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Full Length Research Paper

Fuzzy automata based on Atanassov fuzzy sets and applications on consumers' advertising involvement

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Fuzzy automata are regarded as an important device in fuzzy systems due to the advantage of powerful mathematical computation. This dynamic machine has been applied in various fields, such as pattern recognition, controller design, etc. However, most of the applications pertain to the engineering. Considering higher-order sets with imprecision, this paper makes use of Atanassov fuzzy sets and proposes the Atanassov fuzzy automata to solve problems in social science. The Atanassov fuzzy automata are powered by the implication and the composition to deal with Atanassov fuzzy relations. Derived from different *t*-norms and *t*-conorms, several types of implications and compositions are presented and compared. Based on the Atanassov fuzzy automata, an integrated model of advertising involvement is constructed as an illustration in order to manifest the feasibility and effectiveness of the proposed dynamic system. The presented device explores new directions for the permeation of Atanassov fuzzy automata to application areas in social science.

Key words: Atanassov fuzzy set, Atanassov fuzzy automata, implication, composition, advertising involvement.

INTRODUCTION

The automata theory was first examined by Turing (1936) for the study of limits of human ability to solve mathematical problems in formal ways. Considering the imprecise nature in reality, Wee (1967) introduced the concept of fuzzy automata. Lee and Zadeh (1969) further defined fuzzy finite-state automata as a dynamic fuzzy system operating in discrete time. Fuzzy finite-state automata can model the dynamics of discrete event systems (DES) which transform sequences of input states received at the input of the system into sequences of output states produced at the output of the system by a dynamically changing internal state (Klir and Yuan, 1995; Omlin et al., 1998; Rigatos, 2009). Li and Pedrycz (2005) indicated that fuzzy finite-state automata can be viewed as a mathematical model of computation in fuzzy systems. Recently, a higher order set with imprecision has been extended to automata. Based on Atanassov's intuitionistic fuzzy sets, Jun (2006) proposed intuitionistic fuzzy finite state machines. Zhang and Li (2009) presented the properties of intuitionistic fuzzy recognizers and intuitionistic fuzzy finite automata. Moreover, Srivastava and Tiwari (2010) studied the relationship between intuitionistic fuzzy topologies and intuitionistic fuzzy automata.

Due to the merit of appropriately modeling vagueness and hesitation, the extension of fuzzy sets to a pair of membership degree and non-membership degree proposed by Atanassov (1986) has been discussed widely. Several theorems carry Atanassov's theory to an extreme, such as implication (Cornelis et al., 2004), correlation (Gerstenkorn and Manko, 1991; Hung, 2001), composition (Bustince and Burillo, 1996b; Deschrijver and Kerre, 2003), entropy measure (Burillo and Bustince,

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1996; Szmidt and Kacprzyk, 2001), distance measure (Atanassov, 1999; Szmidt and Kacprzyk, 2000; Grzegorzewski, 2004), similarity measure (Liang and Shi, 2003; Hung and Yang, 2007), and divergence measure (Chaira and Ray, 2008; Hung and Yang, 2008). The terminological debate about the appropriateness of “intuitionistic” is also regarded as a vital issue when it comes to Atanassov’s work.

Dubois et al. (2005) pointed out there is a misunderstanding between Atanassov’s intuitionistic fuzzy sets and the intuitionistic fuzzy logic of Takeuti and Titani (1984). More relevant discussions on the terminological deficiency are presented in the studies of Atanassov (2005) and Tizhoosh (2008). In order to abstain from the controversial point, this paper will hereinafter use Atanassov fuzzy set (AFS, for short), which is employed by Montero et al. (2007), to refer to Atanassov’s intuitionistic fuzzy sets.

Based on sound theoretical development, AFSs have been applied in many areas, including edge detection (Chaira and Ray, 2008), product design (Chen, 2009), pattern recognition (Hung and Yang, 2008; Khatibi and Montzaer, 2009), medical diagnosis (De et al., 2001; Kharal, 2009), and decision making (Lin et al., 2007; Li et al., 2009), etc. It is worthwhile to mention that AFSs are equivalent to other imprecise sets.

Gau and Buehrer (1993) propounded the concept of vague sets. Bustince and Burillo (1996a) showed that vague sets are AFSs. Tizhoosh (2008) suggested that AFSs and interval-valued fuzzy sets proposed by Gorzlczany (1987) and Turksen (1996) constitute a mathematical isomorphism, but perform with distinct semantics. For an explicit relationship among various sets, Deschrijver and Kerre (2007) investigated the position of AFSs in the framework of the different theories modelling imprecision.

Fuzzy finite-state automata have great importance with significant applications such as the model of computing with words, learning systems, pattern recognition and data base theory (Benson, 2002; Neven and Schwentick, 2002; Westerdale, 2002; Ying, 2002), but most of these applications focus on the engineering aspect. Furthermore, although some studies of the automata based on AFSs have been presented, these works only had initial development in theory without enough practice. Since the expression of an AFS characterized by a pair of membership degree and non-membership degree is similar to human thinking logic with pros and cons, this paper tries to combine AFSs with the finite-state automata for solving problems in social science.

The advertisement is regarded as a pivotal channel of marketing communication in business. Krugman (1967) suggested that a consumer accepts advertising messages actively or passively in accordance with the degree to which he/she is involved in the advertisement. Affected by some antecedent stimuli such as source credibility, an individual’s advertising involvement will vary dynamically. Besides, the advertising involvement will result in consequent outcomes such as attitude toward the advertisement. The causal framework of advertising involvement coincides with the intrinsic operation of automata which are composed of input, internal and output states. The antecedents and consequences of advertising involvement are analogous to the input sates and output states in the automata, respectively; the advertising involvement could be considered to be the internal states.

Hence, this paper introduces a novel type of Atanassov fuzzy automata (AFA, for short) based on AFSs and makes use of this dynamic machine to develop an integrated model of advertising involvement in which several antecedents and consequences are included. Moreover, because the implication and composition dominate the core operation of AFA, numerous Atanassov fuzzy implications and compositions are used and the differences are compared. Finally, an empirical study is employed to validate the effectiveness and feasibility of the proposed AFA.

PRELIMINARIES

Some operators of AFSs

Atanassov (1986) generalized the concept of fuzzy set and defined the concept of AFSs. Let X be a finite universe of discourse. An AFS A in X is an object having the following form:

$$A = \left\{ \langle x, \mu_A(x), \nu_A(x) \rangle \middle| x \in X \right\} \quad (1)$$

where the function $\mu_A : X \to [0,1]$ and $\nu_A : X \to [0,1]$ define the degree of membership and the degree of non-membership of the element $x \in X$ to the set $A \subseteq X$, respectively, such that $0 \le \mu_A(x) + \nu_A(x) \le 1$. The value of:

$$\pi_A(x) = 1 - \mu_A(x) - \nu_A(x) \quad (2)$$

is called the degree of indeterminacy. It is the hesitation degree of the element x to the set A. Obviously, $0 \le \pi_A(x) \le 1$ for all $x \in X$. For two AFSs A and B for all $x \in X$, some operations are defined as follows:

$A \le B$ if and only if $\mu_A(x) \le \mu_B(x)$ and $\nu_A(x) \ge \nu_B(x)$; (3)

