

*Full Length Research Paper*

# Smart crack detection of a cracked cantilever beam using fuzzy logic technology with hybrid membership functions

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Damage detection methods have been considerably increased over the past few decades. A crack in a structural member introduces local flexibility that would affect vibration response of the structure. This property may be used to detect existence of a crack together with its location and depth in the structural member. The presence of damage leads to changes in some of the lower natural frequencies and mode shapes. Damage detection is one of the important aspects in structural engineering both for safety reasons and because of economic benefits that can result. This technique has been used in the present investigation for crack detection. Here the crack is transverse surface crack. The crack is analyzed using fuzzy logic system and finite element analysis. The fuzzy controller uses the hybrid membership functions (combination of triangular, trapezoidal and Gaussian) as input and trapezoidal membership functions as output. The input parameters to the fuzzy controller are the first three natural frequencies. The output parameters of the fuzzy controller are the relative crack depth and relative crack location. Finite element analysis has been done for modeling the cracked cantilever beam. By using Several fuzzy rules the results obtained for crack depth and crack location in the Matlab Simulink environment and have been compared with the results obtained from finite element analysis. It is observed that the fuzzy controller can predict the depths and locations accurately close to the finite element analysis. Finally a deviation of the results obtained by comparing the finite element analysis results and fuzzy controller result.

**Key words:** Damage, vibration, natural frequency, fuzzy logic, membership function, fuzzy controller.

## INTRODUCTION

Many researchers have been carried out in an attempt to find methods for non-destructive crack detection in structural members. Vibration-based methods have been proved as a fast and inexpensive means for crack identification. A crack in a structure induces a local flexibility which affects the dynamic behavior of the whole structure to a considerable degree. It results in reduction of natural frequencies and changes in mode shapes. An analysis of these changes makes it possible to determine the position and depth of cracks. Most of the researches used in their studies are open crack models, that is, they assume that a crack remains always open during vibration. The assumption of an open crack leads to a constant shift of natural frequencies of vibration. Various

studies investigated over the last decades, however, indicate that a real fatigue crack opens and closes during vibration. It exhibits non-linear behavior due to the variation of the stiffness which occurs during the response cycle. As a result, a breathing crack gives rise to natural frequencies falling between those corresponding to the open and closed states. Therefore, if an always open crack is assumed, the decrease in experimental natural frequencies will lead to an under-estimation of the crack depth. Harish and Parhi, 2009 have performed analytical studies on fuzzy inference system for detection of crack location and crack depth of a cracked cantilever beam structure using six input parameters to the fuzzy membership functions. The six input parameters are percentage deviation of first three natural frequencies and first three mode shapes of the cantilever beam. The two output parameters of the fuzzy inference system are relative crack depth and relative

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crack location. Experimental setup has been developed for verifying the robustness of the developed fuzzy inference system. The developed fuzzy inference system can predict the location and depth of the crack in a close proximity to the real results. (Mohammad and Vakil, 2008) have proposed a method in which damage in a cracked structure was analyzed using genetic algorithm technique. For modeling the cracked-beam structure an analytical model of a cracked cantilever beam was utilized and natural frequencies were obtained through numerical methods. A genetic algorithm is utilized to monitor the possible changes in the natural frequencies of the structure. The identification of the crack location and depth in the cantilever beam was formulated as an optimization problem. Norhisham et al. (2007) applied Artificial Neural Network for damage detection. In his investigation an ANN model was created by applying Rosenblueth's point estimate method verified by Monte Carlo simulation, the statistics of the stiffness parameters were estimated. The probability of damage existence (PDE) was then calculated based on the probability density function of the existence of undamaged and damaged states. The developed approach was applied to detect simulated damage in a numerical steel portal frame model and also in a laboratory tested concrete slab. The effects of using different severity levels and noise levels on the damage detection results are discussed. Saridakis (2008) applied neural networks, genetic algorithms and fuzzy logic for the identification of cracks in shafts by using coupled response measurements. In this research the dynamic behavior of a shaft with two transverse cracks characterized by three measures: position, depth and relative angle. Both cracks were considered to lie along arbitrary angular positions with respect to the longitudinal axis of the shaft and at some distance from the clamped end. A local compliance matrix of two degrees of freedom (bending in both the horizontal and the vertical planes) was used to model each crack. Ganguli (2001) has developed a fuzzy logic system (FLS) for ground based health monitoring of a helicopter rotor blade. Structural damage is modeled as a loss of stiffness at the damaged location that can result from delamination. The fuzzy system is trained by a batch least squares algorithm based on desired input-output data so that the trained fuzzy system can behave like the training data. Parhi and Amiya (2009) have presented comprehensive review of methodologies in the domain of dynamic vibration of cracked structures using energy methods, finite element methods, fuzzy inference techniques, neural networks, neuro-fuzzy adaptive techniques and genetic algorithms for identifying the intensity and location of cracks.

