Full Length Research Paper

Investigation on the effect of stent in unsteady blood flow

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In this work, we tried to determine the fact that an interest in optimal stents shapes helps to minimize fluid swirl and to maximize shear stress on the wall artery in order to reduce the risk of the restenosis in stented segments. Our study deals with three geometric parameters of the stent, namely the strut spacing \( l \), the strut height \( h \) and the strut width \( w \). These parameters have direct effect on blood flow. A multi-objective optimization based on genetic algorithm is used to determine an optimal stent.

**Key words:** Multiobjective optimization, stent, Navier-Stokes, genetic algorithm.

**INTRODUCTION**

Stent placement in stenosis artery perturbs is more often in blood flow. Stent shape in artery can provoke the presence of recirculation zones, blood stagnation zones, thrombosis and embolism. The aim of this work is to find optimal stents shapes in order to reduce blood stagnation and recirculation zones. As it has already been observed Berry et al. (2000), Moore et al. (2002), a stent associated with a higher value of shear stress is preferred because it lowers the risk of the late restenosis by reducing the presence of blood stagnation. A former paper Quarteroni et al. (2003), Blouza et al. (2008) had shown that a stent associated with a lower value of vorticity is preferred. Qualitative study Blouza et al. (2008) shows that the variation of vorticity and shear stress depend entirely on the variation of three parameters \( l \), \( h \) and \( w \). Using multiobjective optimization approach we minimize the vorticity and maximize the shear stress on the wall artery simultaneous. Indeed, we propose the multiobjective optimization approach because our two criteria compete.

A former paper Blouza et al. (2008), we had supposed that the fluid is modeled by the stoke equation for steady flow. In the work, the fluid is modeled by two dimensional Navier-Stokes equations for unsteady flow and we also suppose that the wall artery is rigid.

**MATHEMATICAL FRAMEWORK**

Let \( \Omega_f \) the domain occupied by the fluid (Figure 1). The boundary of \( \Omega_f \) is decomposed by:

\[
\partial \Omega_f = \Sigma_{in} \cup \Sigma_{out} \cup \Sigma_{stent}
\]

The fluid which is viscous, incompressible and Newtonian is modeled by two dimensional Navier Stokes equations.

Let \( v : [0, T] \times \Omega_f \rightarrow R^2 \)

The fluid velocity vector and \( p : [0, T] \times \Omega_f \rightarrow R \)

The fluid pressure.

We will find the couple \((v, p)\) such as:

\[
\rho \frac{\partial v}{\partial t} + (v \cdot \nabla) v - \mu \Delta v + \nabla p = f \text{ in } [0, T] \times \Omega_f \quad (1)
\]

\[
\nabla v = 0 \text{ in } [0, T] \times \Omega_f \quad (2)
\]

\[
v(t = 0) = v_0 \text{ in } \Omega_f \quad (3)
\]

\[
-p \ln + \mu \nabla v \cdot n = p_{in} \ln \text{ on } [0, T] \times \Sigma_{in} \quad (4)
\]

\[
-p \ln + \mu \nabla v \cdot n = 0 \text{ on } [0, T] \times \Sigma_{out} \quad (5)
\]

\[
v = 0 \text{ on } [0, T] \times \Sigma_{wall} \quad (6)
\]

Where

1) \( I \) is the identity matrix,
2) \( n \) is the unit outward vector normal to \( \partial \Omega_f \).

3) \( v_0 \) is the initial condition of the fluid.

4) \( \mu \) is the viscosity of the fluid.

5) \( \rho \) is the density of the fluid.

6) \( f \) is the volume force of the fluid.

7) \( p_m \) is the boundary conditions imposed of the pressure.

8) Concerning the outflow \( \Sigma_{out} \), we impose free boundary condition of the pressure.

The symmetric properties of the problem imply us to take in the following work the half fluid domain \( \Omega \) (Figure 2) to solve the Navier-Stokes equations.

Let the Navier-Stokes equations in \( \Omega \):

\[
\rho \left( \frac{\partial v}{\partial t} + (v \cdot \nabla)v \right) - \mu \Delta v + \nabla p = f \quad \text{in} \ [0,T] \times \Omega \quad (1)
\]

\[
\nabla . v = 0 \quad \text{in} \ [0,T] \times \Omega \quad (2)
\]

\[
v(t = 0) = v_0 \quad \text{in} \ \Omega \quad (3)
\]

\[
-p n + \mu \nabla v.n = p_m I_n \quad \text{on} \ [0,T] \times \Sigma_m \quad (4)
\]

\[
-p n + \mu \nabla v.n = 0 \quad \text{on} \ [0,T] \times \Sigma_{out} \quad (5)
\]

\[
v = 0 \quad \text{on} \ [0,T] \times \Sigma_{stent} \quad (6)
\]

\[
v = 0 \quad \text{on} \ [0,T] \times \Sigma_{sym} \quad (7)
\]

\[
\frac{\partial v_i}{\partial x_i} = 0 \quad \text{on} \ [0,T] \times \Sigma_{sym} \quad (8)
\]