$A = B$ if and only if $A \le B$ and $B \le A$; (4)

$$A^C = \left\{ \langle x, \nu_A(x), \mu_A(x) \rangle \middle| x \in X \right\}; \quad (5)$$

$$A \cap B = \left\{ \langle x, \min\{\mu_A(x), \mu_B(x)\}, \max\{\nu_A(x), \nu_B(x)\} \rangle \middle| x \in X \right\}; \quad (6)$$

$$A \cup B = \left\{ \langle x, \max\{\mu_A(x), \mu_B(x)\}, \min\{\nu_A(x), \nu_B(x)\} \rangle \middle| x \in X \right\}. \quad (7)$$

In (6) and (7), the "min" operator adopts *t*-norm (fuzzy intersection) and "max" operator adopts *t*-conorm (fuzzy union). The "min" and "max" are one of *t*-norms and *t*-conorms. Four basic *t*-norms and four basic *t*-conorms for two sets *a* and *b* are presented as follows:

T-norm (fuzzy intersection):

Standard intersection: $i(a,b) = \min(a,b)$, (8)

Algebraic product: $i(a,b) = ab$, (9)

Bounded difference: $i(a,b) = \max(0, a+b-1)$, (10)

Drastic intersection: $i(a,b) = \begin{cases} a & \text{when } b = 1, \\ b & \text{when } a = 1, \\ 0 & \text{otherwise.} \end{cases}$ (11)

T-conorm (fuzzy union):

Standard union: $u(a,b) = \max(a,b)$, (12)

Algebraic sum: $u(a,b) = a + b - ab$, (13)

Bounded sum: $u(a,b) = \min(1, a+b)$, (14)

Drastic union: $u(a,b) = \begin{cases} a & \text{when } b = 0, \\ b & \text{when } a = 0, \\ 1 & \text{otherwise.} \end{cases}$ (15)

Correlation and distance between AFSs

Gerstenkorn and Manko (1991) proposed the correlation between AFSs based on the concept of information energy. For AFSs *A* and *B*, let $X = \{x_1, x_2, \ldots, x_n\}$ be a finite universe of discourse, and the correlation coefficient *K* between two AFSs is defined as follows:

$$K(A,B) = \frac{\sum_{i=1}^{n} (\mu_A(x_i) \cdot \mu_B(x_i) + \nu_A(x_i) \cdot \nu_B(x_i))}{\sqrt{\sum_{i=1}^{n} (\mu_A^2(x_i) + \nu_A^2(x_i)) \cdot \sum_{i=1}^{n} (\mu_B^2(x_i) + \nu_B^2(x_i))}} \quad (16)$$

Hung (2001) used a statistical viewpoint to develop the correlation coefficient. The coefficients enable us to obtain not only the strength of the relationship between two AFSs, but the positive or negative direction as well. The coefficient is defined as follows:

$$\rho(A,B) = \frac{1}{2}(\rho_1 + \rho_2), \quad (17)$$

where

$$\rho_1 = \frac{\sum_{i=1}^{n} (\mu_A(x_i) - \overline{\mu_A})(\mu_B(x_i) - \overline{\mu_B})}{\left\{\sum_{i=1}^{n} (\mu_A(x_i) - \overline{\mu_A})^2\right\}^{1/2} \left\{\sum_{i=1}^{n} (\mu_B(x_i) - \overline{\mu_B})^2\right\}^{1/2}}$$

$$\rho_2 = \frac{\sum_{i=1}^{n} (\nu_A(x_i) - \overline{\nu_A})(\nu_B(x_i) - \overline{\nu_B})}{\left\{\sum_{i=1}^{n} (\nu_A(x_i) - \overline{\nu_A})^2\right\}^{1/2} \left\{\sum_{i=1}^{n} (\nu_B(x_i) - \overline{\nu_B})^2\right\}^{1/2}}.$$

Atanassov (1999), based on the Hamming distance, proposed a distance measure to calculate the separate degree between two AFSs:

$$d(A,B) = \frac{1}{2} \sum_{i=1}^{n} \left(|\mu_A(x_i) - \mu_B(x_i)| + |\nu_A(x_i) - \nu_B(x_i)| \right). \quad (18)$$

Implication for AFSs

Because the AFA are powered by the functions of implications and compositions, the selection of implication and composition is crucial in the process. The implication is the function for joining two sets together and yielding a relation.

In classical logic, the implication J can be expressed with several different forms. The nature of these forms is equivalent in classical logic, but dissimilar in fuzzy logic. As a result, we introduce four types of fuzzy implications and further generate Atanassov fuzzy implicators in order to bring about Atanassov fuzzy relations (AFR, for short).

S-implication

The equation is extended from classical logic to fuzzy logic by a fuzzy union and a fuzzy complement. Let *c*(*a*) denote the conventional fuzzy complement and *c*(*a*)=1-*a*:

$$J(a,b) = u(c(a), b). \quad (19)$$

R-implication

The equation is extended from classical logic to fuzzy logic by a fuzzy intersection. The "sup" denotes the supremum operation:

$$J(a,b) = \sup\{x \in [0,1] \mid i(a,x) \le b\}. \quad (20)$$

QL1-implication

In classical logic, equations can be transformed by De Morgan laws. Nevertheless, it does not work in fuzzy logic. Equations 21 and 22 are extended from classical logic to fuzzy logic by a fuzzy union, fuzzy intersection, and fuzzy complement:

$$\mathrm{J}(a,b)=u\left(c(a),i(a,b)\right). \tag{21}$$

QL2-implication

$$\mathrm{J}(a,b)=u\left(i(c(a),c(b)),b\right). \tag{22}$$

Equations 19 to 22 are constructed by the fuzzy union and the fuzzy intersection. In other word, each implication can evolve four different implicators by (8) to (11) or (12) to (15). Due to the dual characteristic, in (21) and (22), if "*i* " adopts the standard intersection, "*u*" has to use the standard union to be paralleled. Table 1 demonstrates sixteen Atanassov fuzzy implicators derived from two AFSs *A*=(a_1, a_2) and *B*=(b_1, b_2).

Composition for AFSs

The composition is a combination of a fuzzy set and a fuzzy relation or a combination of two fuzzy relations. Bustince and Burillo (1996a) analyzed the structures on AFRs. Assume that an AFR *Q* from a universe *X* to a universe *Y* is an AFS in *X*×*Y*, that is, an expression *Q* given by:

$$Q=\left\{\left\langle (x,y),\mu_Q(x,y),\nu_Q(x,y)\right\rangle \middle| x\in X, y\in Y\right\} \tag{39}$$

where $\mu_Q : X\times Y\to[0,1]$ and $\nu_Q : X\times Y\to[0,1]$ satisfy the condition

$0\le \mu_Q(x,y)+\nu_Q(x.y)\le 1$ for all $(x,y)\in X\times Y$.

Let α, β, λ, *γ*be *t*-norms or *t*-conorms. $Q\in AFR(X\times Y)$ and $T\in AFR(Y\times Z)$. The composition of *Q* and *T* is the AFR from *X* to *Z* defined by:

$$T^{\alpha}_{\lambda}\circ{}^{\beta}_{\gamma}Q=\left\{\left\langle (x,z),\mu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z),\nu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z)\right\rangle \middle| x\in X, z\in Z\right\}$$

where

$$\mu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z)=\underset{y}{\alpha}\left\{\beta[\mu_Q(x,y),\mu_T(y,z)]\right\},$$

$$\nu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z)=\underset{y}{\lambda}\left\{\gamma[\nu_Q(x,y),\nu_T(y,z)]\right\},$$

whenever

$$0\le \mu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z)+\nu_{T^{\alpha}_{\lambda}\circ^{\beta}_{\gamma}Q}(x,z)\le 1,\ \forall (x,z)\in X\times Z.$$

Burillo and Bustince (1995) have proved that for $\alpha=\vee$, β *t*-norm, $\lambda=\wedge$, γ *t*-conorm, the composition of AFRs satisfies the largest number of properties. Hence, we can generate four types of compositions according to different *t*-norm and *t*-conorm from (8) to (15).