## FINITE ELEMENT FORMULATION

### Theory

The beam with a transverse edge crack is clamped at left end, free

at right end and has uniform structure with a constant rectangular cross-section of  $800 \times 50 \times 6$  mm as shown in Figure 1. The Euler-Bernoulli beam model is assumed for the finite element formulation. The crack in this particular case is assumed to be an open crack and the damping is not being considered in this theory. Both single and double edged crack are considered for the formulation.

### Governing equation of free vibration

The free bending vibration of an Euler-Bernoulli beam of a constant rectangular cross section is given by the following differential equation as given in:

$$EI \frac{d^4 y}{dx^4} - m\omega_i^2 y = 0 \quad (1)$$

where 'm' is the mass of the beam per unit length (kg/m), ' $\omega_i$ ' is the natural frequency of the  $i$ th mode (rad/s), E is the modulus of elasticity (N/m<sup>2</sup>) and I is the moment of inertia (m<sup>4</sup>).

By defining  $\lambda^4 = \frac{m\omega_i^2}{EI}$  equation is rearranged as a fourth-order differential equation as follows:

$$\frac{d^4 y}{dx^4} - \lambda^4 y = 0 \quad (2)$$

The general solution to the equation is:

$$y = A \cos \lambda_i x + B \sin \lambda_i x + C \cosh \lambda_i x + D \sinh \lambda_i x \quad (3)$$

where A, B, C, D are constants and ' $\lambda_i$ ' is a frequency parameter. Adopting Hermitian shape functions, the stiffness matrix of the two-noded beam element without a crack is obtained using the standard integration based on the variation in flexural rigidity. The element stiffness matrix of the uncracked beam is given as:

$$[K^e] = \int [B(x)]^T EI [B(x)] dx \quad (4)$$

$$[B(x)] = \{H_1(x)H_2(x)H_3(x)H_4(x)\} \quad (5)$$

where are the Hermitian shape functions defined as:

$$H_1(x) = 1 - \frac{3x^2}{l^2} + \frac{2x^3}{l^3} \quad (6a)$$

$$H_2(x) = x - \frac{2x^2}{l} + \frac{x^3}{l^2} \quad (6b)$$

$$H_3(x) = \frac{3x^2}{l^2} - \frac{2x^3}{l^3} \quad (6c)$$

$$H_4(x) = -\frac{x^2}{l} + \frac{x^3}{l^2} \quad (6d)$$

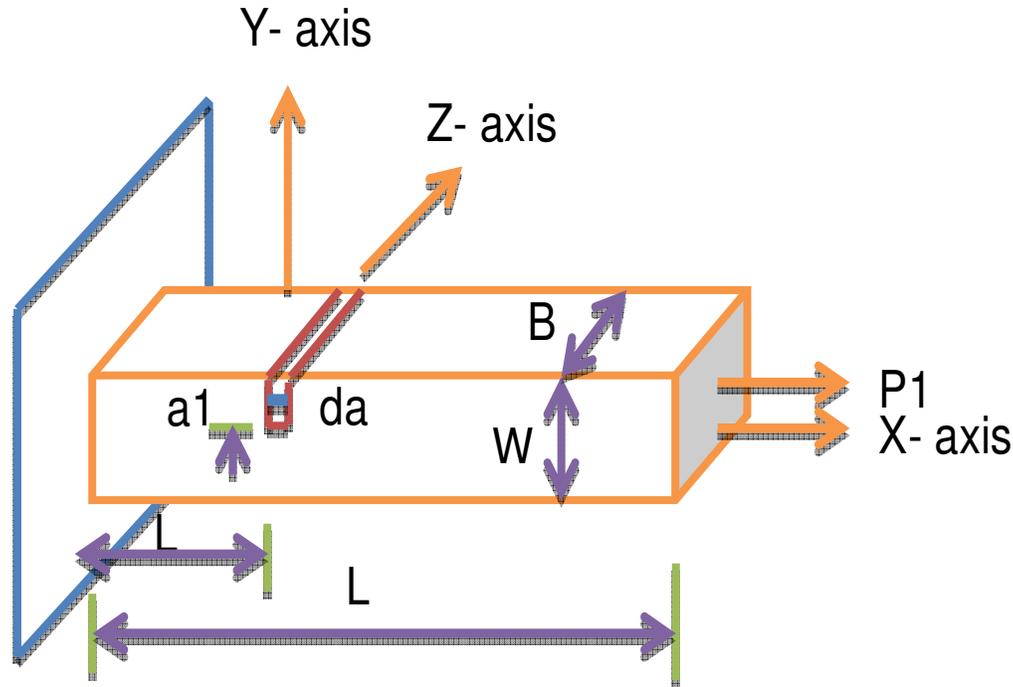


Figure 1. Geometry of Cantilever beam.

