An accurate estimation of the lateral force is an important key point for the stability analysis, because the lateral force influences both pile and slope stabilities. Therefore, overestimation of the lateral forces on the pile will naturally lead to conservative pile design and un-conservative slope stability design and vice versa. This is because the estimated lateral loads are used as extra resisting force(s) on slope stability calculations due to the pile reaction. In this paper, 2 different methods of lateral force calculations namely plastic deformation and visco-plastic flow are obtained on a pile that installed into slope which is susceptible to failure. First, the case of a slope without piles is considered and the methods are then used to calculate the safety factor for the slope. Then, the stability of slopes reinforced with piles is analyzed. Expressions are derived allowing the force needed to increase the safety factor to a desired value and the most suitable location of piles within the slope to be evaluated.

Key words: Lateral soil movement, soil-pile interaction, finite difference, slope stability, safety factor.

INTRODUCTION

Failures of mass movements often result in extensive property damage and loss of human life. It is recognized that ensuring the stability of both natural and man-made slopes continues to be a fundamental issue in geotechnical engineering. There is no universally accepted method for the prevention or correction of landslides. Each slide is unique and should be considered on the basis of unique inherent characteristics.

Stabilization of a slope may depend on a number of factors such as; its geometry, surface and groundwater conditions, strength of materials and the reason for stabilization. A number of techniques have been developed to stabilize slopes considering the above mentioned conditions (Abramson et al., 1996).

Piles have been used successfully in many situations in order to stabilize slopes or to improve slope stability (Ito and Matsui, 1975; Anagnastopoulos et al., 1991; Popescu, 1991; Poulos, 1995; Chen and Poulos, 1997 and Won et al., 2005) and numerous methods have been developed for the analysis of piled slopes.

The limit equilibrium method was used by Ito et al. (1979) to deal with the problem of the stability of slopes containing piles. In this approach the safety factor of the piled slope was defined as the ratio of the resisting moment to the overturning moment acting on the potentially unstable soil mass. The driving moment and the resisting moment due to soil shearing resistance were obtained applying the ordinary slice method. To calculate the resisting moment due to the piles, Ito et al. (1979) proposed the use of the theoretical equation, derived previously by Ito and Matsui (1975), to evaluate the lateral force acting on a row of piles due to soil movement.

A similar approach was developed by Lee et al. (1995) in which Bishop's simplified method (Bishop, 1955) was employed to find the critical sliding surface for the slope as well as the driving moment and resisting moment due to soil shearing resistance. These forces were calculated using a procedure based on the boundary element method.

Hassiotis et al. (1997) have extended the friction circle method to incorporate the pile reaction in slope stability analysis. The Ito and Matsui equation (1975) has been used to evaluate the lateral force that the failing soil mass exerts on a row of piles.

Ausilio et al. (2001) have used the kinematic approach of limit analysis for the stability of slopes that are reinforced with piles. The case of a slope without piles is first considered, and a solution is proposed to determine the safety factor of the slope, which is defined as a reduction coefficient for the strength parameters of the soil. Then, the stability of a slope containing piles is analyzed. To account for the presence of the piles, it is assumed that a lateral force and a moment are applied at the depth of the potential sliding surface. For simplicity, the effect of pore
water pressure on slope stability is not considered.

The influence of the one row of pile groups on the stability of the weathered slope was investigated by Jeong et al. (2003) based on an analytical study and a numerical analysis. An uncoupled analysis, in which the pile response and slope stability are considered separately, was performed by quantifying the load transfer of passive pile groups subjected to lateral soil movements in slope.

Recently, Martin and Chen (2005) have used displacement method to evaluate the response of piles caused by an embankment slope in a translational failure mode, induced by a weak layer or a liquefied layer beneath the embankment. The analysis includes the kinematic loading acting on the bridge piles caused by lateral soil movements and the effects of spatial variation of soil displacement on the response of piles.

