Approach to calculate solar radiation inside a semi-cylindrical greenhouse

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A semicylindrical greenhouse is simple in construction, requires less material and can withstand high wind pressure. The transmission of solar radiation through the curved surface is dependent on angle of incidence of solar radiation on the cover. A calculation procedure has been developed for the transmission of obliquely incident beam radiation through a semi-cylindrical greenhouse. An analysis has also been presented for total solar radiation entering the greenhouse on a typical day for N-S and E-W orientations of the greenhouse for semi humid climate of Udaipur (27°42' N, 75°33' E) Rajasthan, India. It is found that E-W oriented greenhouse receives more solar radiation as compared to N-S oriented greenhouse for typical winter month of December by about 7.67%, whereas N-S oriented greenhouse receive less radiation energy by about 2.56% as compared to E-W oriented greenhouse for the typical summer month of May.

Key word: Angle of incidence, greenhouse, semicylindrical surface, solar radiation, transmittance.

INTRODUCTION

The controlled environment inflated plastic greenhouses can be considered a mean of solar energy utilisation, for better plant growth. Greenhouse structure comes in a variety of shapes and styles. The selection depends on the availability of resources, ease in construction and maintenance, climatic conditions, type of crop to be grown etc. The semicylindrical shape is most commonly used because it is; simple in construction, require less material and can withstand high wind pressure (Grag, 1993; Nelson, 1991; Panwar et al., 2003). The angle of incidence of solar radiation will be different at different elements on circular surface of semicylindrical cover. The transmittance of solar radiation depends on angle of incidence (Rathore et al., 2007; Kothari et al., 2006). To properly analyse performance of greenhouse it is required to determine angle of incidence of solar radiation on each element of greenhouse cover and calculate the transmittance corresponding to the angle of incidence. The geometrical analysis of a Nubian vault and hemispherical dome shaped building was presented by (Bansal et al., 1997). They compared the solar radiation at different times and at various latitudes, the solar radiation intercepted by a unit area of each of this building geometry and a flat geometry. Finally, they reported that for latitudes up to 40°N, a Nubian vault form building envelope with East-West orientation receives minimum solar radiation. Similarly (Felske, 1979) developed calculation procedure for determining the transmission of incident beam radiation through a cylindrical glass shell and on to an enclosed flat absorbing plate. The developed procedure is useful for evacuated cylindrical solar collector. Finally it was concluded that, depending upon the conditions, the cylinder might have either a greater or lesser transmittance than a flat plate.

Further East-West orientation of the cylinder’s axis should be avoided.

In the present study an elementary strip of small thickness has been considered. A procedure has been developed to determine the angle of incidence of solar radiation on elementary strip, solar radiation received by this strip and transmitted to the greenhouse. Total radiation entering the greenhouse is determined by integrating...
over the entire surface of semicylindrical cover.

**Analysis**

To find out the amount of beam radiation, which will be transmitted through the cylindrical cover, the angle of incidence, which the sun rays make with each element of the arbitrarily oriented cylindrical surface, must first be determined and then integrated over the surface to yield the total transmitted energy. This in turn requires specification of the vector position of the sun and the local vector normal to each element of the cylindrical cover. The unit vector (Figure 1) may express the position of sun as seen from the ground (Garg and Kumar, 1998).

\[
\mathbf{n}_{\text{sun}} = (\cos \alpha \cos \gamma_s) \mathbf{i} + (\cos \alpha \sin \gamma_s) \mathbf{j} + (\sin \alpha) \mathbf{k}
\]

where,

\[
\sin \alpha = \cos \phi \cos \delta \cos \omega + \sin \phi \sin \delta
\]

\[
\sin \gamma_s = -\frac{\cos \delta \sin \omega}{\cos \alpha}
\]

The local vector normal to the cylindrical surface is determined by using the co-ordinate system shown in Figure 2. The unit vectors for the various co-ordinates are expressed in terms of the unit vectors in the reference position as follows,

\[
\mathbf{i}' = \cos \gamma \mathbf{i} + \sin \gamma \mathbf{j}
\]

\[
\mathbf{j}' = -\sin \gamma \mathbf{i} + \cos \gamma \mathbf{j}
\]

\[
\mathbf{k}' = \mathbf{k}
\]

Surface azimuth angle \(\gamma\) is the counter clockwise rotation of the cylinder about the x-axis. The general expression for the vector normal to the cylindrical surface for any point on cylinder is then given by,

\[
\mathbf{n}_{\text{cyl}} = \sin \beta \mathbf{i}' + \cos \beta \mathbf{k}'
\]

\[
= (\sin \beta \cos \gamma) \mathbf{i} + (\sin \beta \sin \gamma) \mathbf{j} + (\cos \beta) \mathbf{k}
\]

Where, \(\beta\) defines the direction of an arbitrary radius vector in the X’-Z’ pane as shown in Figure 3.

