Kinematics and kinetic analysis of the slider-crank mechanism in otto linear four cylinder Z24 engine

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Nissan Z24 is one of the numerous vehicles in Iran. MegaMotor’s reports show high rate damaging in the crankshaft and connecting rod of this engine vehicle. It is necessary to carry out a complete research about slider-crank mechanism because of high expensive repairs and replacement of these parts and their reverse effects on the other parts such as cylinder block and piston. Results of initial researches show that an important reason of these parts' damaging is using of downshifting in driving. In this research, we are concerned on the analysis of kinematics and kinetic of slider-crank mechanism of the engine in maximum power, maximum torque and downshifting situation. The influence of different parameters such as engine RPM and downshifting effects were investigated on crankshaft and connecting rod loads. Two methods for analyzing of slider-crank mechanism were used: solving Newton's law and MSC/Adams/Engine software.

Key words: Engine, slider-crank mechanism, kinematics and kinetic, downshifting, Newton's Law, engine rpm.

INTRODUCTION

The base of dynamic mechanism operation of engine is slider-crank mechanism, which consist of crankshaft, connecting rod and piston. Combustion pressure transferred from piston (the part merely has reciprocating motion) to the connecting rod (the part has both linear and rotation motion) and finally to the crankshaft (the part has merely rotation motion).

Cveticanin and Maretic (2000) have studied dynamic analysis of a cutting mechanism which is a special type of the crank shaper mechanism (Cveticanin and Maretic, 2000). The influence of the cutting force on the motion of the mechanism was considered. They used Lagrange equation to obtain boundary values of the cutting force analytically and then numerically.

Ha et al. (2006) have derived the dynamic equations of a slider-crank mechanism. They, for this purpose, used Hamilton’s principle, Lagrange multiplier, geometric constraints and partitioning method (Ha et al., 2006). Their formulation was expressed by only one independent variable. Finally to obtain the best dynamic modeling, they compared obtained results and numerical simulations.

Also, a new identification method based on the genetic algorithm was presented to identify the parameters of a slider-crank mechanism. Koser (2004) investigated on kinematic performance analysis of a slider-crank mechanism based on robot arm performance and dynamics (Koser, 2004). He analyzed kinematic performance of the robot arm using generalized Jacobian matrix. It was obtained that the slider-crank mechanism based robot arm had almost full isotropic kinematic performance characteristics and its performance was much better than the best 2R robot arm. He used complex algebra to solve that classical problem and he
obtained solution as the root of a cubic equation within a defined range.

Another research about transmission angle was carried by Shrinivas and Satish (2002). They have summarized importance of the transmission angle for most effective force transmission. In this regard, they investigated 4-, 5-, 6- and 7-bar linkages, spatial linkages and slider-crank mechanisms.

One of the numerous vehicles in Iran is Nissan Z24, produced by MegaMotor’s company. This company reports show high rate damaging in the crankshaft and connecting rod of this engine vehicle. As investigation on phenomena like vibration, resonance, fatigue, noise . . . , and optimization of these parts, kinematics and kinetic of slider-crank mechanism must be known, it is necessary to carry out a complete research about slider-crank mechanism because of high expensive repairs and replacement of these parts and their reverse effects on the other parts such as cylinder block and piston. Results of initial researches show that an important reason of these parts' damaging is using of downshifting in driving. So, in this research, we concerned on analysis of kinematics and kinetic of slider-crank mechanism of the engine in maximum power, maximum torque and downshifting situation. The influence of different parameters such as engine rpm and downshifting effects were investigated on crankshaft and connecting rod loads.

MATERIALS AND METHODS

At first stage the combustion chamber pressure curve of Nissan Z24 engine was measured in MegaMotor’s power test lab (Engine, Gearbox and Axel Manufacturing located in Tehran province in Iran, info@megamotor.ir). These experimental data have been shown in Figure 1.

Because of different type of motion in this mechanism; such as: linear, linear-rotation and rotation, there is inertial force in the system. The inertial force has important role in engine slider-crank mechanism, so behavior of this force must be known. For analyzing of inertia force, the kinematics of mechanism should be defined.

