might have such an influence.

Students' responses to the third question as well as the findings from the interview transcripts indicate that active group students remembered what they learned more clearly than that of the traditional group students about the topics "there is a larger infinite set than any infinite set" and "the infinity of cardinal numbers." This may be due positive effects of active learning approach employed, and particularly brainstorming and group work utilized during the instruction of these topics on students' retention of knowledge in these topics.

Another interesting point in this study was that a problem-based learning activity based on a scenario called "magic hotel" was developed to teach the concept of countability. Students indicated positive comments about this activity in the interviews. An excerpt from student interviews illustrates this idea:

"...*The most useful example I remember about Cantorian set theory is that hotel example. In addition, countability of rational numbers was quite interesting for me, I remember these two*... ."

The effect of problem-based learning on students' retention of knowledge in countability will be investigated in a different study.

Whether the effect of an active learning approach on learning is due to the method itself or due to a positive increase in students' motivation because of the utilization of a new and different approach is disputable. Previous studies reported positive effects of active learning approaches utilized in this study on student achievement and ideas (Narli et al., 2008, 2010). Results of this study indicate that even 14 months after instruction, the active group students remember subjects more clearly than those of the control group. Therefore, it may be argued that constructivist learning environment is really effective on retention of student knowledge in this study. This effect of constructivist learning environment may be explained as follows: Students utilize intuitive knowledge when they start learning a new concept. These primary intuitive ideas are formed in daily life and by previous experiences. Mathematics education literature showed that this primary intuitive knowledge influences students' and pre-service teachers' performance in many mathematics subjects (Ball, 1990; Fischbein, 1987, 1993; Tall, 1990; Tall and Vinner, 1981; Tirosh, 1991; Tsamir, 1999).

In this study, students' intuitional ideas about the concept infinity were taken into consideration in the development of constructivist learning environment. In other words, the method was prepared in such a way to prevent students from being influenced negatively by their primary intuitions. It should be reminded that sole awareness of intuitions is not enough (Tsamir, 1999). There should also be formal knowledge of Cantorian set theory. This study tried to process formal knowledge in a manner to help students internalize it in the constructivist learning environment. This process may have improved the efficiency of the active learning approach.

REFERENCES

Abacıoglu H, Akalın E, Atabey N, Dicle O, Miral S, Musal B, Sarıoğlu S (2002). Problem Based Learning, Dokuz Eylul Publications, Edition 1, Izmır.

Acikgoz KU (2002). Active Learning, Education World Press, Kanyılmaz Press, Izmır.

Allen K, Kwon ON, Rasmussen C (2005). Students' retention of mathematical knowledge and skills in differential equations. School Sci. Math., 105, 227.

Anderson J, Austin K, Barnard T, Jagger J (1998). Do third-year mathematics undergraduates know what they are supposed to know? Int. J. Math. Edu. Sci. Technol., 29: 401–420.

Arrigo G, D'Amore B (1999). I see it but I do not believe it: Episthemologic and didactic obstacles in the understanding of current infinity. Educacio´n Matema´tica, 11(1): 5–24.

Albanese M, Mitchell S (1993). Problem-Based Learning: A Review of Literature on Its Outcomes and Implementation Issues, Acad. Med., 68(1).

Bagni TG (1997). Didactics of Infinity: Euclid's proof and Eratosthenes' sieve Prime numbers and potential infinity in High School. Didactics of Mathematics-Technology in Education (1997), D'Amore, B. and Gagatsis, A. (Eds.), Erasmus ICP-96-G-2011/11, Thessaloniki, pp. 209-218.

Ball DL (1990). Prospective elementary and secondary teachers' understanding of division. J. Res. Math. Educ., 21(2): 132–144.

Bednar AK, Cunningham D, Duffy TM, Perry JD (1992). Theory into practice: How do we link? In T. M. Duffy and D. H. Jonassen (Eds.), Constructivism and the technology of instruction Hillsdale, NJ: Erlbaum, pp. 17-34.

Bonwell CC, Eison JA (1991). Active Learning: Creating Excitement in the Classroom, ASHEERIC Higher Education Report No. 1, George Washington University, Washington, DC.

Brewer WF (1987). Schemas versus mental models in human memory. In Modelling Cognition edited by P. Morris, Wiley, NewYork, pp. 187-197.

Brooks JG, Brooks MG (1993). In search of understanding: The case for constructivist classrooms. Alexandria, VA: Association for Supervision and Curriculum Development.

Chi MT, Feltovich PJ, Glaser R (1981). Categorization and representation of physics problems by experts and novices. Cogn. Sci., 5: 121-152.