The cosine of the angle of incidence between the solar beam radiation and an element of the cylindrical surface is then given as

\[
\cos \theta_i = \mathbf{n}_{\text{sun}} \cdot \mathbf{n}_{\text{cyl}}
\]

\[
= \cos \gamma \cos \phi \cos \gamma_s + \sin \gamma \cos \phi \sin \gamma_s + \sin \phi \cos \omega
\]

To determine which rays directly strike the floor of greenhouse, consider the views looking down the axis of the cylinder in Figure 3. Depicted there are the limiting intercept angles for the two basic cases. In Figure 3, \(\beta_2\) denotes the angle between the Z’-axis and the projection of the solar vector onto the X’-Z’ plane and is given by

\[
\tan \beta_2 = \frac{\mathbf{\hat{r}} \cdot \mathbf{n}}{\mathbf{\hat{r}} \cdot \mathbf{\hat{n}}_{\text{cyl}}}
\]

\[
= \cos \gamma \cos \phi \cos \gamma_s + \sin \gamma \cos \phi \sin \gamma_s
\]

and

\[
\beta_1 = \frac{\pi}{2} - 2\beta_2
\]
The power of the direct solar radiation incident on an elementary strip of the semicylinder (Figure 3) parallel to its axis, is

\[ dw = I_b \cdot L \cdot r \cdot d\beta \cdot \cos \theta \]

where \( I_b \) is the direct solar radiation; \( L \) and \( r \) the length and radius of the semicylinder, \( \beta \) is the angle between the horizontal and the elementary semicylinder strip; \( \theta \) is the angle of incidence of the solar rays on the elementary strip. The length of greenhouse is 10 m and radius is about 3 m.

The power of the energy entering the greenhouse through this strip is

\[ dw = \tau(\theta) \cdot dw = t_\tau \cdot I_b \cdot L \cdot r \cdot d\beta \cdot \cos \theta \]

where \( \tau(\theta) \) is the transmittivity of the radiant energy incident at the angle \( \theta \), \( \beta_1 \) is the angle made by the solar radiation striking the centre of greenhouse floor with vertical and \( \beta_2 \) is the angle made by the solar radiation at extreme position of semicylinder with vertical. The energy, which will be transmitted through the greenhouse cover can be determined by doubling the integration from \( \beta_1 \) to \( \beta_2 \):

\[ w_i = 2 \cdot L \cdot r \cdot I_b \cdot \int_{\beta_1}^{\beta_2} \tau(\theta) \cdot \cos \theta \cdot d\beta \]

\[ = 2 \cdot L \cdot r \cdot I_b \cdot \int_{\beta_1}^{\beta_2} \tau(\theta) \cdot (x \cdot \sin \beta + y \cdot \sin \beta + z \cdot \cos \beta) \cdot d\beta \]

where

\[ x = \cos \alpha \cdot \cos \gamma, \quad x = \cos \alpha \cdot \sin \gamma, \quad y = \sin \alpha \]

Transmittance for the beam radiation at different angle of incidence has been calculated as method shown in Appendix-I

The transmittance at an incidence angle \( \theta \), is given by

\[ \tau(\theta) = \tau_a \cdot \tau_r \]  

With the help of equations 1 and 2 we can find out the direct solar radiation energy entering the solar greenhouse of the semicylindrical type at various times of the day and then, summing these values we can calculate the total amount of energy entering during the sunshine hours.

In addition to direct solar radiation, diffuse radiation also enters the greenhouse. Since the magnitude of the scattered radiation is approximately same in all direction, its transmittivity can be calculated by defining equivalent angle for beam radiation that gives the same transmittance as for diffuse radiation. For a wide range of conditions encountered in solar collector applications this equivalent angle is taken as 60° (Duffie and Beckman, 1991). The diffuse radiation energy entering the greenhouse is given by,

\[ w_d = \tau(60) \cdot \pi \cdot r \cdot L \cdot I_d \]

Where \( I_d \) is the diffuse radiation.

