Full Length Research Paper

Chaotic oscillator design and realizations of the Rucklidge attractor and its synchronization and masking simulations

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In this paper, Rucklidge attractor’s chaotic oscillator, synchronization and masking communication circuits were designed and simulated. The electronic circuit oscilloscope outputs of the realized Rucklidge system is also presented. Simulation and oscilloscope outputs are used to illustrate the accuracy of the designed and realized Rucklidge chaotic oscillator circuits. The Rucklidge system is addressed suitable for chaotic synchronization circuits and chaotic masking communication circuits using Matlab-Simulink® and Orcad-PSpice® programmes. Simulation results are used to visualize and illustrate the effectiveness of Rucklidge chaotic system in synchronization and masking communication. All simulations results performed on Rucklidge chaotic system are to verify the application of secure communication.

Key words: Rucklidge attractor, chaotic oscillator, synchronization, chaotic masking, secure communication.

INTRODUCTION

Over the last three decades, chaos has been extensively studied within the scientific, engineering and mathematical communities. Chaos behavior can occur every where, even in very simple and low-dimensional nonlinear systems. The well known Poincare'-Bendixon theorem (Nayfeh and Balanchandran, 1994) requires an autonomous continuous time state space model to be at least three-dimensional in order to have bounded chaotic solutions. On the other hand, for non-autonomous systems, chaos can appear in two-dimensional models. There are many examples, such as Lorenz (Lorenz, 1963), and Rossler (Rossler, 1976, 1979) systems that have been widely studied. Electronic circuits that consist of two nonlinear elements can be used to verify theoretical predictions. As an example, the nonlinear duffing forced oscillators have been experimentally studied (Hayashi, 1964). Another popular example is the nonlinear Chua’s circuit, built and experimentally examined (Madan, 1993). Up till now, various chaotic systems and their electronic circuits are introduced (Rucklidge, 1992; Sprott, 1994; Ashwin and Rucklidge, 1998; Lü et al., 2004; Chen et al., 1999; Soliman and Elwakil, 1999; Madhekar, 2006; Chandra et al., 2001; Pehlivan and Uyaroglu, 2010).

Chaos and chaotic systems have many fields of applications. One of the popular practical applications is secure communication. Synchronization of chaotic systems and chaos based secure communication has become an area of active research in recent years (Pecora and Carroll, 1990, 1991; Pehlivan and Uyaroglu, 2007a; Bai et al., 2002; Kocarev et al., 1992; Cuomo et al., 1993; Hayes et al., 1993; Cuomo and Oppenheim, 1993; Wu and Chua, 1993; Itoh, 1999; Pehlivan and Uyaroglu, 2007b; Uyaroglu and Pehlivan, 2010 In Press; Li et al., 2003; Miliou et al., 2007).

Different approaches are proposed and being pursued. Among themes the technique of Pecora-Carroll who show that, when a state variable from a chaotically evolving system is transmitted as an input to a replica of part of the original system, the replica subsystem (receiver) sometimes synchronizes to the original system...
(sender). Thus, they suggest that this phenomenon of chaos synchronization may serve as the basis for new ways to achieve secure communication (Pecora and Carroll, 1990, 1991). Chaotic signals depend very sensitively on initial conditions, have unpredictable features and noise like wideband spread spectrum. So, it can be used in various communication applications because of their features of masking and immunizing information against noise.

Chaos-based secure communication systems have been alternative of the standard spread-spectrum systems, since they are able to spread the spectrum of the information signals and simultaneously encrypt the information signals with chaotic circuitry which is simple and inexpensive. Many researchers have investigated the implications of chaotic signals in communication systems. For example, Kocarev et al. (1992) and Cuomo et al. (1993) have used chaotic signals in communication security, and spread spectrum communication.

This paper focuses on the identical synchronization and its applications in signal masking communications. The brief is organized as follows. In Section II, Simulink, PSpice simulations and electronic circuit realization oscilloscope outputs of the chaotic Rucklidge system are obtained. In Section III the Pecora-Carroll method is applied to synchronize Rucklidge system. In Section IV, the chaotic masking communication method of Rucklidge system is realized also using Simulink and PSpice. Section V contains a discussion and conclusions.

