Fatigue failure fault prediction of truck rear axle housing excited by random road roughness

Meng Qinghua¹*, Zheng Huifeng² and Lv Fengjun³

¹School of Mechanical Engineering, Hangzhou Dianzi University, Hangzhou, China.
²College of Metrology and Measurement Engineering, China Jiliang University, Hangzhou, China.
³School of Automobile, Zhejiang Institute of Communications, Hangzhou, China.

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A premature fatigue failure that occurred prior to the expected load cycles during the vertical fatigue tests of a truck rear axle housing prototype was studied. A seven degree of freedom dynamic model of truck was presented and vibrating equations on automobile as a whole was also built based on the model. Random road roughness data in time history that is generated by random phase method were applied to vibration differential equations to acquire dynamic load imposed on the rear axle housing. Fatigue failure of the rear axle housing finite element model was predicted after the dynamic load was imposed on it. The simulation results showed that there were premature fatigue failure regions at Banjo transition of the rear axle housing. Results provided from tests were compared with the analyses. The simulation results were same as the fatigue tests. Design enhancement solutions were proposed to increase the fatigue life of the housing according to the simulation and test results.

Key words: Fatigue failure, finite element analysis, automobile dynamics, solid axle, rear axle housing.

INTRODUCTION

Due to their higher loading capacity, solid axles are typically used in the commercial vehicles. Solid axles are loaded with dynamic load when vehicle runs on road. Dynamic stress is generated in the solid axles. The dynamic load is the major cause leading to solid axle's fatigue failure when vehicle runs high-speed or carries large weight (Zheng et al., 2009). Load bearing of the solid axle accounts for 60% of the whole vehicle’s load bearing. Thus the designed solid axle must have the ability to resisting fatigue failure.

In this paper, according to premature fatigue failure of a truck's rear axle housing, a seven degree of freedom dynamical model of vehicle was constructed, and the vehicle vibration differential equations based on the model were presented also. According to the road power spectrum data issued in national standard, a random phase method was used to generate random road roughness data of time history. Input the random road roughness data into the vibration differential equations, then the dynamic load generated in solid axle could be acquired. The dynamic load was applied into the rear axle housing finite element model and fatigue failure of the rear axle housing was analyzed. Results provided from tests were compared with the analyses. It showed that the simulation results were correct. In this paper, the reasonable solutions preventing the premature fatigue failure of the rear axle housing were presented also.

CONSTRUCTION OF VEHICLE DYNAMICAL MODEL

At present, when designers design the vehicle solid axle housing, the dynamic load generated in axle housing is regarded as static load or equivalent static load and the influence of random road roughness is not included generally. This processing method is applicable to a vehicle running slow and loading low. When vehicle runs high-speed or carries heavy load, the inertia load caused by vibration and impact load increase greatly. It can not reflect the actual load of vehicle caused by road roughness if still use static load model. For this reason, when design a new type solid axle, the influence of road roughness and variation of the loading must to be taken into consideration(Gao et al., 2008; Cong et al., 2009).
In Figure 1, \( m \) is suspension mass. \( m_A \) and \( m_B \) are left and right front wheel mass. \( m_C \) and \( m_D \) are left and right rear wheel mass. \( l_1 \) and \( l_2 \) are distance between front axle and suspension center of mass and distance between rear axle and suspension center of gravity. \( 2l_1 \) and \( 2l_4 \) are front wheel-track and rear wheel-track. \( z \) is vertical displacement of center of gravity. \( z_A \) and \( z_B \) are vertical displacements of left and right front wheel. \( z_C \) and \( z_D \) are vertical displacements of left and right rear wheel. \( \theta \) is roll angle of suspension mass around center of gravity. \( \varphi \) is pitch angle of suspension mass around center of gravity. \( k_{A1} \) and \( k_{B1} \) are stiffness coefficients of left and right front suspension. \( k_{C1} \) and \( k_{D1} \) are stiffness coefficients of right and left rear suspension. \( c_{A1} \) and \( c_{B1} \) are damping coefficients of left and right front suspension. \( c_{C1} \) and \( c_{D1} \) are damping coefficients of right and left rear suspension. \( k_{A2} \) and \( k_{B2} \) are stiffness coefficients of left and right front wheel. \( k_{C2} \) and \( k_{D2} \) are stiffness coefficients of right and left rear wheel. \( c_{A2} \) and \( c_{B2} \) are damping coefficients of left and right front wheel. \( c_{C2} \) and \( c_{D2} \) are damping coefficients of right and left rear wheel. \( q_1 \) and \( q_2 \) are road surface irregularity excitation of left and right front wheel. \( q_3 \) and \( q_4 \) are road surface irregularity excitation of right and left rear wheel.