Assuming the beam rigidity EI is constant and is given by EI0 within the element, and then the element stiffness is Equation (6):

$$[K^e] = \frac{EI_0}{l^3} \begin{bmatrix} 12 & 6l & -12 & 6l \\ 6l & 4l^2 & -6l & 2l^2 \\ -12 & -6l & 12 & -6l \\ 6l & 2l^2 & -6l & 4l^2 \end{bmatrix} \quad (7)$$

$$[K_c^e] = [K^e] - [K_c] \quad (8)$$

Here,  $[K_c^e]$  = Stiffness matrix of the cracked element,  $[K^e]$  = Element stiffness matrix,  $[K_c]$  = Reduction in stiffness matrix due to the crack.

According to Peng et al. (2007), the matrix  $[K_c]$  is:

$$[K_c] = \begin{bmatrix} K_{11} & K_{12} & -K_{11} & K_{14} \\ K_{12} & K_{22} & -K_{12} & K_{24} \\ -K_{11} & -K_{12} & K_{11} & -K_{14} \\ K_{14} & K_{24} & -K_{14} & K_{44} \end{bmatrix} \quad (9)$$

where:

$$K_{11} = \frac{12E(I_0 - I_c)}{L^4} \left[ \frac{2l_c^3}{L^2} + 3l_c \left( \frac{2L_1}{L^2} - 1 \right)^2 \right] \quad (9a)$$

$$K_{12} = \frac{12E(I_0 - I_c)}{L^3} \left[ \frac{l_c^3}{L^2} + l_c \left( 2 - \frac{7L_1}{L} + \frac{6L_1^2}{L^2} \right)^2 \right] \quad (9b)$$

$$K_{14} = \frac{12E(I_0 - I_c)}{L^3} \left[ \frac{l_c^3}{L^2} + l_c \left( 2 - \frac{5L_1}{L} + \frac{6L_1^2}{L^2} \right)^2 \right] \quad (9c)$$

$$K_{22} = \frac{12E(I_0 - I_c)}{L^3} \left[ \frac{3l_c^3}{L^2} + 2l_c \left( \frac{3L_1}{L} - 2 \right)^2 \right] \quad (9d)$$

$$K_{24} = \frac{12E(I_0 - I_c)}{L^2} \left[ \frac{3l_c^3}{L^2} + 2l_c \left( 2 - \frac{9L_1}{L} + \frac{9L_1^2}{L^2} \right)^2 \right] \quad (9e)$$

$$K_{44} = \frac{12E(I_0 - I_c)}{L^2} \left[ \frac{3l_c^3}{L^2} + 2l_c \left( \frac{3L_1}{L} - 1 \right)^2 \right] \quad (9f)$$

Here,  $l_c = 1.5W$ ,  $L$  = Total length of the beam,  $L_1$  = Distance between the left node and crack,  $I_0 = \frac{BW^3}{12}$  = Moment of inertia of the beam cross section,  $I_c = \frac{B(W-a)^3}{12}$  = Moment of inertia of the

beam with crack. It is supposed that the crack does not affect the mass distribution of the beam. Therefore, the consistent mass matrix of the beam element can be formulated directly as:

$$[M^e] = \int_0^1 \rho A [H(x)]^T [H(x)] dx \tag{10}$$

$$[M^e] = \frac{\rho A l}{420} \begin{bmatrix} 156 & 22l & 54 & -13l \\ 22l & 4l^2 & 13l & -3l^2 \\ 54 & 13l & 156 & -22l \\ -13l & -3l^2 & -22l & 4l^2 \end{bmatrix} \tag{11}$$

The natural frequency then can be calculated from the relation:

$$[-\omega^2 [M] + [K]] \{q\} = 0 \tag{12}$$

where: q=displacement vector of the beam.

### ANALYSIS OF FUZZY LOGIC SYSTEM FOR CRACK DETECTION

Fuzzy logic is a tool for Embedding Human structured knowledge (Experience, Expertise and Heuristic). P. L. Zadeh says: "Fuzzy logic may be viewed as a bridge over the excessively wide gap between the precision of classical crisp logic and the imprecision of both the real world and its human interpretation". Fuzzy logic attempts to model the way of reasoning that goes in the human brain. Almost all of human experience is stored in the form of the If-Then rules. Human reasoning is pervasively approximate, non-quantitative, linguistic and dispositional. Fuzzy logic can be explained in the following steps.

#### Fuzzy set

A fuzzy set, as the name implies, is a set without a crisp boundary. That is the transition from "belongs to a set" to not belong to a set is gradual and this smooth transition is characterized by membership functions that give fuzzy sets flexibility in modeling commonly used linguistic expressions. A membership function assigns to each element in the set under consideration a membership grade, which is a value in the interval [0, 1].