**SIMULINK, PSPICE SIMULATIONS AND ELECTRONIC CIRCUIT REALIZATION OF THE RUCKLIDGE ATTRACTOR**

The following nonlinear autonomous ordinary differential equations comprise the Rucklidge chaotic system:

\[
\dot{x} = -K \cdot x + L \cdot y - y \cdot z \\
\dot{y} = x \\
\dot{z} = -z + y^2
\]  

(1)

The Lyapunov exponents (LE) of the Rucklidge Attractor are 0.193, 0 and -3.193. Namely, only one positive LE is present.

Figure 2 shows the circuit schematic for implementing the Rucklidge Attractor. We use TL081 opamps, the Analog Devices AD633JN multipliers, appropriate valued resistors and capacitors for PSpice simulations. The circuit is supplied ±15 V power supplies. Acceptable inputs to the AD633 multiplier IC are -10 to +10 V. The resistors R1-R12, are all shown with nominal values in Figure 2. Figure 2 also shows the PSpice simulation results of this circuit. Matlab-Simulink and PSpice simulation results of the Rucklidge attractor (Figures 1 - 2) give the same conclusions.

The experimental electronic circuit realization of the Rucklidge system is implemented for $K = 2$, $L = 6.7$ parameters and initial conditions $x_0 = 1$, $y_0 = 0$, $z_0 = 4.5$. Oscilloscope outputs of the Rucklidge oscillator are shown in Figure 3 (a), (b) and (c) for xy, xz, and yz attractors, respectively.

**SYNCHRONIZATION OF THE RUCKLIDGE ATTRACTOR**

Synchronization between chaotic systems has received considerable attention and led to communication applications. There are two major methods for coupling and synchronizing identical chaotic systems, the cascading method and the one-way coupling method. With these methods, a message signal sent by a transmitter system can be reproduced at a receiver under the influence of a single chaotic signal through synchronization.

This paper presents the study of numerical simulation of chaos synchronization for chaotic Rucklidge attractor. The method of synchronization is Pecora-Carroll (P-C) method; drive subsystem and response subsystem were constructed. Figure 4 shows block diagram of a cascaded synchronization system, pointed out simulation modelling and outputs of P-C Synchronization of Rucklidge attractor.

Synchronization of chaotic motions among coupled dynamical systems is an important generalization from the phenomenon of the synchronization of linear system, which is useful and indispensable in communications. There are two major methods in chaos synchronization of coupled identical systems; The cascading method and the one way coupling method. The idea of the methods is to reproduce all the signals at the receiver under the influence of a single chaotic signal from the driver.

Therefore, chaos synchronization provides potential applications to communications and signal processing. However, to build secure communications system, some other important factors, need to be considered. Simulations of synchronization of Rucklidge system are presented as shown in Figure 4.

The initial values of the two subsystem are different, the initial value of the drive system is (1, 0, 4.5), the initial value of the response subsystem1 is (0, 4.5) and subsystem 2 is (-2, 4.5). Simulation results show that the two subsystems synchronize well. Figure 5 shows the circuit schematic for implementing the P-C Synchronization of Rucklidge system. We use TL081 opamps, the AD633JN multipliers, appropriate valued resistors and capacitors for PSpice simulations. The circuits are supplied ±12 V power supplies. Figure 5 also shows PSpice simulation results of this circuit. Matlab-Simulink
and PSpice simulations of Rucklidge system (Figures 4 and 5) give the same conclusions.

**CHAOTIC MASKING COMMUNICATION OF THE RUCKLIDGE SYSTEM**

Due to the fact that output signal can recover input signal, it indicates that it is possible to implement secure communication for a chaotic system. Figure 6 which contains the principle and Simulink scheme of the Rucklidge attractor showed a general secure communication system that employs the masking technique pointed out as follows.

The presence of the chaotic signal between the transmitter and receiver has proposed the use of chaos in secure communication systems. The design of these systems depends as we explained earlier on the self-synchronization property of the Rucklidge attractor. Transmitter and receiver systems are identical except for their initial values, in which the transmitter system is 1, 0, 4.5 and the receiver system is -2, 0, and 4.5 as shown in Figure 6.