Kinematics analysis of slider-crank mechanism

The engine slider-crank mechanism has been shown in Figure 2. The piston has linear motion in x direction in this figure:

$$x = r \cos(\theta) + l \cos(\beta)$$

Where, \( r \) is the crank radius, \( L \) is the connecting rod length, \( \theta \) is the crank rotation angle and \( \beta \) is the connecting rod angle with x axis. From Figure 1, one can obtain that:

$$r \sin(\theta) = l \sin(\beta)$$

$$n = \frac{r}{l}$$

And thus:

$$x = r \cos(\theta) + l \sqrt{1 - n^2 \sin^2(\theta)}$$

Using Taylor series in Equation 4:

$$\sqrt{1 - n^2 \sin^2(\theta)} = 1 - \frac{1}{2} n^2 \sin^2(\theta) - \frac{1}{8} (n \sin(\theta))^4 - \frac{1}{16} (n \sin(\theta))^6 + ...$$

$$= 1 - \frac{1}{2} n^2 \sin^2(\theta) + \frac{n^4}{8} \sin(\theta)^4 - \frac{n^6}{16} \sin(\theta)^6 + ...$$

Because \( n \) is less than 1 (about 0.3), we can eliminate high degree sentences. Thus:

$$\sqrt{1 - n^2 \sin^2(\theta)} \equiv 1 - \frac{1}{2} n^2 \sin^2(\theta) = 1 - \frac{1}{4} n^2 - \frac{1}{4} n^2 \cos(2\theta)$$

From Equations 1 and 6, one can obtain:

$$x = r \cos(\theta) + l(1 - \frac{1}{4} n^2 - \frac{1}{4} n^2 \cos(2\theta))$$

For simplification in calculation we use these notations:

$$Q_1 = n l$$

$$Q_2 = -\frac{n^2 l}{4}$$

$$Q_3 = l(1 - \frac{n^2}{4})$$

$$Q_4 = 1 + \frac{n^2}{4}$$

$$Q_5 = \frac{n^2}{4}$$

Equation 7 and 8 will result:

$$x = Q_1 \cos(\theta) + Q_2 \cos(2\theta) + Q_3$$

Where \( \omega \) is the crankshaft rotational velocity. Piston speed obtained from derivation of Equation (9):

$$v_p = -Q_1 \cos(\theta) \omega - 2Q_2 \cos(2\theta) \omega$$

And for piston acceleration:

$$a_p = -Q_1 \cos(\theta) \omega^2 - Q_1 \sin(\theta) \alpha - 4Q_2 \cos(2\theta) \omega^2 - 2Q_2 \sin(2\theta) \alpha$$

where \( \alpha \) is the crankshaft rotational acceleration.

From Equation 11 and Taylor series, for other parts, can reached the following result:
\begin{align*}
\lambda &= n\omega \cos(\theta)(Q_4 - Q_5 \cos(2\theta)) \\
\eta &= n\alpha \cos(\theta)(Q_4 - Q_5 \cos(2\theta)) - n\omega^2 \sin(\theta)
\end{align*}

(12)

(13)

That is, \( \lambda \) is the connecting rod rotational velocity and \( \eta \) is the connecting rod rotational acceleration.

Now the velocity and acceleration of connecting rod’s C.G (Center of gravity) could be calculated. Connecting rod acceleration could be calculated from Figure 3:

\[
\ddot{\mathbf{a}}_g = \ddot{\mathbf{a}}_p + \ddot{\mathbf{a}}_{g/p}
\]

(14)

where \( \ddot{\mathbf{a}}_{g/p} \) is acceleration vector of connecting rod’s C.G relative the piston, as follow:

\[
\ddot{\mathbf{a}}_{g/p} = \dddot{\mathbf{r}}_p \times \dddot{\mathbf{r}}_g + \dddot{\mathbf{R}} \times (\dddot{\mathbf{R}}_g \times \dddot{\mathbf{r}}_g)
\]

(15)

where \( \mathbf{r}_p \) is the displacement vector of connecting rod’s C.G relative the piston, that (Meriam and Kraige, 1998):

\[
\ddot{\mathbf{r}}_g = s(-\cos(\beta) \mathbf{i} + \sin(\beta) \mathbf{j}) = \left(-e(Q_3 + Q_2 \cos(2\theta)) \mathbf{i} + (e Q_1 \sin(\theta)) \mathbf{j}\right)
\]

(16)

where \( s \) is the distance between connecting rod’s C.G and piston, and \( e = s/l \). For vertical acceleration of C.G:

\[
\ddot{a}_g = (-Q_1 \cos(\theta)\omega^2 - Q_1 \sin(\theta)\alpha - 4Q_2 \cos(2\theta)\omega^2
- 2Q_2 \sin(2\theta)\alpha - (-n\alpha \cos(\theta)(Q_4 - Q_5) \cos(2\theta)) +

\frac{n\omega^2 \sin(\theta)(Q_4 - Q_5 \cos(2\theta)) - \frac{1}{2} \frac{n^2 \omega^2}{\cos(\theta)sin(\theta)} - e(Q_3 + Q_2 \cos(2\theta))\mathbf{i})
\]

(17)
Figure 3. Connecting rod acceleration.