Where
1) $v_1$ is the first component of vector $v$,
2) $v_2$ is the second component of vector $v$,
3) $\Sigma_{\text{sym}}$ is the symmetric axis,
4) on $\Sigma_{\text{sym}}$ the no penetration condition $vn = v_2 = 0$ and the continuity stress tensor

$$\sigma.n = \frac{\partial v_1}{\partial x_2} = 0$$

are imposed.

Variational formulation for the fluid equations

Let $W$ and $Q$ the variational spaces:

$$W = \{ w \in (H^2(\Omega))^2 : w = 0 \text{ on } \Sigma_{\text{stent}}, w_2 = 0 \text{ on } \Sigma_{\text{sym}} \}$$

$$Q = L^2(\Omega).$$

Where

1) $w_2$ is the second component of $w$.

Now, we can introduce the variational formulation for the fluid equations for all $t \in [0,T]$, we find the velocity $v \in W$ and the pressure $p \in Q$ such that:

$$\int_{\Omega} \rho \nu.v.w dt + c(v,v,w) + a(v,w) + b(w,p) = F(w), w \in W$$

$$b(v,q) = 0, \forall q \in Q.$$  

Where

$$c(v,v,w) = \int_{\Omega} \rho \nu.(v.\nabla)v.w dx$$

$$a(v,w) = \int_{\Omega} \mu \nabla v.\nabla w dx$$

$$b(w,p) = -\int \nabla w \cdot p dx$$

$$F(w) = \int_{\Omega} f.w dx + \int_{\Sigma_{\text{in}}} p_n w.n d\sigma.$$  

Position of the problem

The vorticity is given by:

$$\nabla \times v = \frac{\partial v_2}{\partial x_1} - \frac{\partial v_1}{\partial x_2},$$

and the shear stress by:

$$\tau_\omega = \mu \left( \frac{\partial v_2}{\partial x_1} + \frac{\partial v_1}{\partial x_2} \right).$$

Let $J_1$ the first cost functional which measures the mean square wall shear stress between two struts, that is:

$$J_1(l,h,w) = \frac{1}{\text{length}(\Gamma_w)} \sqrt{\int_{\Gamma_w} \int_0^l \tau_\omega^2 d\Gamma_\omega dt}$$

and let $J_2$ the second cost functional which measures the mean swirl value near the struts, that is:

$$J_2(l,h,w) = \frac{1}{\text{area}(\omega)} \sqrt{\int_0^l \int_{\omega} \nabla \times v^2 d\Omega dt}$$

Where $\omega$ is the integration domain, Figure 3.

We use the multiobjective optimization approach based
on genetic algorithm Deb et al. (2003) so as to minimize $J_1$ and maximize $J_2$ simultaneously. This method is chosen here because it has already obtained, in many applicative fields, robust and global optimal solutions (Dumas and Alaoui, 2007; Dumas et al., 2005). Moreover, it is well adapted to determine a set of optimal solution located on the Pareto front. More precisely, the ε-multiobjective evolutionary algorithm developed by Deb et al. (2003) and freely available at the site http://www.iitk.ac.in/kangal/soft.htm, is used for all the computations done here. It is based on the $\varepsilon$ dominance principle that relaxes the classic dominance principle with a factor $\varepsilon$ and also on the use of two co-evolving populations (an EA population and archive population). A good diversity is ensured by allowing on the Pareto front only one solution in each hyper box.

**NUMERICAL RESULTS**

**Spatial discretisation**

Let $W_h$ be the finite element approximation spaces for the fluid velocity obtained from $W$ by using the mixed finite element $P^1 + \text{bubble} / P^1$, and let $Q_h$ be also the finite element approximation spaces for the fluid pressure obtained from $Q$ by using the finite element $P^1$

$$W_h = \left\{ w_h \in (C^0(\Omega))^2 : \forall K \in \tau_h, w_h \in P^1 + \text{bubble}, \right. \\
\left. (w_2)_h = 0 \text{ on } \Sigma_{	ext{int}}, w_h = 0 \text{ on } \Sigma_{	ext{ext}} \right\}$$

and

$$Q_h = \left\{ q_h \in C^0(\Omega) : \forall K \in \tau_h, q_h \in P^1 \right\}$$

**Time discretisation**

The time integration scheme is based on the implicit Euler approximation.