The total energy entering the greenhouse is the sum of beam and diffuse radiation transmitted through semicylindrical greenhouse cover and can be mathematically presented as,

\[ w = w_i + w_d \]  

RESULTS AND DISCUSSIONS

To appreciate the developed simulation model, numerical calculations have been carried out for total radiation entering the greenhouse. Theoretical calculations have been made for the critical day of the month of May and December representing summers and winters conditions of udaipur. The 15th day of month May and 10th day of month December considered as critical solar day of the month (Klein, 1997). The hourly values of direct and diffuse radiation have been taken from the handbook on Solar Radiation (Mani and Rangarajan, 1981).

The total radiation energy entering the greenhouse in the month of December for Udaipur in E-W greenhouse is more by 7.67 per cent as compared to N-S placed greenhouse. In summers the energy entering E-W greenhouse is less by 2.72% as compared to N-S greenhouse hence the radiative effect is more important in greenhouse while predicting microclimate inside the greenhouse for crop cultivation as supported by Wang and Boulard (2000).


**Figure 5.** Hourly variation of solar radiation transmitted inside symicylindrical greenhouse for the month of December (10/12/2007) at Udaipur.

**Table 1.** Solar radiation received on floor of greenhouse for E-W and N-S orientation of greenhouse.

<table>
<thead>
<tr>
<th>Day of the month</th>
<th>Solar radiation outside greenhouse (MJ/day-m$^2$)</th>
<th>Solar radiation received on greenhouse floor (MJ/day-m$^2$)</th>
<th>E-W</th>
<th>N-S</th>
<th>per cent change</th>
</tr>
</thead>
<tbody>
<tr>
<td>10-12-2006</td>
<td>13.77</td>
<td>8.84</td>
<td>8.75</td>
<td>7.67</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

**APPENDIX-I**

Radiation transmittance through semicylindrical surface

The transmittance through, reflection and absorption of solar radiation by cover of greenhouse is very important in determining the thermal performance of greenhouse. The transmittance, reflectance, and absorption depend on incoming solar radiation and angle of incidence of solar radiation. It also depends on the thickness, refractive index, and extinction coefficient of the cover material.

Reflection of unpolarised radiation on passing from air with refractive index $n_1$ to cover material with refractive index $n_2$ can be given as (Duffie and Beckman, 1971).

$$r_\perp = \frac{\sin^2(\theta - \theta_i)}{\sin^2(\theta - \theta_i)}$$

$$r_\parallel = \frac{\tan^2(\theta - \theta_i)}{\tan(\theta + \theta_i)}$$

Where $\theta_i$ and $\theta$ are the angles of incidence and refraction, $r_\perp$ and $r_\parallel$ are perpendicular and parallel component of unpolarized radiation. The angles $\theta_i$ and $\theta$ are related to the indices of refraction.

$$\frac{n_1}{n_2} = \frac{\sin \theta}{\sin \theta_i}$$

The value of $n_1$ for air is taken as 1.0 and $n_2$ for cover material is taken as 1.45

The components $r_\perp$ and $r_\parallel$ are not equal (except at normal incidence), and the transmittance of initially unpolarized radiation is the average transmittance of the two components,

$$\tau_\perp = \frac{1}{2} \left[ \frac{1 - r_\parallel}{1 + r_\parallel} + \frac{1 - r_\perp}{1 + r_\perp} \right]$$

The absorption of radiation is based on the assumption that the absorbed radiation is proportional to the local intensity in the medium and the distance $x$ the radiation has travelled in the medium and is given by

$$\tau_a = \exp \left[ -\frac{KL}{\cos \theta} \right]$$

Where $K$ is the constant of proportionality, and $L$ is thickness of cover.
Nomenclature: E-W - east-west, \(I_b\) - beam solar irradiance (Wm\(^{-2}\)), \(I_d\) - diffuse solar irradiance (Wm\(^{-2}\)), \(L\) - length of greenhouse, \(n_{cyl}\) - unit vector normal to cylindrical surface, N-S - north-south, \(n_s\) - unit vector in the direction of the beam radiation, \(r\) - radius, \(w\) - total radiation on greenhouse floor, \(w_{df}\) - diffuse radiation on greenhouse floor, \(w_i\) - beam radiation on greenhouse floor, \(i, j, k\) - unit vector along x, y and z axis, \(i', j', k'\) - unit vector along x', y' and z' axis.

Greek letters: \(\alpha\) - altitude angle of sun, \(\beta\) - angle between arbitrary radius vector and z' axis, \(\gamma\) - azimuth angle of sun, \(\gamma_s\) - azimuth angle of greenhouse cover, \(\tau\) - transmittance, \(\beta_1, \beta_2\) - limiting angles for solar radiation on cover, \(\theta_i\) - angle of incidence of solar radiation on greenhouse cover, \(\delta\) - declination of sun.

REFERENCES


