Full Length Research Paper

Parametric evaluation of tree root water-uptake effect on ground movement

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A model is a tool that simulates the reality through simplification by ignoring what is not important. Therefore, assumptions are usually involved, prompting the need for parametric analysis for identification of the responsive parameters with respect to the numerical simulation results. A negative pore-water pressure was estimated through two-dimensional governing equation for unsaturated soil in an axi-symmetrical form due radial nature tree roots water-uptake. The results of the root water-uptake analysis are then used as an input for the prediction of ground displacements in a stress–deformation analysis. The proposed method was studied and tested against data collected on a case history involving a mature Lime tree on Boulder clay at Stacey Hall, Wolverton, England and mechanical properties of Boulder Clay. The results of the analysis showed that the predicted ground displacement is sensitive to all the parameters tested. Initial time step sizes analysis showed that the results differs not more than ±5% indicating there are no problems with convergence. These results suggest that accurate measurements of these parameters would be necessary for the study of ground displacement due root water-uptake.

Key words: Unsaturated soils, matric suction, water-uptake, simulation, stress-deformation.

INTRODUCTION

Withdrawal of water by plant roots results in change in water pressures and moisture content in the soil. Soil settlement occurs whenever there is an increase in effective confining stress. The variation in the moisture content leads to a change in the effective stress that causes a decrease in porosity and void volume which eventually results into volume change in soil. A horizontal and vertical distribution of roots determines the dispersal of root water-uptake (Ali and Mu’azu, 2010; Mu’azu et al., 2010). A simple concept of sink term for root water-uptake was developed by Rees and Ali (2006) and incorporated to two-dimensional axi-symmetric governing equation for unsaturated soil. Various researchers have done work on root water uptake coupled and uncoupled approach to ground deformation (Fredlund and Hung, 2001; Nivarro et al., 2009; Fatahi et al., 2009; Fatahi et al., 2010; Nyambayo and Potts, 2010). In describing water uptake by plant roots, there are two main approaches (Feddes et al., 1976; Mathur and Rao, 1999). The first strategy typically considers radial soil water flow to a single root and is therefore known as the ‘microscopic’ approach. In contrast, the second approach is based on a ‘macroscopic’ view of the problem and considers the root systems are analyzed as a single unit. These macroscopic models also allow natural interaction with the transpiration process. The inclusion of a volumetric sink term in Richard’s equation to accommodate water-uptake is an approach that has been used quite widely by Molz (1981) and Clausnitzer and Hopmans (1994).

Uncertainties in the assumptions of soil parameters, effect of elapse time, simulation step size time and atmospheric parameters caused most discrepancies in these simulations. A parametric analysis for identification of the responsive parameters with respect to the numerical simulation work and introduction of effects of mechanical properties of the soils in the evaluation are therefore, the main objective of this research. This paper employed two-dimensional axi-symmetrical finite element approach to solve the transient partial coupled flow and stress-deformation equations. The study was

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based on case study of mature single lime tree on a Boulder Clay as reported by Biddle (1998).

WATER-UP TAK E CONCEPTS

In prediction of soil movement, two fundamental stages are generally involved; an assessment of the changes in moisture conditions and the knowledge of the volumetric strains induced by these changes. The first step is dealt with through the use of modified Richard equation (1931) two-dimensional axi-symmetric governing equation for unsaturated soil with sink term.

\[ \frac{\partial \psi}{\partial \mathbf{t}} + \frac{\partial \psi}{\partial \mathbf{z}} = \left[ \frac{K(\psi)}{\partial \mathbf{z}} \right] + \frac{1}{\mathbf{r}} \frac{\partial}{\partial \mathbf{r}} \left[ K(\psi) \frac{\partial \psi}{\partial \mathbf{r}} \right] + c_1 \frac{\partial}{\partial \mathbf{z}} S(\mathbf{r}, \mathbf{z}) \]  

Where \( K(\psi) \) is the unsaturated hydraulic conductivity, \( t \) is the time, \( r \) and \( z \) are the coordinate, \( \psi \) is the volumetric moisture content and \( S \) is the sink term \( S(\psi, z, r) \) in Equation 1, for water-uptake for two-dimensional axi-symmetric (Rees and Ali, 2006), comprising of vertical and radial components, incorporating water stress function when soil moisture is limiting:

\[ S(\mathbf{r}, \mathbf{z}, \mathbf{r}) = \frac{4T_a}{z_r r_r} \alpha(\psi) \left[ 1 - \frac{\mathbf{z}}{z_r} \left( 1 - \frac{\mathbf{r}}{r_r} \right) \right] \]

Where \( \alpha(\psi) \) (dimensionless) is a prescribed function of the capillary potential referred to as a water-stress function, \( T_a \) is actual transpiration rate, \( z_r \) is maximum rooting depth in the radial direction, \( z \) is maximum rooting depth, \( r \) is radial distance from the origin of the plant trunk and \( z \) is depth in the soil profile.