The interaction behaviour between pile and soil is a complicated phenomenon due to its 3-dimensional nature and can be influenced by many factors, such as the characteristics of deformation and the strength parameters of both pile and soil. In addition, movement of the failure mass (landslide) involves a complicated mechanism. It is recognized that the pile design for slope stabilization is too simplified for the complicated event. Nevertheless, there is no precise design method for stabilizing piles so as to utilize effectively the pile effect in slope stabilization. In fact, many slopes failed or continuously moved even after the installation of stabilizing piles (Ito et al., 1982).

In this paper three different methods of lateral force estimations have been identified. 2 of these methods are codified to provide predictive numerical values for the lateral forces on piles installed for stabilizing slopes. Although historically there are different methods for lateral force estimations, only the developments due to Ito and Matsui (1975) and De Beer and Carpentier (1977) and current study are presented. Soil reinforcement by means of piles bored through the sliding mass to the stable underlying soil is often used as a countermeasure to stabilize landslides. With this case the following design steps applied; Evaluating the total shear force needed to increase the safety factor of the slope by the desired value and calculating the maximum lateral force that each pile can receive from the sliding soil and transmits to the stable underlying soil.

**Forces on piles in soil undergoing lateral movements**

**Ito and Matsui method**

Ito and Matsui (1975) and Ito et al. (1979, 1982) have developed an analysis to show that piles placed in plastically deforming ground, such as landslide mass movement, can prevent further plastic deformations.

The theory proposed by the above authors, called the theory of plastic deformation, estimates the lateral force between the 2 extremes of zero and large landslide mass movement. It is assumed that no reduction in shear resistance along the sliding surface has taken place as a result of strain-softening caused by the movement in a potential landslide. For this reason it is assumed that the state of plastic equilibrium occurs only in the soil just around the piles (Figure 1), satisfying the Mohr-Coulomb yield criterion. Therefore, the lateral force acting on the pile may be estimated neglecting the change of equilibrium condition of the whole slope. When the stress distribution in the soil AEBB’E’A’ is considered, the frictional forces on surfaces AEB and A’E’B’ are neglected.

Therefore, a theoretical equation for the lateral force \( p \) per unit length may be given as follows:

\[
p = A e \left[ \frac{N_{\phi} \log \left( \frac{N_{1}^{1/2} + N_{\phi} \tan(\pi/4) \phi}{N_{1}^{1/2} \tan(\pi/4) \phi - 2 \phi} \right)}{D_{1}^{1/2} - D_{2}^{1/2}} \right] - 2 N_{\phi} \tan(\pi/4) \phi - 2 N_{\phi} \tan(\pi/4) \phi - 1
\]

\[
= \frac{2 N_{\phi} \tan(\pi/4) \phi + N_{\phi}^{1/2} - 2 N_{\phi} \tan(\pi/4) \phi}{N_{\phi} \tan(\pi/4) \phi - 1} + 2 N_{\phi} \tan(\pi/4) \phi \frac{N_{1}^{1/2} \tan(\pi/4) \phi}{N_{1}^{1/2} - 1} - N_{\phi} \tan(\pi/4) \phi - 1
\]

(1)

Where \( N_{\phi} = \tan(\pi/4)(\phi/2) \) and

\( A = D_{1} \left[ \frac{N_{1}^{1/2} \tan(\pi/4) \phi + N_{\phi}^{1/2}}{N_{1}^{1/2} - N_{\phi}^{1/2}} \right] \)

Hence, the total corresponding lateral force induced per unit length of the pile due to the plastically deforming soil layer will be obtained by the integration of Equation (1) along the soil depth layer (or until a critical slip surface).