ATANASSOV FUZZY AUTOMATA FOR ADVERTISING INVOLVEMENT

The proposed AFA and algorithm

The fuzzy automaton is one of the fuzzy systems in fuzzy set theory. Taking advantage of AFSs which display the powerful competence to model vagueness, we extended AFSs to fuzzy automata and developed the AFA to deal with advertising involvement. In the AFA, the sequences of input states can be transformed into the sequences of output states in the discrete time. The process of transformation from input states to output states is supported by a changing internal state which is a dynamic situation. The internal state not only determines the output states at the current time, but also stores a specific state to be used at the next time.

The operation of the AFA is similar to the development of consumers' advertising involvement because involvement is a concept of continuity that the current involvement state would induce the next involvement state. The antecedents of advertising involvement can be viewed as input states; the subconstructs of advertising involvement as internal states; the consequences of advertising involvement as output states. A state-transition fuzzy relation connects the advertising involvement and its antecedents. A response fuzzy relation connects the advertising involvement and its consequences. The finite AFA, *A*, are a fuzzy relational system defined by the quintuple:

$$A=[X,Y,Z,R,S]$$

Where *X* is a nonempty finite set of the antecedents for advertising involvement; *Y* is a nonempty finite set of the consequences of advertising involvement; *Z* is a nonempty finite set of subconstructs of advertising involvement; *R* is a response fuzzy relation on *Z*×*Y; S* is a state-transition fuzzy relation on *X*×*Z*×*Z*.

Let $X=\{x_1,x_2,\cdots,x_n\}$, $Y=\{y_1,y_2,\cdots,y_m\}$,

Table 1. Atanassov fuzzy implicators.

Implicator	M (degree of membership)	N (degree of non-membership)	
S 1-1	$\max\{a_2,\ b_1\}$	$\min\{a_1,\ b_2\}$	(23)
S 1-2	$a_2+b_1-a_2b_1$	a_1b_2	(24)
S 1-3	$\min\{1,\ a_2+b_1\}$	$\max\{0,\ a_1+b_2-1\}$	(25)
S 1-4	$\begin{cases} a_2 & \text{when } b_1=0 \\ b_1 & \text{when } a_2=0 \\ 1 & \text{otherwise} \end{cases}$	$\begin{cases} a_1 & \text{when } b_2=1 \\ b_2 & \text{when } a_1=1 \\ 0 & \text{otherwise} \end{cases}$	(26)
R 1-1	$\begin{cases} 1 & \text{when } a_1\le b_1 \\ b_1 & \text{when } a_1>b_1 \end{cases}$	$\begin{cases} b_2 & \text{when } a_1>b_1,\ a_2<b_2 \\ 0 & \text{otherwise} \end{cases}$	(27)
R 1-2	$\begin{cases} 1 & \text{when } a_1\le b_1 \\ \dfrac{b_1}{a_1} & \text{when } a_1>b_1 \end{cases}$	$\begin{cases} 1-\dfrac{b_1}{a_1} & \text{when } a_1>b_1,\ a_2\le b_2,\ \dfrac{a_1}{b_1}<\dfrac{1-a_2}{1-b_2} \\ 0 & \text{when } a_2>b_2 \\ 0 & \text{when } a_1\le b_1,\ a_2<b_2 \\ \dfrac{b_2-a_2}{1-a_2} & \text{otherwise} \end{cases}$	(28)
R 1-3	$\min\{1,\ 1-a_1+b_1\}$	$\begin{cases} 0 & \text{when } b_1>a_1, b_2>a_2 \\ b_2 - a_2 & \text{when } b_1\le a_1, b_2>a_2, b_1+b_2\le a_1+a_2 \\ a_1 - b_1 & \text{when } b_1\le a_1, b_2>a_2, b_1+b_2>a_1+a_2 \\ 0 & \text{when } b_1\le a_1, b_2\le a_2 \end{cases}$	(29)
R 1-4	$\begin{cases} b_1 & \text{when } a_1=1 \\ 1 & \text{otherwise} \end{cases}$	$\begin{cases} b_2 & \text{when } a_1=1,\ a_2=0 \\ 0 & \text{otherwise} \end{cases}$	(30)
QL 1-1	$\max\{a_2,\ \min\{a_1,\ b_1\}\}$	$\min\{a_1,\max\{a_2,\ b_2\}\}$	(31)
QL 1-2	$a_2+a_1b_1-a_1a_2b_1$	$a_1a_2+a_1b_2-a_1a_2b_2$	(32)
QL 1-3	$\begin{cases} a_2 & \text{when } a_1+b_1\le 1 \\ a_1+a_2+b_1\text{-}1 & \text{when } a_1+b_1>1 \end{cases}$	$\begin{cases} a_1 & \text{when } a_2+b_2>1 \\ a_1+a_2+b_2\text{-}1 & \text{when } a_2+b_2\le 1,\ a_1+a_2+b_2\ge 1 \\ 0 & \text{otherwise} \end{cases}$	(33)
QL 1-4	$\begin{cases} a_1 & \text{when } a_2=0,\ b_1=1 \\ a_2 & \text{when } a_1=1,\ b_1=1 \\ a_2 & \text{when } a_1=1,\ b_1=0 \\ a_2 & \text{when } a_1\ne 1,\ b_1\ne 1 \\ b_1 & \text{when } a_1=1,\ a_2=0 \\ 1 & \text{otherwise} \end{cases}$	$\begin{cases} a_1 & \text{when } a_2=1,\ b_2=0 \\ a_1 & \text{when } a_2=0,\ b_2=1 \\ a_1 & \text{when } a_2\ne 0,\ b_2\ne 0 \\ a_2 & \text{when } a_1=1,\ b_2=0 \\ b_2 & \text{when } a_1=1,\ a_2=0 \\ 0 & \text{otherwise} \end{cases}$	(34)

Table 1. Contd.

QL 2-1	$\max\{\min\{a_2,b_2\},\ b_1\}$	$\min\{\max\{a_1,b_1\},b_2\}$	(35)
QL 2-2	$a_2b_2+b_1-a_2b_1b_2$	$a_1b_2+b_1b_2-a_1b_1b_2$	(36)
QL 2-3	$\begin{cases} b_1 & \text{when } a_2+b_2 \le \\ a_2+b_1+b_2\text{-}1 & \text{when } a_2+b_2>1 \end{cases}$	$\begin{cases} b_2 & \text{when } a_1+b_1\ge 1 \\ a_1+b_1+b_2\text{-}1 & \text{when } a_1+b_1<1,\ a_1+b_1+b_2\ge 1 \\ 0 & \text{otherwise} \end{cases}$	(37)
QL 2-4	$\begin{cases} a_2 & \text{when } b_1=0,\ b_2=1 \\ b_1 & \text{when } a_2=0,\ b_2=1 \\ b_1 & \text{when } a_2=1,\ b_2=0 \\ b_1 & \text{when } a_2\ne 1,\ b_2\ne 1 \\ b_2 & \text{when } a_2=1,\ b_1=0 \\ 1 & \text{otherwise} \end{cases}$	$\begin{cases} a_1 & \text{when } b_1=0,\ b_2=1 \\ b_1 & \text{when } a_1=0,\ b_2=1 \\ b_2 & \text{when } a_1=1,\ b_1=0 \\ b_2 & \text{when } a_1=0,\ b_1=1 \\ b_2 & \text{when } a_1\ne 0,\ b_1\ne 0 \\ 0 & \text{otherwise} \end{cases}$	(38)