And for horizontal acceleration of C.G:

\[ a_{xy} = (-n\alpha \cos(\theta)(Q_4 - Q_1 \cos(2\theta)) + n\omega^2 \sin(\theta) \]

\[ e(Q_4 - Q_1 \cos(2\theta)) - \frac{1}{2} n^3 \omega^2 \cos(\theta) \sin(2\theta) \]

\[ e(Q_4 - Q_1 \cos(2\theta))^2 eQ_4 \sin(\theta)) \]

Kinetic analysis of slider-crank mechanism

Kinetic calculation must start from the piston because slider-crank mechanism started from that. The force diagram of piston was shown in Figure 4 and Equations 19 and 20:

\[ \sum F_x = m_p.a_p \]  

\[ R_x = F_g - m_p.a_p \]

The force diagram of connecting rod was shown in Figure 5 and Equations 21 and 22:

\[ \begin{cases} 
\sum F_x = m_c.a_{cx} \\
N_x - R_x = m_c.a_{cx} 
\end{cases} \]  

\[ N_x = R_x + m_p.a_p + m_c.a_{cx} \]

Engine torque can be obtained from Figure 6 as follow:

\[ T = N_x.r_x \sin(\theta) + N_y.r_y \cos(\theta) \]

For \( N_y \):

\[ \sum M_A = I_A.\eta \]

\[ N_y = \frac{N_x.r_x \sin(\theta) - I_A.\eta}{Q_1 + Q_2.\cos(2\theta)} \]

Where \( I_A \) is Inertia of connection rod. But for all journals:

\[ T_c = T_1 + T_2 + T_3 + T_4 \]

That \( T_c \) indicates the crankshaft torque, not engine output torque. Note that the friction force is negligible in comparison with gas force, so it has ignored in calculations.

Engine output torque from flywheel has been calculated considering flywheel inertia and resistance torque of crankshaft end side (\( T_s \)). \( T_s \) consist of fan, alternator, timing chain and oil pump resistance torque, as follow:

\[ T_S = T_{fan} + T_{Timing} + T_{Oilpump} + T_{Alternator} \]

\( T_s \) approximately is 10-15% of total engine output torque.

\[ T_f = T_c - T_s - J_f.\alpha_f \]

For calculation of implied forces over relevant parts like crankshaft and connecting rod, a FORTRAN program was written used considering above equations.

Dynamic analysis by Adams/Engine

For checking the accuracy of program, the extract results of program for sample engine have been compared with results of
simulating the same engine in Adams/Engine software (Figure 7). The results of last section and Adams/Engine's output were compared. The compared results of two methods were shown in next figure.

**Downshifting modeling**

As mentioned, the main reason of failing of Nissan engine's crankshaft is using of downshifting in driving. It means shifting the gear from light gear (like 3) to heavy gear (like 1) and usually is used for speed control of vehicle by the drivers in very steep roads with heavy loads. Before shifting from 3 to 1:

\[ r_{g3} = \frac{\omega_{c3}}{\omega_{g3}} = \frac{\omega_{g1}}{\omega_{g3}} \Rightarrow \omega_{g3} = \omega_{c3} = r_{g3} \cdot \omega_{g3} \quad (29) \]

After disengagement of engine and shifting from 3 to 1 and before releasing the clutch:

\[ r_{g1} = \frac{\omega_{c1}}{\omega_{g1}} = \frac{\omega_{c3}}{\omega_{g1}} \Rightarrow \omega_{c1} = \omega_{c3} = r_{g1} \cdot \omega_{c1} \]

\[ \omega_{g1} = \omega_{e1} = \omega_{g3} = \omega_{e3} = \omega_e \]

\[ \omega_{c1} = \left( \frac{r_{g1}}{r_{g3}} \right) \omega_e \]

where, \( r_{g1} \) is the first gear ratio, \( r_{g3} \) is the third gear ratio, \( r_d \) is the differential ratio, \( \omega_{c1} \) is the clutch plate rotational velocity in gear 1, \( \omega_{c3} \) is the clutch plate rotational velocity in gear 3, \( \omega_{g1} \) is the transmission rotational velocity in Gear 1, \( \omega_{g3} \) is the transmission rotational velocity in Gear 3, \( \omega_{e1} \) is the engine rotational velocity in Gear 1 and \( \omega_{e3} \) is the engine rotational velocity in Gear 3.