**DYNAMICAL MODEL OF VEHICLE**

Three assumptions are given when vehicle vibration model is constructed.

Supposing that the body is a rigid body, the body weight and other weights supported by suspension are simplified as a rigid body which only has lumped mass.

The vehicle keeps constant speed to run in straight line and the wheels keep contact with road all the time.

Suspension stiffness and tyre stiffness are linear relationships with displacement. Suspension damping and tyre damping are linear relationships with relative velocity.

Figure 1 shows the constructed seven degree of freedom model of vehicle. The seven freedom degrees are vertical body’s jumping, front and back pitching, left and right rolling and four wheels’ vertical jumping.

**VIBRATION RESPONSE OF VEHICLE**

In this paper, dynamics equations of the seven degree of freedom were deduced by using of Newton’s law of motion equations.

If pitch angle \( \theta \) and roll angle \( \varphi \) are small, the vertical displacements of body’s four terminal vertexes have the following relationships.

\[
\begin{align*}
z_a &= z - l_1 \cdot \theta + l_3 \cdot \varphi \\
z_b &= z - l_1 \cdot \theta - l_4 \cdot \varphi \\
z_c &= z + l_2 \cdot \theta - l_4 \cdot \varphi \\
z_d &= z + l_2 \cdot \theta + l_4 \cdot \varphi \\
\end{align*}
\]

The dynamic load of \( Z \) axle caused by suspension mass \( m \) is:

\[
\begin{align*}
m \cdot z &= c_{A1} \cdot (\dot{Z}_a - Z_a) + k_{A1} \cdot (Z_a - Z_b) \\
&+ c_{B1} \cdot (\dot{Z}_b - Z_b) + k_{B1} \cdot (Z_b - Z_b) + c_{C1} \cdot (\dot{Z}_c - Z_c) \\
&+ k_{C1} \cdot (Z_c - Z_c) + c_{D1} \cdot (\dot{Z}_d - Z_d) + k_{D1} \cdot (Z_d - Z_d)
\end{align*}
\]

The Equation (3) is gotten from the Equations (1) and (2).

\[
\begin{align*}
m \cdot \ddot{z} + (c_{A1} + c_{B1} + c_{C1} + c_{D1}) \cdot \ddot{z} - c_{A1} \cdot \dot{Z}_a \\
&- c_{B1} \cdot \dot{Z}_b - c_{C1} \cdot \dot{Z}_c - c_{D1} \cdot \dot{Z}_d \\
&- [(c_{A1} + c_{B1}) \cdot l_1 - (c_{C1} + c_{D1}) \cdot l_2] \cdot \theta \\
&+ [(c_{A1} - c_{B1}) \cdot l_3 - (c_{C1} - c_{D1}) \cdot l_4] \cdot \varphi \\
&+ (k_{A1} + k_{B1} + k_{C1} + k_{D1}) \cdot z - k_{A1} \cdot Z_a \\
&- k_{B1} \cdot Z_b - k_{C1} \cdot Z_c - k_{D1} \cdot Z_d \\
&- [(k_{A1} + k_{B1}) \cdot l_1 - (k_{C1} + k_{D1}) \cdot l_2] \cdot \theta \\
&+ [(k_{A1} - k_{B1}) \cdot l_3 - (k_{C1} - k_{D1}) \cdot l_4] \cdot \varphi = 0
\end{align*}
\]

The Equations (4) to (9) can be gotten from analyzing the body’s pitching, rolling and four wheels’ vertical jumping.