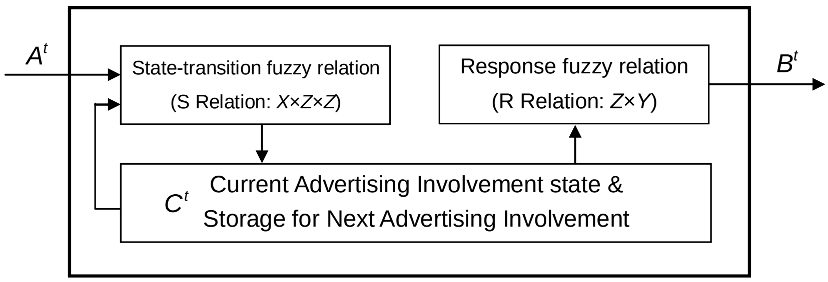

Figure 1. Basic scheme of AFA.

$Z=\{z_1,z_2,\cdots,z_q\}$ be initial parameters in the beginning machine, and A^t, B^t, C^t denote the antecedents, consequences, advertising involvement at time *t*, respectively.

The scheme of AFA is illustrated in Figure 1. Given A^t at some time *t*, the C^t can be generated by state-transition fuzzy relation *S*; B^t can be generated by response fuzzy relation *R*.

Before the AFA start to work, set up parameters *X*, *Y* and *Z* to construct state-transition fuzzy relations *S* and response fuzzy relation *R* by implicators (23) to (38) in advance. When the setup is prepared, a sequence of A^1, A^2,…, A^t enters the machine and then yields B^1, B^2,…, B^t, and C^1, C^2,…, C^t at time *t*.

For any given antecedent A^t, the ternary state-transition relation *S* is converted into a binary relation, S_{A^t}, on *Z*×*Z* by the following equation:

$$S_{A^t}(z_\delta,z_\varphi)=(\max\{\min\{\mu_{A^t}(x_i),\mu_S(x_i,z_\delta,z_\varphi)\}\},\ \min\{\max\{\nu_{A^t}(x_i),\nu_S(x_i,z_\delta,z_\varphi)\}\})\quad i=1,2,...,n,\ ;\ \delta,\varphi=1,2,...,q \tag{41}$$

Assume that the initial advertising involvement *Z* is given, and the advertising involvement C^t and the consequence B^t are determined by:

$$C^t=Z\circ S_{A^1}\circ S_{A^2}\circ\ldots\circ S_{A^t} \tag{42}$$

$$B^t=C^t\circ R \tag{43}$$

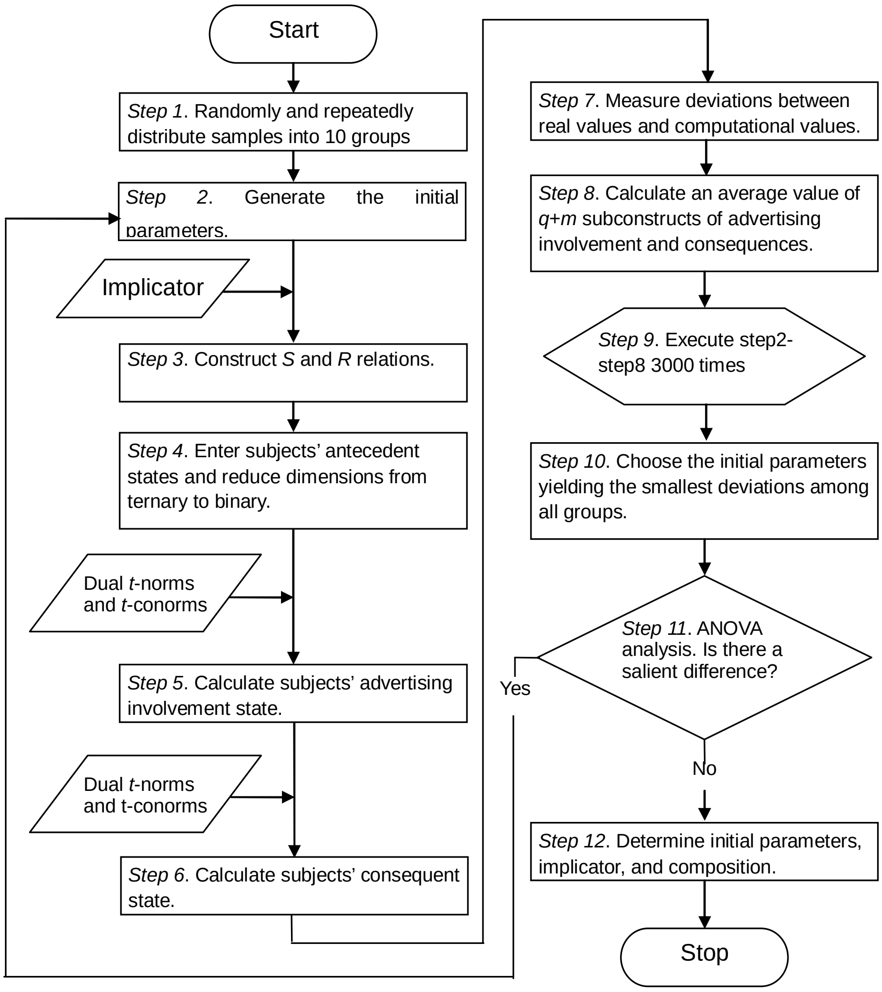

Figure 2. The flowchart of the proposed algorithm.

The proposed algorithm explains how to utilize the AFA to develop an integrated model of advertising involvement. A well-developed model relies on the setup of initial parameters in the machine, the selection of implicators, and the selection of compositions. Therefore, the following steps are executed to look for the best parameters, implicator, and composition. Figure 2 is a flowchart of the proposed algorithm.

Step 1: Randomly and repeatedly distribute experimental samples into 10 groups with equal number of subjects.
Step 2: Generate the initial parameters in the initial AFA with random numbers.

The initial antecedents: $X = \{x_1, x_2, \ldots, x_n \,|\, x_i \in [0,1]\}$,

The initial consequences: $Y = \{y_1, y_2, \ldots, y_m \,|\, y_j \in [0,1]\}$,

The initial advertising involvement: $Z = \{z_1, z_2, \ldots, z_q \,|\, z_k \in [0,1]\}$.

Step 3: Construct a state-transition fuzzy relation S on X×Z×Z and a response fuzzy relation R on $Z \times Y$ by one of implicators (23) to (38).

Step 4: Enter the subjects' antecedent state A^t deriving from the questionnaire, and use (41) to attain S_{A^t}.

Step 5: Use (40) and (42) simultaneously to generate the subjects' advertising involvement state E^t.
Step 6: Use (24) and (27) simultaneously to generate the subjects' consequent state B^t.
Step 7: Use (18) to calculate the deviation between the real values investigated from the questionnaire and the computational values generated from the AFA for the advertising involvement state and the consequent state.
Step 8: Calculate an average value of deviations which contain q subconstructs of advertising involvement and *m* consequences for each subject.
Step 9: Execute from Step 2 to Step 8 1000 times.
Step 10: Choose the initial parameters which can yield the

smallest average value of 1000 deviations among all groups.