In the case of a purely frictional soil (putting the cohesion \( c = 0 \) in Equation (1) the lateral force, \( p \), per unit length may be obtained by integrating Equation (2) along the depth, \( z \):

\[
p = e \left[ D_{1} \left( 3 \log \left( \frac{D_{1}^{1/2} + D_{2}^{1/2} \tan(\pi/8)}{D_{1}^{1/2} - D_{2}^{1/2}} \right) - 2(D_{1} - D_{2}) \right) - \gamma z (D_{1} - D_{2}) \right]
\]

(3)

**De Beer and Carpentier method**

The theory of plastic deformation was first proposed by Ito and Matsui (1975) and later discussed by De Beer and
Carpentier (1977). The latter authors developed comparable equations by modifying the Ito and Matsui’s method and gave the following equations as another way of obtaining the lateral force \( p \) per unit length induced on the piles.

\[
p = \frac{\gamma}{N_\phi} \left[ 1 + \frac{\sin \phi}{2} \right] \left[ D_1 \left( \frac{D_1}{D_2} \right)^{F_1(\phi)} e^{\frac{D_1-D_2}{N_\phi}} - D_2 \right] + c \cot \phi \\
\left[ D_1 \left( \frac{D_1}{D_2} \right)^{F_1(\phi)} e^{\frac{D_1-D_2}{N_\phi}} - D_1 - D_2 \frac{1 + \sin \phi}{2} + D_2 \right]
\]

(4)

Where:

\[ F_1(\phi) = \frac{N_\phi}{\tan \left( \frac{\pi}{4} + \frac{\phi}{2} \right)} \left( 1 - \sin \phi \right) \tan \phi + N_\phi \left( 1 - \sin \phi \right) - 1 \]

and

\[ F_2(\phi) = \frac{1 - \sin^2 \phi}{1 + \sin^2 \phi} \tan \phi \tan \left( \frac{\pi}{8} + \frac{\phi}{4} \right) \]

The lateral force, \( p \), acting on the piles is obtained by integrating Equation (4) along the depth of the soil layer. For the case of cohesionless soil \( c = 0 \) in Equation (4), by integrating along the depth the lateral force per unit length may be obtained as:

\[
p = \frac{\gamma}{N_\phi} \left[ 1 + \frac{\sin \phi}{2} \right] \left[ D_1 \left( \frac{D_1}{D_2} \right)^{F_1(\phi)} e^{\frac{D_1-D_2}{N_\phi}} - D_2 \right] + \left( D_1 \right)
\]

(5)

For the case of cohesive soil, putting \( \phi = 0 \) and by integrating along the depth of the pile, the lateral force per unit length may be obtained as:

\[
p = c \left[ D_1 \left( 2 \ln \frac{D_1}{D_2} + \frac{D_1-D_2}{D_2} \tan \frac{\pi}{8} \right) - \frac{3}{2} \left( D_1 - D_2 \right) \right] + \left( D_1 \right)
\]

(6)

Visco-plastic flow

The visco-plastic flow, past a row of piles, is presented in following sections. The assumptions for which may be summarized as follows (Figure 2):

1) The visco-plastic flow is two-dimensional and is assumed to be uniform in the direction of depth.
2) Flow past a cylinder is stationary, steady and symmetrical.
3) The fluid is deemed incompressible.
4) Coupled non-linear pair of partial differential equations was used to obtain a stream function \( \psi \) and vorticity \( \omega \) using finite difference solution techniques.
5) In order to analyze the flow past a row of piles, a purely viscous fluid model has been used.
6) The soil layer is represented by a Bingham plastic mo-
Far field

Flow velocity (V)

Figure 2. Visco-plastic flow past a cylindrical pile.

Numerical model of visco-plastic flow

Due to rectangular nature of the flow domain, it is convenient to use a finite difference scheme to solve the coupled equations numerically. A mesh is scribed on the flow domain in the \( x \) and \( q \) directions where the interval between grid lines in the \( x \) direction is \( x = \xi / M \) and in the \( q \) directions \( q = \psi / N \), for positive integers \( M \) and \( N \) (Figure 3). On each grid point \((i\xi, j\psi)\) a dependent variable \( Z \) can be written as \( Z_{ij} \). Due to the scaling mechanism \( r = e^\xi \), the \( r \) grid lines are compressed near the cylinder allowing greater field definition.