According Equations 29 and 30 and the difference between clutch and engine rotational speed during releasing the clutch in this shifting, the torque direction will be diverse from driveline to engine. Diverse torque will increase the engine speed and then inertia force, these results will be presented next. By continuing this reaction between engine and transmission line, in short time interval, the vehicle will be established in equilibrium condition in spatial speed. Increment of engine speed has been considered by two methods:

1. By ignoring the heat dissipation in clutch and using energy equation:

\[ \omega_{S0} = \omega_e \cdot \sqrt{\frac{J_e + M_v \cdot \left( \frac{R_w}{r_d \cdot r_{g1}} \right)^2 \left( \frac{r_{g1}}{r_{g3}} \right)^2}{J_e + M_v \cdot \left( \frac{R_w}{r_d \cdot r_{g1}} \right)^2}} \quad (31) \]

where, \( \omega_{S0} \) is the engine rotational velocity in engagement starting
Figure 7. Dynamic model of engine in Adams/Engine methods.

(energy method), \(M_v\) is the vehicle mass, \(R_w\) is the wheel radius and \(J_e\) is the total engine inertia.

2. Numerical solving of engine and transmission system: In this method, it was supposed one degree of freedom for each set of engine, such as: clutch, gearbox, differential and vehicle body, separately. When Engine and clutch are in full engagement:

\[
J_e \ddot{\theta}_e + C_e \dot{\theta}_e + C_i (\dot{\theta}_e - \omega_s) = 0
\]

\[
J_{cl} \ddot{\theta}_1 + k_{cl} (\theta_1 - \theta_2) + C_{cl} (\dot{\theta}_1 - \dot{\theta}_2) + C_i (\dot{\theta}_1 - \omega_s) = 0
\]

When engagement started:

\[
J_e \ddot{\theta}_e + C_e \dot{\theta}_e = T_{cl} + T_e
\]

\[
J_{cl} \ddot{\theta}_1 + k_{cl} (\theta_1 - \theta_2) + C_{cl} (\dot{\theta}_1 - \dot{\theta}_2) = T_{cl}
\]

(32)

For other sets before and after engagement:

\[
J_e \ddot{\theta}_2 + k_p (\theta_2 - \theta_2) + k_{cl} (\dot{\theta}_2 - \dot{\theta}_2) + C_{cl} (\dot{\theta}_2 - \dot{\theta}_2) + C_i (\dot{\theta}_2 - \omega_s) = 0
\]

(33)

where, \(J_e\) is the clutch plat inertia, \(J_g\) is the gearbox inertia (equivalent), \(J_d\) is the differential inertia (equivalent), \(J_v\) is the vehicle inertia (equivalent), \(k_s\) is the clutch spring constant, \(k_p\) is the propeller shaft spring (equivalent), \(k_d\) is the drive shaft spring (equivalent) and \(\omega_s\) is the steady rotational speed in engagement (rpm).

An example of downshifting and way of increasing the engine velocity were shown in Figure 8. This figure shows quick increase in engine velocity and applying torque to engine with maximum capacity of clutch.

RESULTS

Comparison between methods of solution

In Figures 9, 10 and 11, crankshaft output torque and pin journal vertical force for 2800 and 4800 rpm in both Adams/Engine and Newton's Law Results method were compared. As seen in these figures, results of two methods show that each method verifies other one.

Kinematics

The piston and connecting rod acceleration are main results of kinematics analysis of the mechanism. In Figures 12, 13 and 14 acceleration of piston and connecting rod in 2800, 4800, 5700 and 6500 rpm were shown. As seen in these figures, all accelerations increased with increasing of engine velocity. The connecting rod horizontal acceleration is very important because of its major role on the torque.

In Figures 12 and 13, there is an observation that connecting rod vertical acceleration is nearly similar to piston acceleration. This is due to low horizontal displacement of that.

Kinetic

In Figures 15 and 16, the crankpin force (horizontal and vertical), crankshaft torque and flywheel torque were shown in maximum torque and maximum power, respectively. From these figures, one can get the following:

1. Flywheel and clutch design: By average value and fluctuation ratio (flywheel design parameter) could design the suitable flywheel. Majority of this fluctuation is absorbed by flywheel. The clutch could be design after flywheel designation and consideration of output torque etc.