\[
\begin{align*}
J_1 \cdot \ddot{\theta} - [(c_{A1} + c_{B1}) \cdot l_1 - (c_{C1} + c_{D1}) \cdot l_2] \cdot \ddot{z} \\
+ c_{A1} \cdot l_1 \cdot \dot{Z}_a + c_{B1} \cdot l_1 \cdot \dot{Z}_b - c_{C1} \cdot l_2 \cdot \dot{Z}_c - c_{D1} \cdot l_2 \cdot \dot{Z}_d \\
+ [(c_{A1} + c_{B1}) \cdot l_3 + (c_{C1} + c_{D1}) \cdot l_4] \cdot \theta \\
- [(c_{A1} - c_{B1}) \cdot l_1 + (c_{C1} - c_{D1}) \cdot l_2] \cdot \varphi \\
- [(k_{A1} + k_{B1}) \cdot l_1 - (k_{C1} + k_{D1}) \cdot l_2] \cdot z \\
+ k_{A1} \cdot l_1 \cdot Z_a + k_{B1} \cdot l_1 \cdot Z_b - k_{C1} \cdot l_2 \cdot Z_c - k_{D1} \cdot l_2 \cdot Z_D \\
+ [(k_{A1} + k_{B1}) \cdot l_3 + (k_{C1} + k_{D1}) \cdot l_4] \cdot \theta \\
- [(k_{A1} - k_{B1}) \cdot l_1 + (k_{C1} - k_{D1}) \cdot l_2] \cdot \varphi = 0
\end{align*}
\]
According to GB/T 7031-2005 / ISO 8608 1995, mechanical vibration spectrum of road surface roughness measurement data, power spectrum density of road \( G_q (n) \) can be generated using the following formula.

\[
\begin{align*}
J_z \ddot{\phi} + \left[ (c_{Al} - c_{Bl}) \cdot l_3 - (c_{C1} - c_{D1}) \cdot l_4 \right] \ddot{z} \\
- c_{Al} \cdot l_3 \cdot \ddot{z}_A + c_{Bl} \cdot l_3 \cdot \ddot{z}_B + c_{C1} \cdot l_4 \cdot \ddot{z}_C - c_{D1} \cdot l_4 \cdot \ddot{z}_D \\
- \left[ (c_{Al} - c_{Bl}) \cdot l_3 + (c_{C1} - c_{D1}) \cdot l_2 \cdot l_4 \right] \theta \\
+ \left[ (c_{Al} + c_{Bl}) \cdot l_3^2 + (c_{C1} + c_{D1}) \cdot l_2^2 \right] \phi \\
+ \left[ (k_{Al} - k_{Bl}) \cdot l_3 - (k_{C1} - k_{D1}) \cdot l_4 \right] z \\
- k_{Al} \cdot l_3 \cdot z_A + k_{Bl} \cdot l_3 \cdot z_B + k_{C1} \cdot l_4 \cdot z_C - k_{D1} \cdot l_4 \cdot z_D \\
- \left[ (k_{Al} - k_{Bl}) \cdot l_3 + (k_{C1} - k_{D1}) \cdot l_2 \cdot l_4 \right] \theta \\
+ \left[ (k_{Al} + k_{Bl}) \cdot l_3^2 + (k_{C1} + k_{D1}) \cdot l_2^2 \right] \phi = 0
\end{align*}
\]

\[ (5) \]

\[
\begin{align*}
m_A \dddot{z}_A - c_{Al} \cdot l_3 \cdot \dddot{z}_A + (c_{Al} + c_{A2}) \cdot \dddot{z}_A + c_{Al} \cdot \dddot{l_1} \cdot \theta \\
- c_{Al} \cdot l_3 \cdot \dddot{z}_A + (k_{Al} + k_{A2}) \cdot \dddot{z}_A \\
+ k_{Al} \cdot \dddot{l_1} \cdot \theta - k_{Al} \cdot \dddot{l_3} \cdot \phi = c_{A2} \cdot \dddot{q}_1 + k_{A2} \cdot q_1
\end{align*}
\]

\[ (6) \]

\[
\begin{align*}
m_B \dddot{z}_B - c_{Bl} \cdot l_3 \cdot \dddot{z}_B + (c_{Bl} + c_{B2}) \cdot \dddot{z}_B + c_{Bl} \cdot \dddot{l_1} \cdot \theta \\
+ c_{Bl} \cdot l_3 \cdot \dddot{z}_B + (k_{Bl} + k_{B2}) \cdot \dddot{z}_B \\
+ k_{Bl} \cdot \dddot{l_1} \cdot \theta + k_{Bl} \cdot \dddot{l_3} \cdot \phi = c_{B2} \cdot \dddot{q}_2 + k_{B2} \cdot q_2
\end{align*}
\]

\[ (7) \]