Conventional second order accurate finite differences are adopted to approximate the partial derivations involved. The second order Woods formulation which couples \( v \) and \( w \) to set the missing \( w \) boundary value on the cylinder is used as:

\[
d = \frac{1}{2}(v_{ij} + v_{i+1,j}) \]

Further terms used are:

\[
C_{ij} = h^2 (\eta_{ij}^2 + \eta_{i-1,j}^2) + 0.5 Re \eta_{ij} h (v_{i+1,j} - v_{i-1,j}) \quad (12)
\]

\[
D_{ij} = h^2 (\eta_{ij}^2 + \eta_{i+1,j}^2) - 0.5 Re \eta_{ij} h (v_{i+1,j} - v_{i-1,j}) \quad (13)
\]

Where \( q_{ij} \) is given by:

\[
q_{ij} = 2\sqrt{\xi_{ij}^2 + \psi_{ij}^2} \quad (20)
\]

There are special provisions for the use of the latter formulae at the boundaries of the flow domain and at the domain corners. Since the equations are non-linear, an iterative approach will be adopted that requires initial guess starting values for \( v_{ij} \) and \( w_{ij} \). Although it is usual to adopt zero starting values, the solution for the case of a Newtonian zero Re-flow situation leads to a simple solution that is adopted for the general case, and leads to
significant reduction in program execution times. Therefore:

\[
\psi = A \left( e^{2\xi} - 4\xi e^{\xi} - e^{-\xi} \right) + B \left( 1 - 2\xi e^{\xi} - e^{-\xi} \right) \sin \theta \tag{21}
\]

\[
\omega = 4 \left[ 2A \left( e^{\xi} - e^{-\xi} \right) - Be^{\xi} \right] \sin \theta (1 - (\xi / \xi_0)) \tag{22}
\]

Where the condition \( \omega = 0 \) at \( \xi = \xi_0 \) has been enforced; here

\[
A = 1/2 \left( 1 - \xi_0 \right) X_0^2 - (1 + \xi_0) \right) \tag{23}
\]

\[
B = \left( X_0^2 - 1 \right) A \quad X_0 = e^{\xi_0} \tag{24}
\]

Also the viscosities \( \eta_{ij} \) are set to a constant value appropriate to the viscous model used (Fırat et al., 2006). Since an iterative approach needs to be adopted, the following sequential steps were used:

a) Set \( \psi_{ij}^0 \) and \( \omega_{ij}^0 \) as the initial guess values for \( \psi_{ij} \) and \( \omega_{ij} \).

b) Then calculate interior \( \psi_{ij}^1 \) using Equation (8), where the values are the updated currently available, \( \psi \) and \( \omega \) values, in a single sweep of a Gauss-Seidel procedure.

c) Subsequently \( \omega_{ij}^0 \) values are calculated using Equation (9).

d) Then calculate updated \( \eta_{ij} \) values for the whole flow domain.

e) Test for convergence between iterations \( k \) and \( k+1 \) for a quantity \( Z_{ij} \) as:

\[
\left| Z_{ij}^{k+1} - Z_{ij}^k \right| / \left| Z_{ij}^k \right| < TolZ \tag{25}
\]

For all \( i \) and \( j \) with \( Tol\psi \) and \( Tol\omega \) (convergence tolerance values) both set as appropriate small values. This convergence test gives a combination both of absolute and relative comparisons of successive iteration values.

f) Finally a smoothing strategy is adopted by setting:

\[
\overline{Z}_{ij}^{k+1} = S Z_{ij}^k + (1 - S) Z_{ij}^{k+1} \tag{26}
\]

where \( S \) is a smoothing parameter \( (0 < S < 1) \) and the \( \overline{Z}_{ij}^{k+1} \) values are used in subsequent calculations. This helps convergence in difficult cases.