#### Membership function

The basic structure of a fuzzy interface system consists of three components: a rule base, which contains a selection of fuzzy rules, a database which defines the membership functions used in the fuzzy rules and a reasoning mechanism, which performs the interface procedure. The membership function  $\mu_A(x)$  describes the membership of the elements  $x$  of the base set  $X$  in fuzzy

set  $A$ , where by  $\mu_A(x)$  a large class of function can be taken. Reasonable functions are often piecewise linear function, such as triangular or trapezoidal functions. The value for the membership function can be taken in the interval [0, 1]. When the functions are nonlinear the Gaussian membership function will be taken for the smooth operation.

#### Fuzzy logic

In Crisp logic, the truth values acquired by proposition or predicates are 2-valued, namely True, False which may be treated numerically equivalent to (0, 1). However in fuzzy logic, truth values are multivalued such as absolutely false, partly true, absolutely false, and very true and so on and are numerically equivalent to 0-1.

#### Fuzzy linguistic variables

Just like an algebraic variable takes numbers as values, a linguistic variable takes words or sentences as values. The set of values that it can take is called its term set. Each value in the term set is a fuzzy variable defined over a base variable. The base variable defines the universe of discourse for all the fuzzy variables in short. In short the hierarchy is as follows: Linguistic variable → Fuzzy variable → Base variable.

#### Fuzzy if-then rule

A fuzzy if-then rule (also known as fuzzy rule, fuzzy implication or fuzzy conditional statement) assumes the form "if  $x$  is  $A$  then  $y$  is  $B$ ". Where  $A$  and  $B$  are linguistic values defined by fuzzy sets on universes of discourse  $x$  and  $y$  respectively. Often " $x$  is  $A$ " is called the antecedent or premise, while " $y$  is  $B$ " is called the consequence or conclusion (Some of the linguistic terms used are shown in Table 1).

#### Fuzzy mechanism used for crack detection

The fuzzy controller (as shown in Figure 2) has been developed where there are 3 inputs and 2 outputs parameter. The natural linguistic representations for the input are as follows:

- Relative first natural frequency = "FNF"
- Relative second natural frequency = "SNF"
- Relative third natural frequency = "TNF"

- The natural linguistic term used for the outputs are
- Relative crack depth = "RCD"
- Relative crack length= "RCL"

Based on the above fuzzy subset the fuzzy rules are defined in a general form as follows:

If (FNF is FNF<sub>i</sub> and SNF is SNF<sub>j</sub> and TNF is TNF<sub>k</sub>) then

**Table 1.** Linguistic term used for membership functions.

Name of the membership function	Linguistic terms	Description and range of the linguistic terms
L1F1,L1F2,L1F3	fnf <sub>1to3</sub>	Low ranges of relative natural frequency for first mode of vibration in ascending order respectively.
M1F1,M1F2,M1F3	fnf <sub>4,6</sub>	Medium ranges of relative natural frequency for first mode of vibration in ascending order respectively.
H1F1,H1F2,H1F3	fnf <sub>7,9</sub>	Higher ranges of relative natural frequency for first mode of vibration in ascending order respectively
L2F1,L2F2,L2F3	snf <sub>1to3</sub>	Low ranges of relative natural frequency for second mode of vibration in ascending order respectively.
M2F1,M2F2,M2F3	snf <sub>4,6</sub>	Medium ranges of relative natural frequency for second mode of vibration in ascending order respectively.
H2F1,H2F2,H2F3	snf <sub>7to9</sub>	Higher ranges of relative natural frequency for first mode of vibration in ascending order respectively
L3F1,L3F2,L3F3	tnf <sub>1to3</sub>	Low ranges of relative natural frequency for second mode of vibration in ascending order respectively
M3F1,M3F2,M3F3	tnf <sub>4,6</sub>	Medium ranges of relative natural frequency for second mode of vibration in ascending order respectively
H1F1,H1F2,H1F3	tnf <sub>7to9</sub>	Higher ranges of relative natural frequency for first mode of vibration in ascending order respectively
SD1,SD2,SD3	rcl <sub>1to3</sub>	Small ranges of relative crack depth in ascending order respectively.
MD1,MD2,MD3	rcl <sub>4to6</sub>	Medium ranges of relative crack depth in ascending order respectively
LD1,LD2,LD3	rcl <sub>7to9</sub>	Larger ranges of relative crack depth in ascending order respectively.
SL1,SL2,SL3	rcl <sub>1to3</sub>	Small ranges of relative crack location in ascending order respectively.
ML1,ML2,ML3	rcl <sub>4to6</sub>	Medium ranges of relative crack location in ascending order respectively.
BL1,BL2,BL3	rcl <sub>7to9</sub>	Bigger ranges of relative crack location in ascending order.