Step 11: Employ one-way ANOVA to test the average values of deviations among 10 groups. If the significant difference of average values exists, repeat the successive steps from Step 2 to Step 11 until there is no salient difference.

Step 12: Determine the best initial parameters, implicator, and composition.

A numerical example

In order to realize the development of an advertising involvement model by using AFA, a numerical example is illustrated here. Assume that the advertising involvement model comprises 4 antecedents (X_1, X_2, X_3, X_4), , 3 subconstructs of advertising involvement (z_1, z_2, z_3), and 2 consequences (y_1, y_2). The initial parameters with Atanassov fuzzy data are given as follows:

$$X = \{(x_1,\ \text{Interest},\ 0.2,\ 0.7),\ (x_2,\ \text{Pleasure},\ 0.5,\ 0.3),\ (x_3,\ \text{Sign},\ 0.4,\ 0.4),\ (x_4,\ \text{Product Knowledge},\ 0.3,\ 0.6)\};$$

$$Z = \{(z_1,\ \text{Meaningfulness},\ 0.5,\ 0.2),\ (z_2,\ \text{Attractiveness},\ 0.6,\ 0.3),\ (z_3,\ \text{Vitality},\ 0.2,\ 0.7)\};$$

$$Y = \{(y_1,\ \text{Message Attention},\ 0.4,\ 0.5),\ (y_2,\ \text{Recall},\ 0.3,\ 0.5)\}$$

where (x_1, interest, 0.2, 0.7) represents that the degree to which a consumer is interested in some specific object is 0.2; the degree to which a consumer is not interested in some specific object is 0.7; the hesitation degree is 0.1.

Based on the initial parameters, the next step is to construct the state-transition fuzzy relation and response fuzzy relation by an implicator in Table 1. Equation 25 is demonstrated to generate S and R relations. The three-dimensional array of S relation is as follows:

$$S = \left[\begin{array}{cc}
\begin{array}{cc} & \overset{x_1}{\begin{array}{ccc} z_1 & z_2 & z_3 \end{array}} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.5,\ 0.2) & (0.6,\ 0.3) & (0.2,\ 0.7) \\ (0.5,\ 0.2) & (0.6,\ 0.3) & (0.2,\ 0.7) \\ (0.5,\ 0.1) & (0.6,\ 0.2) & (0.2,\ 0.6) \end{bmatrix} \end{array} &
\begin{array}{cc} & \overset{x_2}{\begin{array}{ccc} z_1 & z_2 & z_3 \end{array}} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.5,\ 0.0) & (0.6,\ 0.1) & (0.2,\ 0.5) \\ (0.5,\ 0.1) & (0.6,\ 0.2) & (0.2,\ 0.6) \\ (0.7,\ 0.0) & (0.8,\ 0.0) & (0.4,\ 0.2) \end{bmatrix} \end{array} \\
\begin{array}{cc} & \overset{x_3}{\begin{array}{ccc} z_1 & z_2 & z_3 \end{array}} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.5,\ 0.1) & (0.6,\ 0.2) & (0.2,\ 0.6) \\ (0.5,\ 0.2) & (0.6,\ 0.3) & (0.2,\ 0.7) \\ (0.6,\ 0.0) & (0.7,\ 0.0) & (0.3,\ 0.3) \end{bmatrix} \end{array} &
\begin{array}{cc} & \overset{x_4}{\begin{array}{ccc} z_1 & z_2 & z_3 \end{array}} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.5,\ 0.2) & (0.6,\ 0.3) & (0.2,\ 0.7) \\ (0.5,\ 0.2) & (0.6,\ 0.3) & (0.2,\ 0.7) \\ (0.5,\ 0.0) & (0.6,\ 0.1) & (0.2,\ 0.5) \end{bmatrix} \end{array}
\end{array}\right]$$

where

$$x_1 \times z_1 \times z_1 = (\min\{1,\ 0.7 + 0.5\}, \max\{0,\ 0.2 + 0.2 - 1\}) \times z_1 = (1,0) \times (0.5,0.2)$$
$$= (\min\{1,\ 0 + 0.5\}, \max\{0,\ 1 + 0.2 - 1\}) = (0.5,0.2).$$

The R relation is presented as follows:

$$R = \begin{array}{cc} & \begin{array}{cc} y_1 & y_2 \end{array} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.6,\ 0.0) & (0.5,\ 0.0) \\ (0.7,\ 0.1) & (0.6,\ 0.1) \\ (1.0,\ 0.0) & (1.0,\ 0.0) \end{bmatrix} \end{array}$$

where

$$z_1 \times y_1 = (\min\{1,\ 0.2 + 0.4\}, \max\{0,\ 0.5 + 0.5 - 1\}) = (0.6,0).$$

Let a consumer's antecedent state at t=1 as follows:

$$A^1 = \{(x_1,\ 0.3,\ 0.4),\ (x_2,\ 0.4,\ 0.4),\ (x_3,\ 0.6,\ 0.3),\ (x_4,\ 0.5,\ 0.4)\}.$$

The ternary state-transition fuzzy relation S is converted into a binary relation S_{A^1} by applying (41):

$$S_{A^1} = \begin{array}{cc} & \begin{array}{ccc} z_1 & z_2 & z_3 \end{array} \\ \begin{array}{c} z_1 \\ z_2 \\ z_3 \end{array} & \begin{bmatrix} (0.5,\ 0.3) & (0.6,\ 0.3) & (0.2,\ 0.5) \\ (0.5,\ 0.3) & (0.6,\ 0.3) & (0.2,\ 0.6) \\ (0.6,\ 0.3) & (0.6,\ 0.3) & (0.4,\ 0.3) \end{bmatrix} \end{array}.$$

Use (24) and (26) simultaneously to calculate C^1. Assume that we select the standard intersection and standard union in the composition:

$$C^1 = [(0.5,\ 0.2)\ \ (0.6,\ 0.3)\ \ (0.2,\ 0.7)] \circ \begin{bmatrix} (0.5,\ 0.3) & (0.6,\ 0.3) & (0.2,\ 0.5) \\ (0.5,\ 0.3) & (0.6,\ 0.3) & (0.2,\ 0.6) \\ (0.6,\ 0.3) & (0.6,\ 0.3) & (0.4,\ 0.3) \end{bmatrix}$$
$$= [(0.5,\ 0.3)\ \ (0.6,\ 0.3)\ \ (0.2,\ 0.5)]$$

where (0.5,0.3) is the degree of membership and non-membership for the first subconstruct of meaningfulness, respectively. The computation is scored by:

$$(0.5,0.3) = (\max\{\min\{0.5,0.5\}, \min\{0.6,0.5\}, \min\{0.2,0.6\}\},$$
$$\min\{\max\{0.2,0.3\}, \max\{0.3,0.3\}, \max\{0.7,0.3\}\}).$$

Use (24) and (27) simultaneously to calculate B^1. Assume that we select the standard intersection and standard union in the composition:

$$B^1 = [(0.5,\ 0.3)\ \ (0.6,\ 0.3)\ \ (0.2,\ 0.5)] \circ \begin{bmatrix} (0.6,\ 0.0) & (0.5,\ 0.0) \\ (0.7,\ 0.1) & (0.6,\ 0.1) \\ (1.0,\ 0.0) & (1.0,\ 0.0) \end{bmatrix}$$
$$= [(0.6,\ 0.3)\ \ (0.6,\ 0.3)].$$

The final results imply that if a consumer's antecedent state is {(interest, 0.3,0.4), (pleasure, 0.4,0.4), (sign,0.6,0.3), (product knowledge, 0.5,0.4)}, his/her advertising state is {(meaningfulness, 0.5,0.3), (attractiveness, 0.6,0.3), (vitality, 0.2,0.5)} and the consequent state is {(message Attention, 0.6,0.3), (Recall, 0.6,0.3)} by applying the AFA.