Once satisfactory convergence has been achieved the dimensionless force per unit length on the cylinder may be calculated using a numerical procedure. Since the methods adopted thus far have been second order, the Trapezium second order integration rule was used to yield:

\[
p = 2 d\theta \sum_{j=1}^{N-1} \left\{ \eta_{ij} \omega_{ij} - \left[ \partial \omega / \partial \xi \right]_{ij} \sin (j d\theta) \right\} \tag{27}
\]

Where the derivative term was calculated using the second order forward difference approximation for a quantity \( Z \) as:

\[
(\partial Z / \partial \xi)_{ij} = \left[ 4Z_{ij} - Z_{ij+1} - 3Z_{ij-1} / 2 d \xi \right] \tag{28}
\]

Noting that at \( \theta = 0 \) and \( \theta = \pi \) \( \omega \) and \( \partial \omega / \partial \xi \) are both zero, simplifies the calculation of force on the pile, \( p \), above.

**Comparison of forces for benchmark results**

Three different lateral force estimations are shown in Figure 4 which illustrates the variation of the lateral force on the pile versus the ratio \( D_2 / D_1 \). It can be seen that the visco-plastic flow displays a trend where lateral load predictions lie between the data given by Ito and Matsui (1975), De Beer and Carpentier (1977). It is seen that in addition to parametric variation of inter-pile distance, \( D_2 / D_1 \), the mechanical properties of the flowing soil mass has also been varied from \( \tau_c = 10 \text kN/m}^2 \) to 50 and then 100. KONS (KONS = \( V_{\eta_{ij}} \)) is a product of flow velocity and plastic viscosity. The following numbers represent the relevant research work and can be found in the legend of Figure 4:

2) Represents Ito and Matsui in terms of passive earth pressure (1978).
3) Represents Ito and Matsui in terms of active pressure (1975).
4) Represents current study.

It is also noted that lateral forces converge in all four methods as shear stress \( \tau_c \) tends to zero.

According to the results obtained by the present visco-plastic flow, the Ito and Matsui (1975) and the De Beer and Carpentier (1977) methods the lateral force on the pile becomes infinite when \( D_2 = 0 \), that is, when the interspaces between the piles becomes zero (Figure 1). This does not correspond to reality because the forces generated by the piles cannot be larger than those needed for the equilibrium of the soil mass which is located "upstream" of the pile row (in the event of a potential landslide). For this reason it is suggested that the gap \( (D_2) \) between two piles should be \( 2/3 a \), where \( a \) is the diameter of the pile used.

**Stability analysis of slopes reinforced with piles**

When the safety factor for a slope is considered to be inadequate, slope stability may be improved installing a
retaining structure such as a row of piles (Figure 5). The piles should be designed to provide the stabilizing force needed to increase the safety factor to a selected value. For the slope stability problem containing piles in a row, two separate analyses have been carried out in terms of slope and pile stability. In practical applications, the study of a slope reinforced with piles is usually carried out by extending the methods commonly used for the stability analysis of slopes to incorporate the reaction force exerted on the unstable soil mass by the piles. To date, the limit equilibrium method is the most widely used approach to analyze slope stability due to its simplicity of use. Moreover, this method allows for the effect of seepage, loading and general soil conditions without requiring additional computational efforts. Major criticism of the limit equilibrium method is that it is generally based on simplified assumptions and the results obtained from this method are, in the light of limit analysis, neither upper bounds nor lower bounds on the true solution (Ausilio et al., 2001).

Figure 4. Comparison of lateral load calculations for different theories.
The slope instability can be analyzed by dividing resisting moments/forces (either together or separately, since it depends on method of analysis used) and disturbing moments/forces acting on the soil mass DBCAD, shown in Figure 5. Due to pile installation, the extra resisting force provided by piles at the plane AB is added to resisting moments/forces within the parameters of normal slope stability calculations. Pile stability may be analyzed using forces acting on a single pile at the plane AB, as shown in Figure 5. This force is used as an extra resisting force for the slope stability but reactively it is also used as a design force to calculate pile integrity and stability.