\[
\begin{align*}
m_C \dddot{z}_C - c_{C1} \cdot l_3 \cdot \dddot{z}_C + (c_{C1} + c_{C2}) \cdot \dddot{z}_C - c_{C1} \cdot \dddot{l_2} \cdot \theta \\
+ c_{C1} \cdot l_4 \cdot \dddot{z}_C + (k_{C1} + k_{C2}) \cdot \dddot{z}_C \\
- k_{C1} \cdot \dddot{l_2} \cdot \theta - k_{C1} \cdot \dddot{l_4} \cdot \phi = c_{C2} \cdot \dddot{q}_3 + k_{C2} \cdot q_3
\end{align*}
\]

\[ (8) \]

\[
\begin{align*}
m_D \dddot{z}_D - c_{D1} \cdot l_3 \cdot \dddot{z}_D + (c_{D1} + c_{D2}) \cdot \dddot{z}_D - c_{D1} \cdot \dddot{l_2} \cdot \theta \\
- c_{D1} \cdot l_3 \cdot \dddot{z}_D + (k_{D1} + k_{D2}) \cdot \dddot{z}_D \\
- k_{D1} \cdot \dddot{l_2} \cdot \theta - k_{D1} \cdot \dddot{l_4} \cdot \phi = c_{D2} \cdot \dddot{q}_4 + k_{D2} \cdot q_4
\end{align*}
\]

\[ (9) \]

ROAD ROUGHNESS

According to GB/T 7031-2005 / ISO 8608 1995, mechanical vibration spectrum of road surface roughness measurement data, power spectrum density of road \( G_q (n) \) can be generated using the following formula.

\[
G_q (n) = G_q (n_0) \left( \frac{n}{n_0} \right)^w
\]

\[ (10) \]

In the formula, \( n \) is space frequency (reciprocal of wave( m\(^{-1}\)). \( n_0 \) is reference space frequency (\( n_0 = 0.1 \) m\(^{-1}\)). \( w \) is exponent of fit power spectrum density. \( G_q (n_0) \) is road spectrum of reference frequency \( n_0 \).

According to the given road spectrum in GB/T 7031-2005 / ISO 8608, phase random method was used to generate random road roughness in time-history. The method uses the formula to generate random road roughness.

\[
q(t) = \sum_{i=1}^{m} \sqrt{4G_q (n_i)} \Delta n \cdot \cos (n_i vt - \theta)
\]

\[ (11) \]

In this equation, \( n_i \) is space frequency. \( \Delta n \) is increment of space frequency. \( \theta_i \) is the random number which distributes uniformly in \([0,2\pi]\). \( v \) is speed of vehicle. \( t \) is vehicle travel time.

The software Matlab is used to simulate the road roughness when vehicle travels under 50 km/h. The F level road roughness got from the simulation is showed in Figure 2.

BUILDING FINITE ELEMENT MODEL OF AXLE HOUSING

3D Solid model of axle housing

The housing essentially consists of two equivalent thin walled shells, which have a uniform thickness of 8 mm and welded along the neutral axis of the rear axle. An erection loop is welded on the housing’s front side to increase stiffness. Differential is fixed in the housing by bolts. For sealing reasons, a dome is welded to the rear side. The CAD model of the axle housing was imported into ANSYS Workbench V11.0 preprocessing environment to constitute the finite element model. To building the finite element model, housing was meshed using SOLID187, a higher order three-dimensional solid element, which has a quadratic displacement behavior and is well suited to model irregular meshes. Finite element model consisted of 448,003 elements.

Because of the vertical acceleration of lumped mass of the vehicle body due to the road surface roughness, maximum dynamic loading on each coil spring seat is estimated about twice as much as static loading. According to the dynamics analysis aforementioned, vertical loading range was obtained as 126 to 1670
Finite element analysis and results

Figure 3 shows equivalent von Mises stress distribution provided from the FE analysis. Results show that there are tensile stress concentrated regions A1 and A2 at transition area of axle spindle mountings and Banjo transition area of the carrier mounting side of the lower shell. Locations of the critical regions and the premature fatigue failure are the same as seen in Figure 4. The calculated maximum von Mises stress is 15.3 MPa far below the material’s yielding stress. This means that housing prototype satisfies the safety conditions for maximum loading if it is exerted statically.