The numerical solution of Equation 1 via the finite element spatial discretization procedure and a finite-difference time-stepping scheme, particularly adopting a Galerkin weighted residual approach which yields the discretized matrix form with added deformation component (Rees and Ali, 2006)

\[ K \psi + C \psi + J + S = 0 \]

The parabolic shape functions and eight-node isoparametric elements were employed (Zienkiewicz and Taylor, 1989). The time-dependent nature of Equation 3 was dealt with via a mid-interval backward difference technique, yielding

\[ K(n+1)^{n+1} \psi(n+1) + C(n+1)^{n+1} (\frac{\psi(n+1) - \psi(n)}{\Delta t}) + J(n+1)^{n+1/2} + S(n+1)^{n+1/2} = 0 \]  

GROUND MOVEMENT CONCEPTS

The second step was tackled through stress-deformation formulation considering unsaturated soil mechanics concept in ground water field concept. Fredlund and Hung (2001) stated that the volume change constitutive relations for the unsaturated soils are formulated using the two stress state variables, namely; net normal stress \( (\sigma - u) \) and matric suction \( (u_a - u_w) \).

Constitutive relationships are to compliment governing flow equation providing additional relationship between stress-deformation and stress state variables. It was also assumed that

the pore-air pressure is the same with atmospheric pressure, so that the distribution of pore-water pressure is equivalent to the matric suction distributions. Changes in the negative pore-water pressure occur as a result of root water-uptake and can be related to changes in soil volume through the use of constitutive relations. Swelling in the field occurs along the rebound curve at an overburden pressure of \( (\sigma - u_a) \) and matric suction \( (u_a - u_w) \). Shrinkage occurs along either a recompression curve or the virgin compression curve. The mathematical expression for the recompression curve can be expressed as

\[ de = c_i' d(\sigma - u_a) + c_z' d(u_a - u_w) \]

Where \( de \) is change in void ratio, \( c_i' \) is coefficient of compressibility with respect to a change in stress \( (\sigma - u_a) \) and \( c_z' \) is coefficient of compressibility with respect to a change in stress \( (u_a - u_w) \), \( (\sigma - u_a) \) is net mean stress, \( (u_a - u_w) \) is matrix suction, \( \sigma \) is total effective stress; \( u_a \) is pore water pressure and \( u_w \) is pore air pressure. The form of the constitutive equation for the rebound curve is similar in expression to Equation 5 except that the moduli are from the rebound curve. The net normal stress state within the soil mass can be computed using Equation 6 while horizontal net normal stresses can be estimated from the vertical stresses and \( K \) using Equation 7:

\[ \sigma_z = \int_{0}^{H} \rho g dy \]

\[ \sigma_r = K_0 \sigma_z \]

Where \( \sigma_z \) is vertical net normal stress, \( \sigma_r \) is horizontal stress, \( K_0 \) is coefficient of earth pressure at rest and \( H \) is depth of soil under consideration. While the soil is a normally consolidated clay with a consolidation behavior that can be described by

\[ de = C_r \ln \left( \frac{\sigma_v + \Delta \sigma_v - u_{wf}}{\sigma_v - u_a + (u_a - u_w)} \right) \]

Where \( de \) is the change of void ratio in the element, \( C_r \) is the recompression index, \( \sigma_v \) is the vertical total stress, \( \Delta \sigma_v \) is the change in the total vertical stress, \( u_{wf} \) is the final pore water pressure, \( u_a - u_w \) is the matric suction equivalent (Fredlund and Rahardjo, 1998). The elasticity parameters are functions of the stress state of the soil, net normal stress and the matric suction. These elastic moduli \( E \) and \( H \) could be estimated from volume change indices, initial void ratio and Poisson’s ratio (Fredlund and Hung, 2001). The method of volume change prediction is based on one-dimensional oedometer test. The expression for estimating elastic moduli \( E \) and \( H \) are as follows:

\[ C_i' = \frac{de}{\partial(\sigma - u_a)} \]

\[ C_z = C_i' (\sigma - u_a)_m \frac{0.435}{435} \]
\[ \sigma_m = \sigma - u_a \]  
\[ n_i = \frac{4.605(1 + \mu)(1 - 2\mu)(1 + e_0)}{C_i} \]  
\[ E = n_i(\sigma)_m \]  
\[ C_2^s = \frac{\partial e}{\partial(u_a - u_w)} \]  
\[ C_m = C_2^s \frac{(u_a - u_w)_m}{0.435} \]  
\[ n_m = \frac{4.605(1 + \mu)(1 + e_0)}{C_m} \]  
\[ H = n_m(\psi)_m \]