When a row of piles is installed into the slope, the factor of safety (FOS) changes due to the additional resisting force, \( p_p \), provided by the piles. To evaluate this force per unit width of the failure mass, the total force may be integrated along the depth of the pile (until reaching the slip surface). Then, the result, \( p \), is divided by centre to centre distance, \( D_1 \) where \( P_p = p / D_1 \).

According to Ito and Matsui (1975), the force acting on the slope is equal to \( P_p \) regardless of the state of the equilibrium of the slope. This force is added into the FOS calculations. Nevertheless, an overestimation of the force \( P_p \) may lead to un-conservative results in the design of the slope. To remain on the safe margin, Hassiotis et al. (1997) suggested the use of mobilized lateral force, \( P_m \) so that:

\[
P_m = P_p / \text{co}
\]

(29)

Where co is greater than 1.0. In this research, \( P_p \) is scaled by the un-reinforced FOS of the slope where it is then reinforced by a row of piles (i.e. \( \text{co} = \text{FOS} \)). This mobilized force, \( P_m \) is used to evaluate stabilized FOS, but the total lateral force per unit length, \( p \) is still used to design the piles.

Two distinct methods of lateral load estimations above were used to evaluate lateral loads on the pile in a row. With the piles in place and with the restraining forces of the piles against the sliding soil mass (Figure 5), a second analysis was performed to find the new stabilized FOS against sliding without changing the failure surface. The reinforced FOS values are given in Tables 1 and 2. The case study was examined by two different mass divisions namely method of slices (MOS) and Gauss quadrature (GQ).

Method of slices (MOS)

In this method, a possible failure slip surface is divided by vertical or inclined planes into a series of slices (Figure 6).
Table 1. Un-reinforced FOS values.

<table>
<thead>
<tr>
<th>Programs</th>
<th>Methods of analysis for FOS calculations (un-reinforced)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fellenius</td>
</tr>
<tr>
<td>Reese et al.</td>
<td>-</td>
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<tr>
<td>SLOPE program</td>
<td>1.073</td>
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<tr>
<td>This study</td>
<td>MOS</td>
</tr>
<tr>
<td>GQ</td>
<td>1.085</td>
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</tbody>
</table>

Table 2. Reinforced FOS values by using two different lateral load estimations

<table>
<thead>
<tr>
<th>Lateral load calculations</th>
<th>Methods of mass division</th>
<th>Pile locations</th>
<th>Methods of analysis for FOS calculations (reinforced)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
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<td></td>
<td>Fellenius simpli</td>
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<td>Plastic deformation</td>
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<td>1.397</td>
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<td>1.680</td>
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<td>1.453</td>
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</tbody>
</table>

The width between slices can be equal or unequal and of necessity each slice has a variable height, measured along the centre line. Using a sufficiently large number of slices the base and top of each slice can be approximated to be a straight line. The potential slip surface is divided into slices only for ease of analysis. The factor of safety is taken to be the same for each slice. This implies that interslice normal and shear forces must act between each slice.

Gauss quadrature (GQ)

One of the most accurate numerical methods in ordinary use for integrating polynomials is the Gauss quadrature formulae. The great mathematician Karl Friederich Gauss (1777-1855) discovered that by a special placement of the nodes, the accuracy of the numerical integration process could be greatly increased. Consider the definite integral:

$$ I = \int_{-1}^{1} f(\xi) d\xi $$

(30)

to be evaluated numerically from a given number of Gauss points. Gauss considered the problem of determining the values of $\xi$ should be chosen in order to get the greatest possible accuracy. In other words, how should one subdivided the interval (-1, +1) so as to get the best possible results? Gauss found that the points in the interval should not be equally spaced but should be symmetrically placed with respect to the mid-point of the interval (Figure 7). The forces acting on a single Gauss slice are identical with the forces considered with the method of slices (Fırat, 1999).