Causal variables of advertising involvement

The integrated model is composed of antecedents, advertising involvement, and consequences. With an extensive literature review, there are sixteen antecedents and eight consequences included in the integrated model. Based on the framework of involvement presented by Zaichkowsky (1985), the antecedents can be categorized into three factors. The personal factor consists of interest, pleasure, sign, risk probability, risk importance (Laurent and Kapferer, 1985), need for cognition (Andrews et al., 1990), source credibility (Gotlieb and Sarel, 1991), product knowledge, product involvement (Laczniak et al., 1999), and information expectancy (Lee, 2000). The type of advertisement belongs to the stimulus factor (Krugman, 1967; Muehling et al., 1990). The situational factor consists of occasion-location, occasion-objective, occasion-time, occasion-person (Schiffman and Kanuk, 2004; Howard and Kerin, 2006) and purchase importance (Zaichkowsky, 1985).

There are three popular scales for measuring advertising involvement, including the revised personal inventory involvement (RPII) developed by Zaichkowsky (1994), the reaction profile (RP) presented by Wells (1964), and the view response profile (VRP) proposed by Schlinger (1979). Each scale of advertising involvement can be divided into several subconstructs. The RPII contains the subconstructs of cognition and affection; the RP consists of meaningfulness, attractiveness and vitality; the VRP is composed of entertainment, confusion, relevant news, and brand reinforcement. The three major scales are individually used as an internal state to test which one has the best effect in the AFA.

Eight consequences of advertising involvement are attitudinal acceptance (Wright, 1974), cognition response (Laczniak et al., 1989; Andrews and Durvasula, 1991), recall (Petty et al., 1983; Gardner et al., 1985; Andrews and Durvasula, 1991; Laczniak et al., 1999), message attention (Laczniak et al., 1989), brand evaluation, non-brand evaluation (Laczniak et al., 1989), belief strength, and attitude toward the advertisement (Laczniak and Muehling, 1993). Figure 3 illustrates all variables for the integrated model of advertising involvement.

Measurement design and subjects

The questionnaire helps us attain consumers' antecedent state, advertising involvement state and consequent state. Since interval-valued fuzzy sets are mathematically equivalent to AFSs (Dubois et al., 2005), subjects are asked to give an interval score for each item in the questionnaire. The degree of membership equals the lower bound of the interval score. The degree of non-membership equals one minus the upper bound of the interval score. Table 2 shows the sources of all variables in this paper. However, information expectancy (Lee, 2000), occasion-location, occasion-objective, occasion-time and occasion-person (Schiffman and Kanuk, 2004) do not appear in the table because they are measured with only one item. The Cronbach α of each variable reached more than 0.7. The entire questionnaire achieved high reliability.

Type of Ads and source credibility cannot be measured directly and are in need of manipulating. A two-factorial experimental design was employed (types of Ads: comparative advertisement versus non-comparative advertisement; source credibility: expert endorser versus citizen endorser). A 4G cell phone was determined to be the stimulus product on the print advertisement. The speed and the electromagnetic wave are two comparative points. For the comparative advertisement, we compare high speed and low electromagnetic wave between 3G and 4G cell phones. For the non-comparative advertisement, we directly describe the merits of 4G cell phone without any comparison. The expert endorser with a black suit is set up to show the profession and the citizen endorser is a blue-collar worker in the advertisement. To avoid extra-experimental artifacts due to the usage of an existing cell phone brand, a fictitious brand named Bason was featured in the experimental advertisement. Some essential information introducing the product was also included in the advertisement, such as the basic specification, price, figure, color, operational convenience, size, etc.

Calder et al. (1981) suggested that research sample had better possess high homogeneity due to the fact that high homogeneity can obtain more correct inference and reduce the covariance problem yielded from heterogeneous samples. Thus, 169 college students were enrolled by the convenience sampling in our investigation because students had similar education, age, etc. After the elimination of invalid samples, the final samples were 161. The valid rate was 95.27%.

A manipulation check was employed to manifest whether the two-factorial experimental design was appropriate. The result indicated that subjects who received the advertisement with an expert endorser elicited more source credibility than who received the advertisement with a citizen endorser (M=0.49 versus 0.38, p<0.001). The manipulation was successful in source credibility with the expert and citizen endorsers.

AN EMPIRICAL STUDY

Because the AFA were powered by the initial parameters,

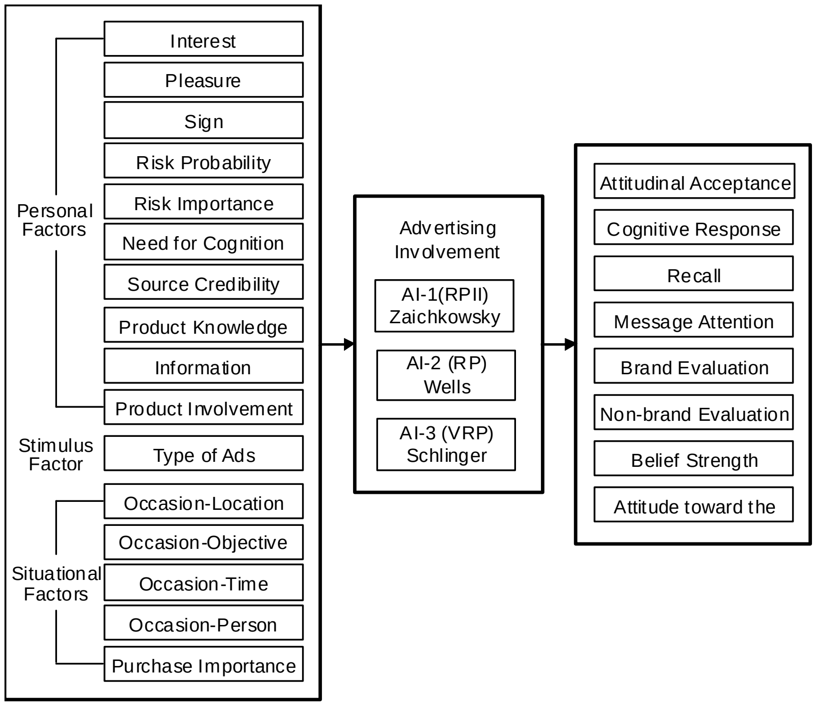

Figure 3. The conceptual framework of advertising involvement.

Table 2. Measurement of variables.