Collins A, Brown JS, Newman SE (1988). Cognitive apprenticeship: Teaching the craft of reading, writing, and mathematics. In L. B. Resnick (Ed.), Knowing, learning and instruction: Essays in honor of Robert Glaser, Hillsdale, NJ: Erlbaum, pp. 453-494.

Colliver J (2000). Effectiveness of Problem-Based Learning Curricula. Acad. Med., 75: 259-265.

Craik FIM, Lockhart RS (1972). Levels of Processing: A Framework for Memory Research. J. Verbal Learn. Verbal Behav., 11: 671-684.

Craik FIM, Tulving E (1975). Depth of Processing and the Retention of Words in Episodic Memory". J. Exp. Psychol. Gen., 104: 268-294.

Cui L (2006). Assessing College Students' Retention and Transfer from Calculus to Physics, Doctoral dissertation Kansas State University.

Derry SJ (1996). Cognitive schema theory in the constructivist debate. Edu. Psychol., 31, 163–174.

Dubin R, Taveggia T (1968). The Teaching -Learning Paradox. A Comparative Analysis of College Teaching Methods, University of Oregon, USA: Center for the Advanced Study of Educational Administration.

Dubinsky E, Weller K, McDonald MA, Brown A (2005). Some historical issues and paradoxes regarding the concept of infinity: An APOS analysis: Part 1. Educ. Stus. Math., 58(3): 335-359.

Duval R (1983). L'Obstacle du Dedoublement des Objets Mathematiques, Educ. Stus. Math., 14: 358–414.

Ebbinghaus H (1885). About the mind: Inter tribulations of Experimental Psychology, Leipzig: Duncker and Humboldt. (Translation by HA Ruger and CE Bussenius, 1993). Reissued by Dover Publications, 1964.

Elshorbagy A, Schonwetter DJ (2002). Engineer morphing: Bridging the gap between classroom teaching and the engineering profession. Int. J. Eng. Educ., 18(3): 295-300.

Engelbrecht J, Harding A, Potgieter M (2005). Undergraduate students' performance and confidence in procedural and conceptual mathematics. Int. J. Math. Edu. Sci. Technol., 36: 701–712.

Engelbrecht J, Harding A, Du Preez J (2007). Long-term retention of basic mathematical knowledge and skills with engineering students