Practical example

This particular problem has been examined by Reese et al. (1992) shown in Figure 8. The slope exists along the bank of a river where sudden drawdown is possible. According to the above authors, slides had been observed along the river at numerous places and it was found necessary to stabilize the slope to allow a bridge to be constructed. Using the Spencer’s method (1967), they analyzed the sudden drawdown case taking the undrained
Figure 7. Application of Gauss quadrature to slope stability.

Figure 8. Stabilized failure surface by a row of piles.
analysis and the FOS was given to be 1.060. They stated that the value is in reasonable agreement with observations.

Reese et al. (1992) used drilled shafts which were 0.915 m in diameter and penetrated below the sliding surface. The tops of the drilled shafts were restrained with anchors in stable soil.

The example slope, shown in Fig. 8, was reanalyzed using eight different methods namely as Fellenius (1936), Bishop simplified (1955), Janbu simplified and generalized (1954), Morgenstern and Price (1965), Spencer (1967), Sarma (1973) and Fredlund and Krahn (1977) and available comparisons were made in Tables 1 and 2 in terms of un-reinforced and reinforced FOS values. Also SLOPE (1993) program used for un-reinforced case to compare the results.

Three different pile locations were examined to find out the most suitable place(s) to obtain higher stabilized FOS values.

**Conclusion**

There are now bounded solutions for the lateral forces on the pile depending on the theory and circumstances used. This is to be welcomed, since an engineer has a choice of examining a number of solutions. The resistance that the piles can provide is assessed via an analysis of the response of piles to lateral soil movement. For the slope stability problem containing piles in a row, two separate analyses have been carried out in terms of slope and pile stability. In practical applications, the study of a slope reinforced with piles is usually carried out by extending the methods commonly used for the stability analysis of slopes to incorporate the reaction force exerted on the unstable soil mass by the piles. To date, the limit equilibrium method is the most widely used approach to analyze slope stability due to its simplicity of use. From the findings of this study, the following conclusions are drawn:

1) The present visco-plastic flow displays a trend where lateral load predictions lie between the data given by Ito and Matsui (1975) De Beer and Carpentier (1977).
2) Similar FOS values are obtained by commercially available SLOPE (1993) program and this study for un-reinforced case.
3) Eight different methods of analysis were carried out to calculate FOS values of a slope reinforced by a row of piles. Relationships have been observed between stabilized FOS and the location of the pile row.
4) For slopes containing piles, analytical expressions have been derived that allow the force needed to increase the safety factor to a desired value and the most suitable pile location within the slope to be evaluated. The calculations carried out using the methods obtained show that installing a row of piles is an effective remedy to improve slope stability especially when the sliding surface for the un-reinforced slope is relatively shallow. Piles appear to be very effective when they are installed in the region around the middle of the slope.

**Nomenclature**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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</table>
| a      | Pile (cylinder) diameter, c; so-called cohesion for soil, D; centre to centre distance between two piles, D; clear distance between two piles, E; pile stiffness, K; modulus of subgrade reaction, M; number of grid points in the r direction, N; number of grid points in the θ direction, θ; lateral force on the pile, Re; Reynolds number, Re; distance from a pile, S; smoothing parameter, \(T_{m}, T_{m0}, T_{m1}\); extra stress tensors, Tol; tolerance value for iterations, Tol(f); tolerance value for convergence of vorticity function, Tol(ψ); tolerance value for convergence of stream function, V; flow velocity, v; velocity component in the θ direction, z; depth from ground surface, γ; unit weight of soil, α_s; \(\alpha_{0}, \alpha_{00}, \alpha_{000}\); rates of strains in polar coordinates, η; apparent viscosity, \(\eta_{0}\); Newtonian flow viscosity, \(\eta_{n}\); Bingham plastic viscosity, φ; angle of internal friction of soil, \(\tau_{v}\); yield stress, \(\theta\); \(\tau\); polar co-ordinates, \(\omega\); vorticity function, \(\zeta\); effective distance in the vicinity of a pile, \(\xi\); scaling factor for r and ψ; stream function.

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