(CD is CD<sub>ijk</sub> and CL is CL<sub>ijk</sub>)

Where i= 1to 9, j=1 to 9, k=1 to 9

(13)

Because of “FNF”, “SNF”, “TNF” have 9 membership functions each.

From the above expression (13), two set of rules can be written:

If (FNF is FNF<sub>i</sub> and SNF is SNF<sub>j</sub> and TNF is TNF<sub>k</sub>) then CD is CD<sub>ijk</sub> (14a)

If (FNF is FNF<sub>i</sub> and SNF is SNF<sub>j</sub> and TNF is TNF<sub>k</sub>) then CL is CL<sub>ijk</sub> (14b)

According to the usual Fuzzy logic control method (Harish and Parhi, 2008), a factor Wijk is defined for the

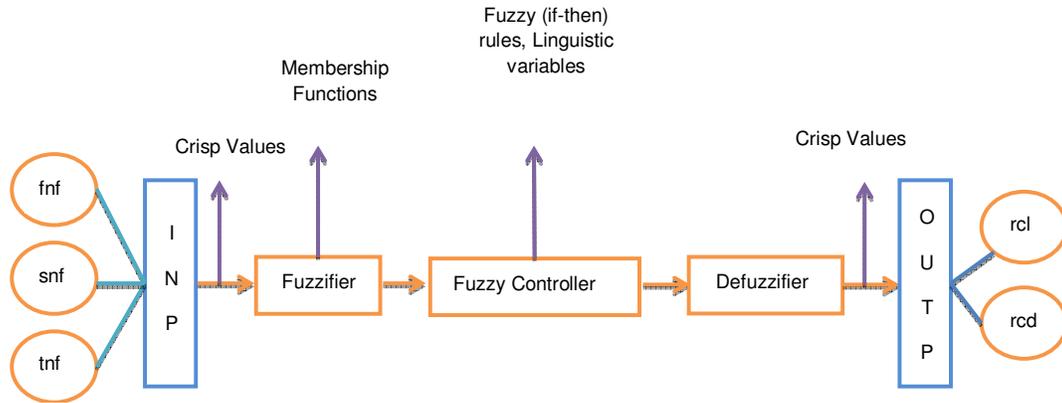


Figure 2. Fuzzy controller.

rules as follows:

$$W_{ijk} = \mu_{fnf_i}(freq_i) \wedge \mu_{snf_j}(freq_j) \wedge \mu_{tnf_k}(freq_k)$$

where  $freq_i$ ,  $freq_j$  and  $freq_k$  are the first, second and third natural frequency of the cantilever beam with crack respectively; by applying composition rule of interference (Harish and Parhi, 2008) the membership values of the relative crack location and relative crack depth (location)CL:

$$\mu_{rcl}(location) = W_{ijk} \wedge \mu_{rcl}(location) \text{ length CL}$$

As:

$$\mu_{rcl}(depth) = W_{ijk} \wedge \mu_{rcl}(depth) \text{ depth CD}$$

The overall conclusion by combining the output of all the fuzzy can be written as follows:

$$\mu_{rcl}(location) = \mu_{rcl111}(location) \vee \dots \vee \mu_{rclijk}(location)$$

$$\vee \dots \vee \mu_{rcl999}(location) \tag{15a}$$

$$\mu_{rcl}(location) = \mu_{rcl111}(depth) \vee \dots \vee \mu_{rclijk}(depth)$$

$$\vee \dots \vee \mu_{rcl999}(depth) \tag{15b}$$

The crisp values of relative crack location and relative crack depth are computed using the center of gravity method (Das et al., 2008) as:

$$Relative\ crack\ location = rcl = \frac{\int location \cdot \mu_{rcl}(location) \cdot d(location)}{\int \mu_{rcl}(location) \cdot d(location)} \tag{16a}$$

$$Relative\ crack\ depth = rcd = \frac{\int depth \cdot \mu_{rcd}(depth) \cdot d(depth)}{\int \mu_{rcd}(depth) \cdot d(depth)} \tag{16b}$$

### WHY WE USE FUZZY LOGIC

- (i) Provides an easy to use interface for applying modern fuzzy logic techniques.
- (ii) Easily integrated into Model-Based design through the use of the Simulink blocks.
- (iii) Provides the ability to use fuzzy logic when appropriate with other control techniques.
- (iv) Provides the ability to generate code for various uses.
- (v) Supplies a fuzzy inference engine that can execute the fuzzy system as a stand-alone application.

### DISCUSSION AND CONCLUSION

In this paper a cantilever beam with a single crack has been taken into consideration. The change in local flexibility due to the presence of the crack is used to calculate the change in the natural frequencies of the cantilever beam. For this theoretical analysis has been used.