FATIGUE LIFE PREDICTION

When vehicle is running on the road, the axle housing is inspired by dynamic loading. Thus fatigue analysis is important to axle housing. Fatigue endurance limit is given as (Topac et al., 2009)

$$S_e' = 0.504 S_{ut}$$  \(12\)

Because the ultimate tensile strength of steel $S_{ut}$ is less
than 400 Mpa, the fatigue endurance limit of steel is about $10^6$ cycles. $S_e$ stands for the stress life endurance limit of ideal laboratory samples. In order to predict the true fatigue strength $S_e$ for a mechanical component, $S_e$ has to be multiplied by several modifying factors which represent various design, manufacturing and environmental influences on the fatigue strength (Smith M et al., 2007). $S_e$ is given as

$$S_e = k_a k_b k_c k_d S_{st}$$  \hspace{1cm} (13)

where $k_a$, a surface factor which depends on surface roughness is given as

$$k_a = a S_{st}$$  \hspace{1cm} (14)

Size factor $k_s$ can be assumed as 0.75. Load factor $k_c$ is given as 1 for bending. Temperature factor $k_d$ is 1 for the range of the ambient temperature of 0 to 250°C (Shigley, 1977). Because there are stress concentrated regions, in addition to the modifying factors aforementioned, a fatigue strength modifying factor $k_e$ must be taken into account that is related to fatigue stress concentration factor $k_f$. Hence $k_e$ is calculated as

$$k_e = 1 / k_f$$  \hspace{1cm} (15)

For safety reason, $k_f$ can be assumed as to be equal to factor of static stress concentration $K_f$ (Shigley et al., 1989). $K_f$ cannot be derived from the standard literature because of the dimensions and shape complexity of the axle housing, however, it is defined as

$$K_f = \frac{\sigma_{\text{peak}}}{\sigma_{\text{nomin}}},$$ \hspace{1cm} (16)

where $\sigma_{\text{peak}}$ is the peak stress and $\sigma_{\text{nomin}}$ is the nominal stress.

To calculate $\sigma_{\text{nomin}}$, the rear axle housing is assumed as a simple beam which has a uniform box profile cross-section along the longitudinal axis X and subjected to pure bending moment.

$$\sigma_{\text{nomin}} = \frac{M}{Z}$$  \hspace{1cm} (17)

where $M$ is bending moment and $Z$ is the section modulus of the critical cross-section.

The fatigue safe coefficient distribution is showed in Figure 4 derived from finite element analysis. It is estimated that crack initiation can occur at the region A2 of the outer shell surface and the symmetric region at about $2.8 \times 10^5$ cycles, which is lower than the expected minimum fatigue life of $5 \times 10^5$ cycles. Here the obtained minimum safe factor value is 0.80. This means, at the two regions A2 and its symmetric region fatigue crack can initiate before $5 \times 10^5$ load cycles as observed in the vertical fatigue tests. The results are in agreement with the results of vertical fatigue tests. The fatigue crack regions derived from vertical fatigue tests are showed in Figure 5.

Enhancement of the fatigue life of the axle housing is dependent on the decrease of the stress concentration. The simplest way to reduce the stress concentration and
To improve the fatigue life is to increase the thickness of the sheet metal. However, except regions A2 and its symmetric region, the housing satisfies the infinite life criteria. An increase of sheet metal thickness causes an unnecessary weight increase. For example, a thickness increase of 0.5 mm enhances the fatigue limit of the housing material at the critical regions up to more than $5 \times 10^5$ cycles which is higher than the desired limit. On the other hand, this also means a weight increase about 5% of unsprung mass of vehicle and can influence vehicle fuel economy and dynamic characteristic. Therefore it is not a practical solution. Smoother transition geometry may offer an enhanced fatigue life without any weight increase. In addition, the use of the ring on the critical regions can decrease stress concentration to enhance fatigue life.

In order to solve the problem of enhancing fatigue life of axle housing, increasing the thickness of sheet metal is not a practical solution because of the weight increase of housing. An application including both redesigning of the Banjo transition area and increasing the thickness of the reinforcement ring may be a good alternative to enhance the fatigue life, which can satisfy minimum design criteria.

**Conclusion**

In order to analyze premature fatigue failure of a truck’s rear axle housing, a seven degree of freedom dynamic model of the truck was presented. Based on the model, vibration differential equations were given. Random road roughness data of time history that is generated by random phase method were applied to vibration differential equations to acquire dynamic load imposed on the drive axle housing according to road power spectrum in the national standard. The acquired dynamic load was applied to the finite element model of axle housing to predict fatigue failure. In the analyses, in which the vertical fatigue test procedure was simulated, stress concentrated regions were predicted at the Banjo transition area and transition area of axle spindle mountings. The regions in which the fatigue cracks originated were well-matched with the results of the analyses. By using the method in this paper, the location of the failure can be predicted.

**REFERENCES**