Variable	Source	Item number	Cronbach α
Antecedent of AI			
Interest	Kapferer and Laurent (1993)	3	0.7012
Pleasure	Kapferer and Laurent (1993)	3	0.7869
Sign	Kapferer and Laurent (1993)	3	0.7960
Risk probability	Kapferer and Laurent (1993)	4	0.9085
Risk importance	Kapferer and Laurent (1993)	3	0.8665
Need for cognition	Cacioppo et al. (1984)	18	0.9099
Source credibility	Gotlieb and Sarel (1991)	6	0.8948
Product knowledge	Smith and Park (1992)	4	0.9346
Purchase importance	Mittal (1989)	3	0.8326
Product involvement	Zaichkowsky (1985)	20	0.9608
Advertising involvement			
AI-1 (RPII)	Zaichkowsky (1994)	10	0.9683
AI-2 (RP)	Wells (1964)	26	0.9667
AI-3 (VRP)	Schlinger (1979)	17	0.8306
Consequence of AI			
Attitudinal acceptance	Wright (1973)	3	0.9161
Cognitive response	Lord and Burnkrant (1993)	6	0.7880
Recall	Ho (1999)	4	0.8729
Message attention	Laczniak et al. (1989)	5	0.9361
Brand strategy	Laczniak et al. (1989)	6	0.9539
Non-brand strategy	Laczniak et al. (1989)	6	0.9294
Belief	Laczniak and Muehling (1993)	5	0.7791
Attitude toward the Ad	Laczniak and Muehling (1993)	5	0.9213

implications, and compositions, several combinations (16 implicators × 4 compositions × 1,000 sets of parameters) were used individually in this dynamic machine. In order to stress the stability of the integrated model, 161 subjects were randomly and repeatedly distributed into 10 groups with the equal number of 80. Each group had to execute all of combinations. An ANOVA was adopted to test the average percentages of deviation among 10 groups. The values in Table 3 are the percentages of deviations which account for the smallest deviation generated by a set of parameter out of 1,000; meanwhile, the value is also the smallest deviation among 10 groups in which the percentages of deviation have no significant difference by the ANOVA.

In Table 3, we preset the optimal percentages of deviation yielded by one of 1,000 sets of parameters under different combinations of the implicator and composition. Take the first percentage of deviation in the table for an instance. The value of 15.2% represents the smallest deviation between the real values and computational values. The real values were obtained via the questionnaire, and computational values were calculated by applying the S 1-1 implicator and the standard composition when the internal state in the AFA adopts the RPII to be the subconstructs of advertising involvement. On average, the selection of compositions seems more important than that of implicators because applying the drastic composition in the AFA obviously gains larger deviations than other types of compositions. By contrast, applying different implicators does not lead to a prominent difference to the percentages of deviations. Although the use of the drastic composition causes great percentages of deviations, the drastic composition which goes with the S 1-3, R1-1,R1-2, orR 1-3 can reduce the deviation. It is because the Atanassov fuzzy relations constructed by above four implicators are often characterized as 0 or 1 in the membership degree and non-membership degree; meanwhile the nature of the drastic composition is to deal with the values of 0 and 1. Hence, the drastic composition merely performs well when the Atanassov fuzzy relations are generated by some specific implicators.

Based on different *t*-norms and *t*-conorms, 16 implicators are derived from 4 implications, including S, R, QL 1, and QL 2, and the composition are also extended to 4 types. Since the implication and composition are the keys to the AFA, Table 4 presents their overall effects on developing an integrated model of advertising involvement. On average, regardless of insignificant difference among 4 implications, the use of the R implication in the AFA can attain relatively small deviations. In the aspect of compositions, the standard composition is superior to other compositions, while applying the drastic composition causes prominently large deviations. Except some special case, the computational values calculated by the drastic composition are always 0 or 1; however, the degrees of advertising involvement and consequences investigated from the questionnaire are unlikely to be the extreme values. Therefore, the drastic composition is viewed as an improper operator in the AFA for developing an advertising involvement model. Furthermore, Table 4 also indicates that AI-2 (Wells' RP) serves as a preferable internal state on account of smaller deviations.

The advertising involvement model consisting of 16 antecedents and 8 consequences is regarded as a full model. Now, we consider another model which only contains the antecedent and consequent variables correlating with the advertising involvement. Two methods were utilized to calculate correlation coefficients between AFSs; one is based on information energy and the other is based on a statistical viewpoint. The correlation analysis is shown in Table 5. In contrast with the statistical method, the correlation coefficients are exceedingly higher in terms of the information energy no matter what the advertising involvement scale is employed in the AFA. If comparing the advertising involvement scales by the priority, we can observe that variables highly correlate with Zaichkowsky's scale according to the viewpoint of statistical method, but only have the worst correlation in terms of information energy. From Tables 4 and 5, the results of correlation analysis by means of the information energy method seem in accordance with the extent of deviations because the Wells' RP which highly correlates with antecedents and consequences has smaller deviations, and the Zaichkowsky's RPII yields larger deviations due to its poor correlation with variables. The viewpoint of energy information to deal with correlation is more meaningful and has plausible persuasion in this paper.

However, because the information energy method is incapable of judging a significance correlation between two variables, the statistical method assists in choosing the correlated variables to develop a new model. Since Wells' RP is the best internal state in the AFA, we use it for the following analysis. Table 6 presents the correlations between Wells' scale and causal variables. According to Table 6, all of consequent variables are significant, but merely seven antecedent variables can be selected in the model, including interest, pleasure, sign, source credibility, product knowledge, and product involvement. Implementing the same proposed algorithm in Figure3, asd we utilized the AFA to develop another model which was constructed by the significant relationship between Wells' advertising involvement and the causal variables. The optimal results are shown in Table 7. The selection of compositions is still important than that of implications. It is worthwhile to notice that the deviations generated by the AFA with all variables or only correlated variables are similar. In other world, even if there are unconcerned variables in the model, the AFA have the competence to distinguish and work well.

CONCLUSIONS

With an empirical study, the presented AFA succeed in operating and developing an integrated model of

Table 3. The optimal percentages of deviations.

Implicator	Composition											
	AI-1 (RPII) (%)				AI-2 (RP) (%)				AI-3 (VRP) (%)			
	Standard	Algebraic	Bounded	Drastic	Standard	Algebraic	Bounded	Drastic	Standard	Algebraic	Bounded	Drastic
S 1-1	15.2	17.3	18.9	44.7	13.0	15.2	18.5	45.9	13.8	15.5	16.6	44.6
S 1-2	15.1	16.2	17.5	44.7	13.2	13.4	17.3	45.9	15.0	14.6	16.2	44.6
S 1-3	15.1	14.9	16.4	30.7	13.2	13.4	15.1	25.7	14.5	13.9	14.9	27.2
S 1-4	15.0	15.0	15.2	44.7	13.4	12.9	13.1	45.9	13.3	13.7	13.6	44.6
R 1-1	15.7	15.9	18.0	23.9	13.5	14.1	15.8	21.7	15.0	15.6	15.8	23.6
R 1-2	15.0	15.3	16.4	29.1	13.3	13.9	15.5	26.1	13.9	15.0	15.5	25.5
R 1-3	14.9	15.0	15.9	31.0	13.2	13.5	15.0	26.6	14.5	13.9	14.7	27.3
R 1-4	14.9	15.0	15.1	44.7	13.0	12.9	13.3	45.9	14.4	13.6	13.3	44.6
QL 1-1	15.2	16.9	19.1	44.7	13.2	15.1	17.0	45.9	15.0	15.8	17.8	44.6
QL 1-2	15.2	17.9	20.2	44.7	13.2	15.6	19.0	45.9	15.0	16.2	19.2	44.6
QL 1-3	15.2	17.6	20.4	44.7	13.2	16.1	20.4	45.9	15.0	17.2	21.8	44.6
QL 1-4	15.2	15.9	16.7	44.7	13.2	14.2	14.7	45.9	15.0	15.9	16.7	44.6
QL 2-1	15.2	17.8	20.1	44.7	13.4	15.6	18.0	45.9	14.2	14.6	20.0	44.6
QL 2-2	15.2	17.2	19.5	44.7	13.3	15.5	18.0	45.9	14.7	14.5	19.3	44.6
QL 2-3	15.2	17.7	20.1	44.7	13.3	15.7	18.0	45.9	14.7	14.6	20.0	44.6
QL 2-4	15.2	17.8	20.1	44.7	14.0	15.7	18.0	45.9	15.7	15.8	20.0	44.6

advertising involvement. Several Atanassov fuzzy implicators are generated in this paper by different *t*-norms and *t*-conorms to construct Atanassov fuzzy relations. According to the analytic results, the effect of compositions is greater than that of implications because applying an inappropriate composition in AFA fails to obtain the intrinsic degrees of advertising involvement and consequences.