Knowing $v^n_h \in W_h$, we find $v^{n+1}_h \in W_h$ and $p^{n+1}_h \in Q_h$ such as:

$$\frac{d}{dt} \int_{\Omega} (v^{n+1} - v^n \circ \chi^n) w dx + a(v^{n+1}, w) +$$

$$b(w, p^{n+1}) = F^{n+1}(w), \forall w \in W$$

$$b(v^{n+1}, q) = 0, \quad \forall q \in Q$$

Where

$$F^{n+1}(w) = \int_{\Omega} f^{n+1} w dx + \int_{\Sigma} p^{n+1} w nd\sigma$$

1) $\Delta t$ is the time step,

2) The term $v^n \circ \chi^n = v^n(x - v^n(x)\Delta t)$ will be computed by the operator “convect” Hecht et al. (2005).

**Boundary conditions**

**Boundary conditions imposed to the pressure**

$$p_n(x, y, t) = 10^4 (1 - \cos(\pi i t / 0.005)), t \in [0,0.005] \quad p_n(x, y, t) = 0, t \in [0.005, T]$$

(Mbaye and Murea, 2008).

**Parameters values related to fluid**

The fluid viscosity is $\mu = 0.353 g / cm s$, the density of the fluid is $\rho = 1g / cm^3$, the volume forces $f = (0,0)$ and the width of artery is $D = 0.4cm$ and the length of the artery is between $[0.492, 3.944]$. Time step is $\Delta t = 0.1 ms$ and $v_0 = 0$ (Mbaye and Murea, 2008).

**Parameters values related to stent**

The three stent design parameters $l, h, w$ lie in a rather large domain $(l, h, w) = [0.02, 0.2] \times [0.01, 0.04] \times [0.007, 0.009]$ Andreas et al. (2002).

**Parameter values related to genetic algorithm**

We use the following parameters for the genetic algorithm: a population of 120 individuals, number of generations 100, the probability of mutation $p_m = 0.33$, the probability of crossover $p_c = 0.9$ and the seed $p_s = 0.123$.

**NUMERICAL SIMULATIONS**

Figure 4 displays the initial population and the Pareto front after 100 generations and 50 iterations in time (5 ms).

Figure 5 displays Pareto front after 100 generations and 50 iterations in time (5 ms). Figure 6 displays the initial population and the Pareto front after 100 generations and 100 iterations in time (10 ms). Figure 7 displays Pareto front after 100 generations and 100 iterations in time (10 ms). Figure 8 displays the initial population and the Pareto front after 100 generations and 150 iterations in time (15 ms). Figure 9 displays Pareto front after 100 generations and 150 iterations in time (15 ms). Figure 10 represents Pareto front at $t = 5, 10$ and 15 ms, respectively after 100
Figure 4. Initial population and pareto front after 100 generations.

Figure 5. Pareto front after 100 generations.
Figure 6. Initial population and Pareto front after 100 generations.

Figure 7. Pareto front after 100 generations.
Figure 8. Initial population and Pareto front after 100 generations.

Figure 9. Pareto front after 100 generations.
Some optimal stent shapes at $t = 5$ ms after 100 generations

According to optimal parameters $l$, $h$ and $w$, we represent in Figures 11 to 18 after 100 generations some optimal stents.

1) For $l = 0.021360, w = 0.008317, h = 0.038598$.

2) For $l = 0.187011, w = 0.007466, h = 0.039841$.

3) For $l = 0.020842, w = 0.008683, h = 0.012138$. 

Figure 10. Pareto front at $t=5$, 10 and 15 ms.

Figure 11. Velocity streamlines.
Figure 12. Velocity streamlines between two struts.

Figure 13. Velocity streamlines.
Figure 14. Velocity streamlines between two struts.

Figure 15. Velocity streamlines.

Figure 16. Velocity streamlines between two struts.
4) For $l = 0.049559$, $w = 0.007901$, $h = 0.018499$.

**Conclusion**

A multiobjective optimization based on genetic algorithm is used to find optimal stent. A curve of exchange named Pareto front is given. The latter gives a large inquiry during the phase of design when the designer submits several criteria for arbitration. The current results using a 2d computational fluid modeling and multiobjective optimization loop indicate that there exists some important rules to fulfill in order to design appropriate stents which may avoid or at least reduce restenosis. In this paper, we suppose that the wall artery is rigid. That hypothesis simplifies the problem. In a future work, the wall artery will be modeled by an elastic structure.

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**REFERENCES**