2. Resonance phenomena: Resonance phenomena are a main factor of damaging. The applying of torque to the crankshaft is motivational factor to engine and even transmission system (after the fluctuation absorbed by flywheel). By using frequency and amplitude of fluctuation and natural frequency of parts such as cylinder block and crankshaft (with flywheel), the design can be optimized for prevention from resonance.

3. Stress analysis: As the engine is damaged, stress analysis of different situation for some parts is essential. The fatigue analysis of moving part such as connecting rod and crankshaft could be done by using the diagrams.

Downshifting

In downshifting, as the combustion pressure is very low,
Figure 8. Downshift gear from 3 to 1.

Figure 9. Comparison of crankshaft output torque between Adams/Engine and Newton's law results.

Figure 10. Comparison of pin journal vertical force between Adams/Engine and Newton's law results (2800 rpm).
Figure 11. Comparison of pin journal vertical force between Adams/Engine and Newton's law results (4800 rpm).

Figure 12. Piston acceleration in different rpms.

Figure 13. Connecting rod vertical acceleration in different rpms.
Figure 14. Connecting rod horizontal acceleration in different rpms.

Figure 15. The horizontal and vertical crankpin force, crankshaft and flywheel torque in maximum torque.

Figure 16. The horizontal and vertical crankpin force and crankshaft and flywheel torque in maximum power.
the inertia forces are important. In Figures 17, 18 and 19 vertical and horizontal force of crankpin and crankshaft torque in different velocities was shown.

From the figures, it could be understood that the inertia forces are quick increased unsteadily with velocity increasing. The forces and torques are sinusoidal with average value near to zero. Another result of velocity increasing is applying high load to all journals. This is clearer in the downshifting situation.

From comparison of downshifting situation and full load we could conclude that the applied load in the downshifting is greater than full load and has high frequency because the velocity is high. Thus the possibility of fatigue occurring in this situation is very high.

Conclusion

In this research, the followings were concluded:

1. There is a well agreement between Newton's law results and Adams/Engine methods.
2. High load is applied to engine in downshifting in comparison with full load condition. There is a necessity for stress, fatigue and trailer in these conditions.
3. Training drivers not to use downshifting and out of standard load on vehicle.
4. Defining maximum speed of engine in electronic control unit (ECU) for preventing uncontrollable loading on engine and downshift situation.
5. Referring to issued statistics from Megamotors
Company and analysis results, the main reason of parts failure is abnormal using of vehicle by drivers.

**Terminology**: $a_{cx}$, Vertical acceleration; $a_{cy}$, horizontal acceleration; $a_{g/p}$, acceleration vector of connecting rod's C.G relative piston; $a_p$, piston acceleration; $C.G$, center of gravity; $F_x$, force in X direction; $I_A$, inertia of connection rod; $J_e$, Total engine inertia; $J_c$, clutch plat inertia; $J_g$, gearbox inertia; $J_d$, differential inertia; $J_v$, vehicle inertia; $k_{cl}$, clutch spring constant; $k_d$, drive shaft spring; $k_p$, propeller shaft spring; $L$, connecting rod length; $M_V$, vehicle mass; $m_p$, piston mass; $N_x$, resistance of pin end in X direction; $N_y$, resistance of pin end in Y direction; $R_w$, wheel radius; $R_{cr}$, wheel radius; $R_d$, resistance of crank end in X direction; $R_y$, resistance of crank end in Y direction; $r$, crank radius; $r_{g1}$, first gear ratio; $r_{g3}$, Third gear ratio; $r_d$, differential ratio; $r_{g9}$, displacement vector of connecting rod's C.G relative piston; $s$, distance between connecting rod's C.G and piston; $T_{cr}$, crankshaft torque; $T_f$, flywheel torque; $T_{sp}$, resistance torque of crankshaft end side; $v_p$, piston speed; $\alpha$, crankshaft rotational acceleration; $\beta$, connecting rod angle; $\lambda$, connecting rod rotational velocity; $\eta$, connecting rod rotational acceleration; $\theta$, crank rotation angle; $\omega$, crankshaft rotational velocity; $\omega_{e1}$, engine rotational velocity in gear 1; $\omega_{e3}$, engine rotational velocity in gear 3; $\omega_{c1}$, clutch plate rotational velocity in gear 1; $\omega_{c3}$, clutch plate rotational velocity in gear 3; $\omega_{g1}$, transmission rotational velocity in gear 1; $\omega_{g3}$, Transmission rotational velocity in gear 3; $\omega_s$, steady rotational speed in engagement; $\omega_{s0}$, engine rotational velocity in engagement starting.

**REFERENCES**