Finite element method is used to find out the natural frequencies of the faulty cantilever beam. A fuzzy controller has been designed using trapezoidal, Gaussian as well as triangular membership function to find out the crack depth and crack location (as shown in Figure 3). Table 1 presents the linguistic terms used for membership functions and the range of the linguistic terms. For the fuzzy controller some fuzzy rules have been formulated, which is described in Table 2. The operation of fuzzy controller has been shown through an example as shown in Figure: 4. Table 3 represents the results of fuzzy logic controller as well as the deviation in result.

The above system is modeled and simulated in the Matlab Simulink environment for the validation of the result which is shown above. It has been observed that the applied fuzzy controllers can predict the relative crack location, relative crack depth of the beam with a considerably less amount of computational time as compared to Finite element analysis.

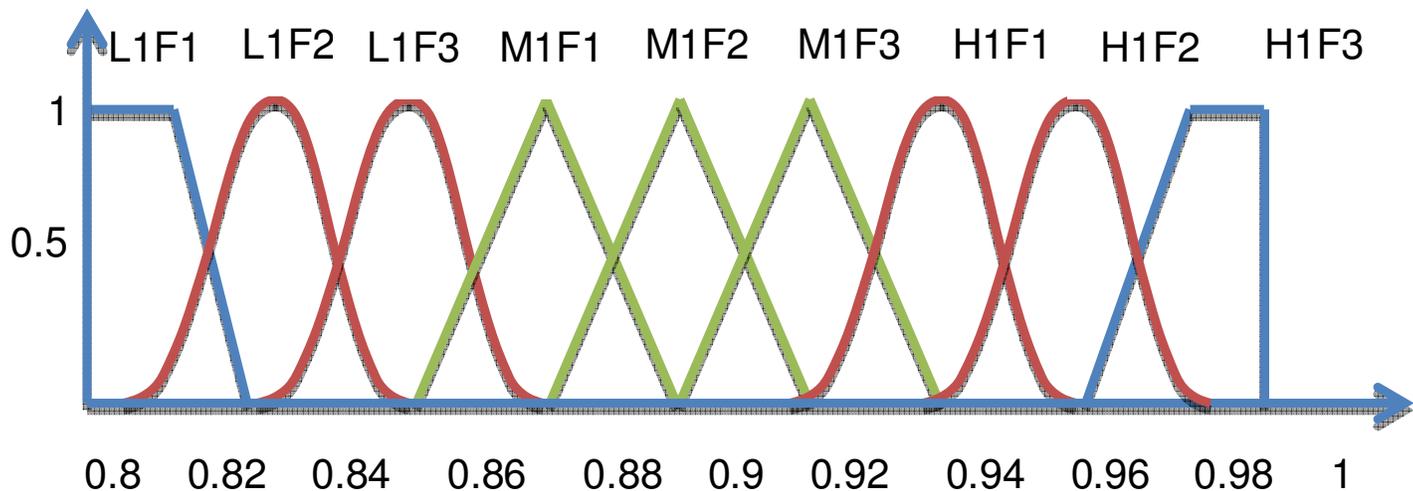


Figure 3(a). Hybrid Membership functions for relative natural frequency for 1st mode of vibration.

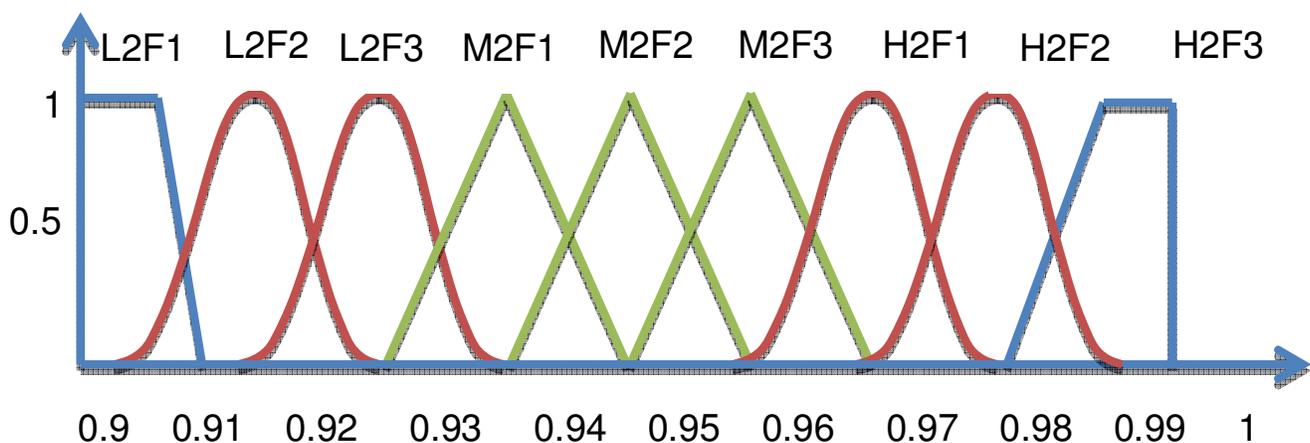


Figure 3(b). Hybrid Membership functions for relative natural frequency for 2nd mode of vibration.