European J. Eng. Educ., 32(6): 735–744.
Falk R (1994). Infinity: A cognitive challenge. Theory and Psychology, 4(1): 35-60.
Falk R, Gassner D, Ben ZF, Ben SK (1986). How do Children Cope With the Infinity of Numbers? Proceedings of the 10th Conference of the International Group for the Psychology of Mathematics Education, London, pp. 13–18.
Finelli CJ, Wicks MA (2000). An instrument for assessing the effectiveness of the circuits curriculum in an electrical engineering program. IEEE Trans. Edu., 43: 137–142.
Fischbein E (1987). Intuition in Science and Mathematics, D. Reidel, Dordrecht, The Netherlands.
Fischbein E (1993). The interaction between the formal and the algorithmic and the intuitive components in a mathematical activity, in R. Biehler, R. W. Scholz, R. Straser and B. Winkelmann (eds.), Didactic of Mathematics as a Scientific Discipline, Kluwer, Dordrecht, The Netherlands, pp. 231–345.
Fischbein E (2001). Tacit Models and Infinity. Educ. Stus. Math., 48: 309-329.
Fischbein E, Tirosh D, Hess P (1979). The intuition of infinity. Educ. Stus. Math., 10: 3–40.
Fosnot CT (1989). Inquiring teachers, inquiring learners: A constructivist approach for teaching. New York: Teachers College Press.
Fosnot CT (1993). Rethinking science education: A defense of Piagetian constructivism. J. Res. Sci. Teach., 30(9): 1189-2201.
Garner B, Garner L (2001). Retention of concepts and skills in traditional and reformed applied calculus. Math. Edu. Res. J., 13: 165–184.
Garbin S (2000). Current infinity: Non-senses of 16-17 years old students, Doctoral Thesis, Universidad Auto´ noma de Barcelona.
Gonzalez G (1995). Students' notions of infinity and their remembrances of mathematics classes: A study with Latino students. (Master's dissertation, Cornell University).
Guney Z (1993). Introduction of Abstarct Mathematics, Dokuz Eylul University Press No: 0.36. DK.93.009.111 Izmır.
Handelsman J, Ebert-May D, Beichner R, Bruns P, Chang A, De Haan R, Gentile J, Lauffer S, Stewart J, Tilghman SM, Wood WB (2004). Scientific teaching. Science, 23: 304, 521–522.
Horng WS (1995). How did Liu Hui perceive the concept of infinity: A revisit. Historia Scientiarum, 4-3: 207-222.
Katona G (1940). Organizing and Memorizing, (Columbia University: NewYork).
Kohen AI, Kipps PH (1979). Factors determining student retention of economic knowledge after completing the principles-of-microeconomics course. J. Econ. Edu., 10: 38–48.
Kvam PH (2000). The Effect of Active Learning Methods on Student Retention in Engineering Statistic. Am. Stat., 3(54): 2.
Kwon ON (2005). Towards inquiry-oriented mathematics instruction in the university. Proceedings of KAIST International Symposium on Enhancing University Mathematics Teaching, Daejeon, Korea.
Mamolo A, Zazkis R (2008). Paradoxes as a window to infinity. Res. Math. Educ., 10(2): 167-182.
Martin WG, Wheeler MM (1987). Infinity concepts among preservice elementary school teachers'. Proceedings of the 11th Conference of the International Group for the Psychology of Mathematics Education, France, pp. 362–368.
McKeachie WJ (1999). McKeachie's Teaching Tips: Strategies, Research, and Theory for College and University Teachers, Houghton Mifflin, Boston.
Miles BM, Huberman AM (1984). Drawing Valid Meaning from Qualitative Data: Towarda a shared Craft. Educ. Res., pp. 20-30.
Monaghan J (2001). Young peoples' ideas of infinity. Educ. Stud. Math., 48: 239–257.
Moreno L, Waldegg G (1991). The conceptual evolution of actual mathematical infinity. Educ. Stus. Math., 22(3): 211-231.
Mura R, Louice M (1997). Is infinity ensemble of numbers? A study on teachers and prospective teachers. Learn. Math., 17(3): 28–35.
Narli S, Baser N (2008). Cantorian Set Theory and teaching prospective teachers. Int. J. Environ. Sci. Educ., 3(2): 99-107.
Narli S, Baser N (2010). The effects of constructivist learning environment on prospective mathematics teachers' opinions. Us-China Educ. Rev,, 7(1): 1-16.
Neisser U (1989). Domains of memory. In Memory: Interdisciplinary approaches, edited by P. R. Soloman, G. R. Goethals CM, Kelley and B. R. Stephens, pp. 67–83, (Springer-Verlag: NewYork).
Norman G, Schmidt H (2000). Effectiveness of Problem-Based Learning Curricula: Theory, Practice and Paper Darts. Med. Educ., 34: 721–728.
Nunez R (1993). Psychocognitive aspects of concept of infinity in mathematics. Editions Universitaires, Fribourg.
Nuthall G (2000), The role of memory in the acquisition and retention of knowledge in science and social studies. Cognit. Instruct., 18: 83–139.
Piaget J (1954). The construction of reality in the child. New York: Basic Books.
Piaget J, Inhelder B (1956). The Child's Conception of Space, Routledge and Kegan Paul, London (originally published in 1948).
Polanco R, Calderón P, Delgado F (2004). Effects of a problem-based learning program on engineering students' academic achievements in a Mexican University. Innovat. Edu. Teach. Int., 41: 145–155.
Prince M (2004). Does Active Learning Work? A Review of the Research. J. Eng. Educ., 93(3): 223-231.
Resnick L (1987). Learning in school and out. Educ. Res., 16(9): 13-20.
Rucker R (1982). Infinity and the Mind (Cambridge, MA: BirkhaÈ user Boston Inc).
Semb GB, Ellis JA (1994). Knowledge taught in school: What is remembered? Rev. Educ. Res., 64: 253–286.
Sierpinska A (1987). Humanities Students and Epistemological Obstacles Related to Limits. Educ. Stus. Math., 18: 371–397.
Singer M, Voica C (2003). Perception of infinity: Does it really help in problem solving. The Mathematics Education into the 21st Century Project Proceedings of the International Conference - The Decidable and the Undecidable in Mathematics Education Brno, Czech Republic, September.
Sousa DA (2000). How the Brain Learns, 2nd edition, (Corwin Press: Thousand Oaks, CA).
St. Clair SW (2004). Assessment of the long-term effects of technology use in the engineering classroom on learning and knowledge retention. Doctoral dissertation, Georgia Institute of Technology.
Steyn TM (2003). A learning facilitation strategy for mathematics in a support course for first year engineering students at the University of Pretoria, Unpublished thesis Philosophiae Doctor: Education, University of Pretoria.
Tall DO (1980). The notion of infinite measuring numbers and its relevance in the intuition of infinity. Educ. Stus. Math., 11: 271-284.
Tall D (1990). Inconsistencies in the learning of calculus and analysis. Focus on Learning Problems in Mathematics 12(4): 49–64.
Tall D (1992). The transition to advanced mathematical thinking: Functions, limits, infinity, and proof. In: D. Grouws. (Ed.). Handbook of research on mathematics teaching and learning. New York: Macmillan, pp. 495-511.
Tall D, Vinner S (1981). 'Concept image and concept definition in mathematics with particular reference to limit and continuity'. Educ. Stus. Math., 12: 151–169.
Tekin VN (2008). Istatistige Giriş. Seckin Yayincilik, Ankara.
Tirosh D (1991). The role of students' intuitions of infinity in teaching the cantorial theory, in D. Tall (ed.), Advanced Mathematical Thinking, Kluwer, Dordrecht, The Netherlands, pp. 199–214.
Tirosh D (1999). Finite and infinite sets: definitions and intuitions. Int J. Math. Educ. Sci. Technol., 30(3): 341-349.
Tirosh D, Tsamir P (1996). The role of representations in students' intuitive thinking about infinity, Int. J. Math. Educ. Sci. Technol., 27(1): 33–40.
Townend MS (2001), Integrating case studies in engineering mathematics: a response to SARTOR 3. Teaching in Higher Education, 6: 203–215.
Tsamir P (1990). Students' Inconsistent Ideas about Actual Infinity, Unpublished thesis for the Master's degree. Tel Aviv University, Tel Aviv, Israel (in Hebrew).
Tsamir P (1999) The Transition From Comparison Of Finite To The Comparıson Of Infinite Sets: Teaching Prospective Teachers. Educ. Stus. Math., 38: 209–234.
Tsamir P (2001). When "the same" is not perceived as such: The case of infinite sets. Educ. Stus. Math., 48: 289-307.