It is necessary to make sure the parameters of transmitter and receiver are identical for implementing the chaotic masking communication. In this masking scheme, a low-level message signal is added to the synchronizing driving chaotic signal in order to regenerate a clean driving signal at the receiver. Thus, the message has been perfectly recovered by using the signal masking approach through cascading synchronization in the Rucklidge attractor. Computer simulation results have shown that the performance of Rucklidge attractor in chaotic masking and message recovery. One disadvantage

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**Figure 1.** Matlab-simulink model and phase portraits of Rucklidge attractor when $K = 2$, $L = 6.7$ and $x_0 = 1$, $y_0 = 0$, $z_0 = 4.5$
Figure 2. PSpice Circuit and PSpice simulation results of the Rucklidge attractor. (a)x-y phase portrait, (b) x-z phase portrait, (c) y-z phase portrait.
Figure 3. Oscilloscope outputs of circuitry of the Rucklidge attractor, (a) x-y phase portrait, (b) x-z phase portrait, (c) y-z phase portrait

Figure 4. Block diagram (a) Simulink modeling (b) Simulation outputs of P-C synchronization of Rucklidge system. (c) Drive and response system chaotic signals before synchronization (d) Unsynchronized case (e) Drive and response system chaotic signals after synchronization (f) Synchronization between X and Xr.
Figure 5. PSpice Circuit and simulations of P-C Synchronization of Rucklidge attractor Circuit  
(a) Drive and response system chaotic signals before synchronization (b) The phase portrait of unsynchronized case  
(c) Drive and response system chaotic signals after synchronization (d) X-Xr Synchronization.
Figure 6. (a) Chaotic signal masking system principle (b) Simulink scheme of the Rucklidge attractor.

Figure 7. Simulink outputs of Masking Communication scheme of Rucklidge attractor (a) Drive (X) and response (Xr) system chaotic signals vs. time (b) Transmitted signal $S(t) = x(t) + i(t)$ (c) Information $i(t)$ and retrieved $ir(t)$ signals (sinus signal) has 0.4V amplitude and frequency 10 KHz.
of using one-way coupling method is that compared to this cascading method, it takes longer to synchronize the coupled systems, especially when the coupling parameter is small. This may cause problems in practical applications such as secure communications since information may be delayed or lost during the first period of matching time. The transmitted signal is a sinus wave of amplitude 0.4 V and of 10 KHz frequency. The sinus wave signal is added to the generated chaotic x signal, and the $S(t) = x + i(t)$ is feed into the receiver. The chaotic x signal is regenerated allowing a single subtraction to retrieve the transmitted signal, $[x+i(t)]-x = i(t)$, if $x = x_r$. This is a result of synchronization as in Figure 7(a). Figure 7 (c) shows the information signal-sinus wave and the retrieved signal output of scope. Figure 8 shows the circuit schematic for implementing the Rucklidge attractor’s

Figure 8. Rucklidge attractor chaotic masking communication circuit.
Chaotic Masking Communication. Figure 9 shows PSpice simulation results of this Chaotic Masking Circuit. The transmitted signal is a sinus wave of amplitude 0.4 V and frequency 10 KHz. Transmitter and receiver circuits are identical except for their initial values, in which the transmitter circuit are 1, 0, 4.5 and the receiver circuit are -2, 0, and 4.5 as shown in Figure 8. Simulink and PSpice simulations (Figures 7 and 9) of Chaotic masking circuit give the same conclusions.

CONCLUSION

This paper focuses on the chaotic oscillator circuit and the identical synchronization of the Rucklidge attractor and its applications in signal masking communications. Rucklidge attractor’s chaotic oscillator circuits were designed and simulated using Matlab-Simulink and PSpice programmes. The real electronically experimental circuit of the Rucklidge attractor was realized. Simulation and oscilloscope outputs illustrate the accuracy of the designed and realized Rucklidge chaotic oscillator circuits. Applied synchronization method is the Pecora-Carroll identical cascading synchronization method. The behaviour of the response system depends on the behaviour of the drive system but not invertible. We have demonstrated in simulations that chaos can be synchronized and applied to secure communications. We suggest that this phenomenon of chaos synchronism may serve as the basis for little known Rucklidge attractor to achieve secure communication. Chaos synchronization and chaos masking were realized using Matlab-Simulink and PSpice programs. Related figures in Figures 4 - 5 for synchronization and Figures 7 - 9 for masking communication, point out that Matlab-Simulink and PSpice outputs prove the same conclusions.

REFERENCES

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