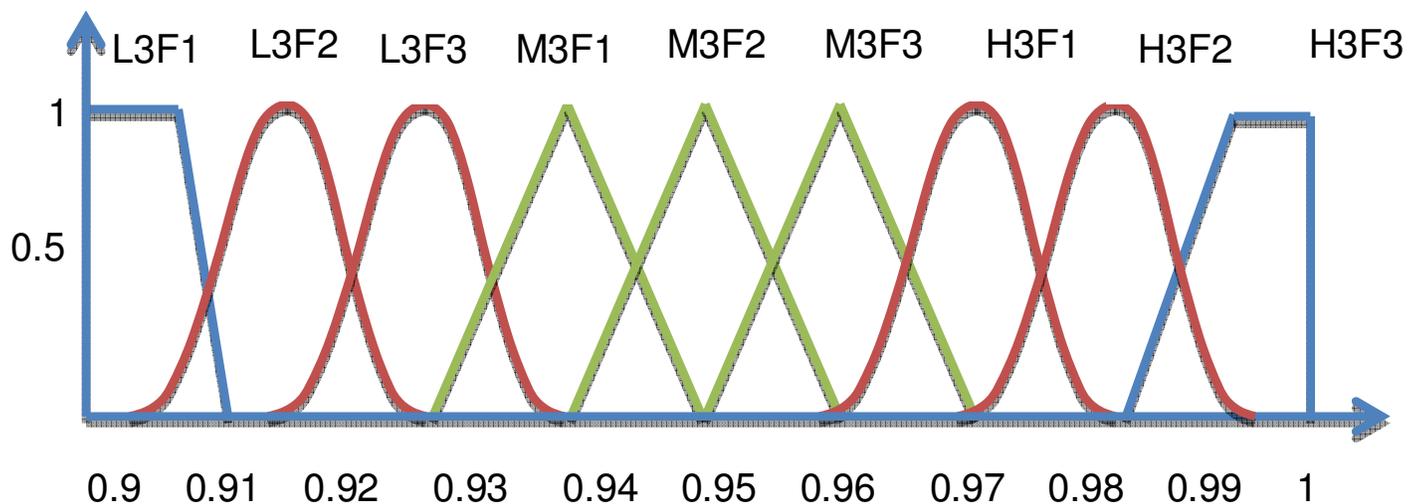


Figure 3(c). Hybrid Membership functions for relative natural frequency for 3rd mode of vibration.

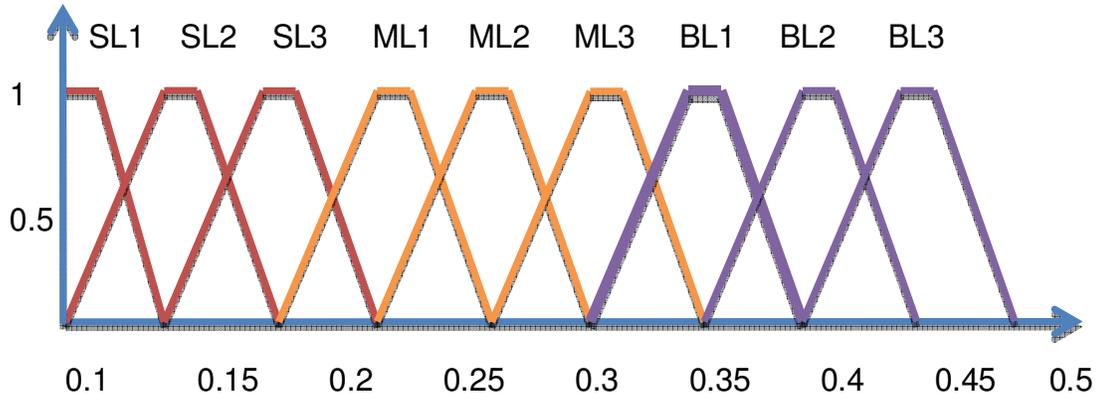


Figure 3(d). Trapezoidal Membership functions for relative crack location.