Tsamir P (2002). Primary and Secondary Intuitions: Prospective Teachers' Comparisons of Infinite Sets, inCERME 2 Proceedings.
Tsamir P, Tirosh D (1992). Students' awareness of inconsistent ideas about actual infinity. Proc. XVI PME Conf., 3: 91.
Tsamir P, Tirosh D (1994). Comparing infinite sets: intuitions and representations. Proceedings of the 18th Annual Meeting for the Psychology of Mathematics Education, Lisbon: Portugal, 4: 345-352.
Waldegg G (1988). Cantor and mathematization of infinity. MATHESIS, 6(1): 75–96.
Waldegg G (1993a). Infinity in Aristotheles work. Educ. Math., 5(3): 20–38.
Waldegg G (1993b). The comparison of infinite sets: A case of instruction resistance, J. Curriculum Cogn. Sci., 5: 19-36.
Waldegg G (2005). Bolzano's approach to the paradoxes of infinity: Implications for teaching. Sci. Educ., 14: 559-577.
Waugh NC, Norman DA (1965). Primary Memory. Psychol. Rev., 72(2): 89-104.
Vernon D, Blake R (1993). Does Problem-Based Learning Work? A Meta-Analysis of Evaluative Research. Acad. Med., 68(7).
Wilson AL (1993). The promise of situated cognition. New Directions Adult Continuing Educ., 57: 71-79.

Appendix 1. Questionnaire

1. *What does it mean for two sets to be equal? Explain.*

2. *Compare the number of members of the following couple of sets and explain your answer.*
The sets to be compared
Number of members
a. A={1,2,3,4,5,…} B={-1,-2,-3,-4,-5,…}
Equal -Unequal
Your explanation: --
b. A={1,2,3,4,5,…} B={50,51,52,…}
Equal -Unequal
Your explanation: --
c. A={1,2,3,4,5,…} B={…,-3,-2,-1,0,1,2,3,…}
Equal -Unequal
Your explanation: --
d. A={2,4,6,8,10,…} B={x: x prime number}={2,3,5,…} Equal -Unequal
Your explanation: --
e. A={1/n: n, positive integer} B={$\sqrt[3]{n}$ | n, positive integer}
Equal -Unequal
Your explanation: --
f. A={points on real segment} B=(2,7)
Equal -Unequal

Your explanation: --
g. A={Points on a circle of radius of 1cm}
Equal -Unequal
B={Points on a square of 2 cm on one side}
Your explanation: --
h. A={1,2,3,4,5,…} B={points on the segment]AB[}
Equal -Unequal

Your explanation: --
3. *In terms of infinite sets, what could be the most important consequence of inequality of a set to its power set? Explain.*