Specifically, the drastic composition is unqualified to handle the process of advertising involvement owing to its nature with 0 or 1. Overall, when Wells' RP serves as an internal state, the combination of S 1-4 and the algebraic composition or the combination of R 1-4 and algebraic composition which operates in the AFA can build the optimal model of advertising involvement.

The correlation between AFSs is measured by statistical perspective and the concept of information energy simultaneously. Although the former emphasizes the merits of strength and direction to thoroughly identify the correlation, the latter is relatively capable of explaining the eventual extent of deviations because the power of correlation coefficients has an influence on the stability of a model. Three advertising involvement scales are individually treated as the internal state in AFA to examine the utility degree. The Wells' RP fits the model well due to high correlation with its antecedents and consequences.

In addition to the examination of three advertising involvement, we present the comparison between the full model and the model which comprises only correlated variables.

The empirical data indicate that there is no significant difference between these two models. This finding implies that the AFA are characterized as the function to cope with any relationship between variables, inclusive of invalid, interaction, collinear problems. By applying this AFA, we can easily gain an individual's degrees of advertising involvement and consequences as long as realizing his/her antecedent state. However, the model is restrictedly applicable to the college students for the time being. We suggest that the future study can not only investigate various respondents in demographics and try different

Table 4. The optimal results under various implication and composition.

Implication and composition	Advertising involvement scale (%)		
	AI-1 (RPII)	AI-2 (RP)	AI-3 (VRP)
Mean of total implication	22.6	21.3	21.7
S implication	22.3	20.9	21.0
R implication	19.7	18.0	18.5
QL1 implication	24.0	23.0	23.7
QL2 implication	24.4	23.2	23.5
Mean of total composition	22.6	21.3	21.7
Standard composition	15.2	13.3	14.6
Algebraic composition	16.5	14.6	15.0
Bounded composition	18.1	16.7	17.2
Drastic composition	40.7	40.7	39.9

Table 5. The results of correlation analysis.

Variable	Statistical method			Information energy method		
	AI-1	AI-2	AI-3	AI-1	AI-2	AI-3
Antecedent of AI						
Interest	0.329(1)	0.294(2)	0.151(3)	0.742(3)	0.772(1)	0.752(2)
Pleasure	0.343(1)	0.323(2)	0.249(3)	0.699(3)	0.734(1)	0.718(2)
Sign	0.180(1)	0.150(2)	0.127(3)	0.814(3)	0.842(2)	0.847(1)
Risk probability	0.144(1)	0.084(2)	0.018(3)	0.820(3)	0.861(2)	0.894(1)
Risk importance	0.174(1)	0.093(3)	0.143(2)	0.709(3)	0.737(2)	0.741(1)
Need for cognition	0.027(1)	0.010(3)	0.065(2)	0.787(3)	0.825(2)	0.840(1)
Source credibility	0.337(3)	0.403(2)	0.442(1)	0.885(3)	0.917(2)	0.928(1)
Product knowledge	0.296(1)	0.275(2)	0.217(3)	0.881(3)	0.896(2)	0.905(1)
Information expectancy	0.077(1)	0.075(2)	0.006(3)	0.663(3)	0.703(1)	0.692(2)
Product involvement	0.368(1)	0.347(2)	0.220(3)	0.719(3)	0.752(1)	0.732(2)
Type of Ads	0.057(3)	0.075(1)	0.065(2)	0.638(3)	0.668(2)	0.673(1)
Occasion- location	0.052(2)	0.096(1)	0.026(3)	0.517(3)	0.567(1)	0.551(2)
Occasion- objective	0.050(2)	0.076(1)	0.041(3)	0.607(3)	0.655(1)	0.646(2)
Occasion- time	0.172(1)	0.140(2)	0.086(3)	0.659(3)	0.694(1)	0.682(2)
Occasion- person	0.101(2)	0.120(1)	0.091(3)	0.671(3)	0.712(1)	0.705(2)
Purchase importance	0.264(1)	0.261(2)	0.122(3)	0.715(3)	0.750(1)	0.728(2)
Consequence of AI						
Attitudinal acceptance	0.635(3)	0.722(1)	0.707(2)	0.937(3)	0.956(1)	0.955(2)
Cognitive response	0.598(2)	0.608(1)	0.589(3)	0.878(3)	0.904(1)	0.897(2)
Recall	0.469(3)	0.489(1)	0.491(2)	0.866(3)	0.893(1)	0.891(2)
Message attention	0.547(1)	0.534(2)	0.518(3)	0.900(3)	0.915(1)	0.912(2)
Brand strategy	0.504(1)	0.494(2)	0.467(3)	0.862(3)	0.883(1)	0.875(2)
Non-brand strategy	0.444(3)	0.472(2)	0.513(1)	0.854(2)	0.883(1)	0.883(1)
Belief strength	0.346(3)	0.401(1)	0.385(2)	0.766(3)	0.807(1)	0.798(2)
Attitude toward the Ad	0.821(2)	0.906(1)	0.798(3)	0.970(2)	0.985(1)	0.970(2)

The number in the brackets is the priority.

t-norms and *t*-conorms to generate new implicators and compositions.

Table 6. Correlation between Wells' scale and causal variables.

Variable	Subconstruct		
	Meaningfulness	**Attractiveness**	**Vitality**
Antecedent of AI			
Interest	0.329***	0.256***	0.281***
Pleasure	0.359***	0.286***	0.299***
Sign	0.183*	0.133	0.110
Risk probability	-0.126	-0.058	-0.069
Risk importance	0.109	0.078	0.088
Need for cognition	-0.016	-0.023	0.036
Source credibility	0.445***	0.365***	0.358***
Product knowledge	0.295***	0.238***	0.285***
Information expectancy	0.087	0.050	0.109
Product involvement	0.378***	0.312***	0.321***
Type of Ads	0.083	0.056	0.098
Occasion- location	0.129	0.060	0.116
Occasion- objective	0.110	0.05	0.057
Occasion- time	0.129	0.139	0.134
Occasion- person	0.109	0.107	0.149
Purchase importance	0.271***	0.233**	0.267***
Consequence of AI			
Attitudinal acceptance	0.719***	0.687***	0.681***
Cognitive response	0.615***	0.573***	0.570***
Recall	0.522***	0.447***	0.452***
Message attention	0.547***	0.508***	0.478***
Brand strategy	0.492***	0.477***	0.449***
Non-brand strategy	0.476***	0.452***	0.429***
Belief strength	0.422***	0.375***	0.360***
Attitude toward the Ad	0.843***	0.906***	0.833***

* p<0.05, ** p<0.01, ***p<0.001.

Table 7. Optimal results for two models.

Implication and composition	Wells' Advertising Involvement (%)	
	All variables	**Correlated variables**
Mean of total implication	21.3	22.1
S-implication	20.9	21.2
R-implication	18.0	21.2
QL 1-implication	23.0	22.5
QL 2-implication	23.2	23.3
	21.3	22.1
Mean of total composition		
Standard composition	13.3	12.4
Algebraic composition	14.6	14.3
Bounded composition	16.7	16.5
Drastic composition	40.7	45.1

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