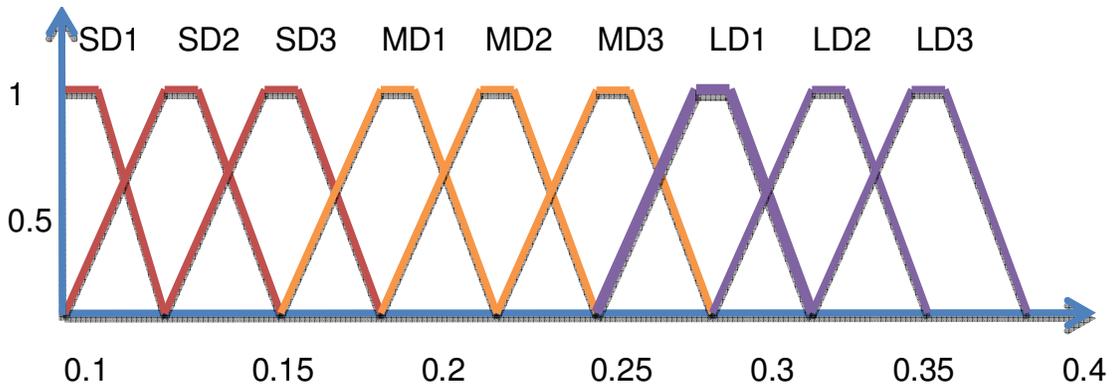


Figure 3(e). Trapezoidal Membership functions for relative crack depth.

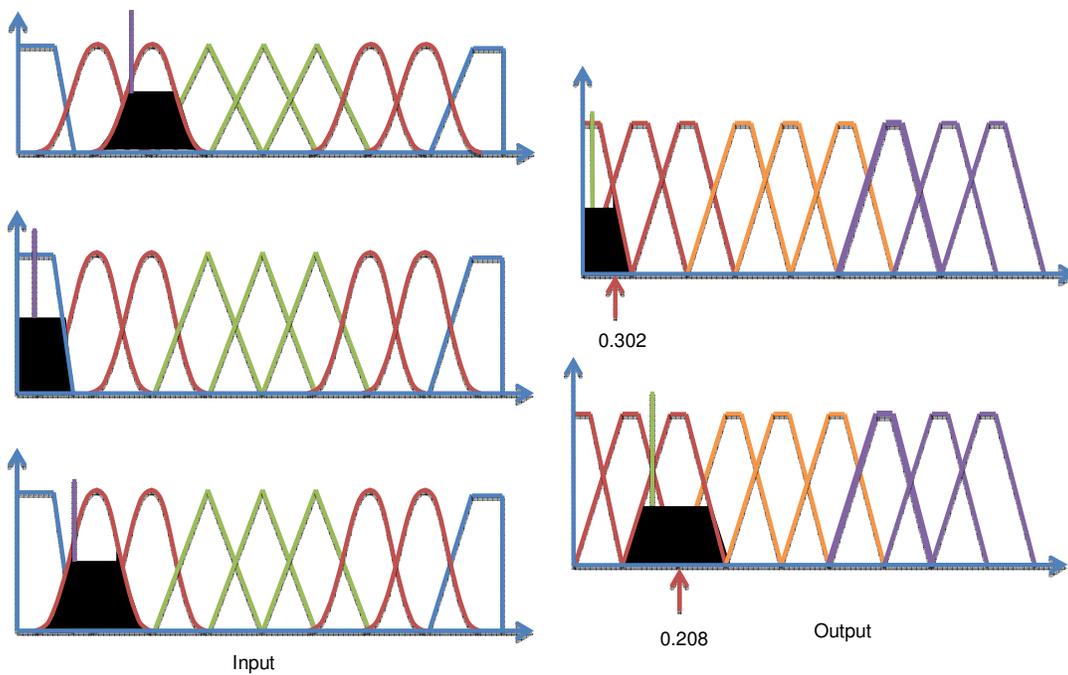


Figure 4. Rule No. 6 of Table 2 is activated.

**Table 3.** Example of Input data and results of fuzzy controller.

Sl.no	Relative first natural frequency	Relative second natural frequency	Relative third natural frequency	Relative crack depth	Relative crack location	Fuzzy controller relative crack depth	Fuzzy controller relative crack location	% Deviation in result (crack depth)	% Deviation in result (crack location)
1	0.870	0.914	0.985	0.225	0.135	0.216	0.139	4	2.96
2	0.878	0.945	0.992	0.312	0.257	0.307	0.265	1.602	3.12
3	0.885	0.969	0.995	0.282	0.220	0.273	0.230	3.19	4.54
4	0.904	0.973	0.974	0.265	0.235	0.257	0.229	3.01	2.55
5	0.915	0.981	0.979	0.230	0.270	0.226	0.267	1.73	1.11
6	0.936	0.991	0.955	0.216	0.3125	0.208	0.302	3.7	3.36
7	0.947	0.995	0.969	0.2	0.375	0.21	0.391	5	4.26
8	0.918	0.929	0.974	0.283	0.4	0.263	0.388	7.06	3
9	0.932	0.955	0.995	0.166	0.237	0.162	0.212	2.4	10.54
10	0.976	0.980	0.929	0.15	0.268	0.161	0.252	7.33	5.97

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