Where \( E \) is elasticity parameter for the soil structure with respect to a change in the net normal stress, \( H \) is elasticity parameter for the soil structure with respect to a change in matric suction, \( C_2 \) is shrinkage indices with respect to change in net normal stress, \( C_m \) is shrinkage indices with respect to change in matric suction, \( n_i \) is coefficient relating to net normal stress with elastic modulus \( (E) \), \( n_m \) is coefficient relating to matric suction with elastic modulus \( (H) \), \( \sigma_m \) is \((\sigma_s - \sigma_0)/2, (s_m - u_a)_m \) is the average of the initial and the final net normal stress for an increment and \((u_a - u_w)_m \) is average of the initial and the final matric suction for an increment. The capillary potential \( (\psi) \) was estimated from Equation 1 which was used as an input for the stress-deformation analysis. This relationship was established to perform the necessary ground displacements estimation

\[ \frac{\partial \varepsilon}{\partial \psi} = \frac{1}{H} \frac{\partial V_r}{\partial \psi} \]  

FINITE ELEMENT SIMULATION

The case considered here relates to a single mature lime tree, 15 m in height, located on a Boulder Clay sub-soil at a site located at Stacey Hall, Wolverton, England. The lime tree under consideration is 15 m tall and close to edge of a field grazed by horses. The mesh consists of 8-noded isoperimetric linear strain quadrilateral elements. The entire finite element mesh consists of 1281 nodes and 400 elements; the axi-symmetric domain is shown in Figure 1. The mesh was configured to offer some refinement within the root zone area, since this is the region where the most significant moisture content variations were expected to occur. Based on the field observations provided by Biddle (1998), the root zone was assumed to extend to a depth of 2.0 m and a radial distance of 5.0 m, for lime tree, the soil profile was assumed to be a homogenous layer of Boulder Clay. The simulation employs a time-step size of 21600 s, which was held constant for the entire period considered. The soil parameters are shown in Table 1 which are typical values for Boulder clay.

Initial matric suction conditions can be measured using field methods and laboratory methods (Fredlund and Rahardjo, 1993) or estimated from theoretical considerations of unsaturated soil conditions. The required soil moisture retention characteristics and unsaturated hydraulic conductivity was simulated from the closed form equation of the developed by van Genuchten (1980), thus

\[ \theta(\psi) = \theta_r + \frac{(\theta_s - \theta_r)}{[1 + (\alpha\psi)^n]^m} \quad \psi \geq 0 \]  
\[ \theta(\psi) = \theta_r - \frac{(\theta_s - \theta_r)}{[1 + (\alpha\psi)^n]^{m+1}} \quad \psi < 0 \]  
\[ K = K_s \frac{[1 + (\alpha\psi)^n]^{m+1} - (\alpha\psi)^n}{[1 + (\alpha\psi)^n]} \]  

Where, \( \theta_r \) is saturated water content, \( \theta_s \) is residual water content, \( \psi \) is suction head (cm), and \( n, m, \alpha \) are empirical shape fitting parameters estimated by fitting Equations 19 and 20 to the experimental data. \( K \) and \( K_s \) are unsaturated and saturated hydraulic conductivities respectively, while \( l \) is a soil specific parameter generally assumed to be 0.5.

Application of the proposed model, requires specification of the water retention curve (hence specific moisture capacity) and the hydraulic conductivity relationship for the Boulder Clay. In fact Equation 19 was again used to determine the water retention curve and Equation 20 was used to estimate the hydraulic conductivity.

Model verification

The numerical results seem to agree with Fredlund and Hung’s (2001) analysis, the difference is about 5%. The slight disparity between the two results is as a result of the fact that two entirely different unsaturated soil models were used in this study and the two different theories influenced the volume change of an unsaturated soil differently. Fredlund and Hung (2001) considered that the water uptake is only time dependent while the current study
Table 1. Parameters used in the analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$k_s$</td>
<td>$10^{-6}$ m/s</td>
<td>Biddle (1998)</td>
</tr>
<tr>
<td>$T_s$</td>
<td>5 mm/day</td>
<td>Biddle (1998)</td>
</tr>
<tr>
<td>$\Psi_d$</td>
<td>1500 kPa</td>
<td>Feddes et al. (1976)</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>21 kN/m³</td>
<td>Indraratna et al. (2006)</td>
</tr>
<tr>
<td>$\varepsilon_0$</td>
<td>0.60</td>
<td>Powrie et al. (1992)</td>
</tr>
<tr>
<td>$C_r$</td>
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<td>Indraratna et al. (2006)</td>
</tr>
<tr>
<td>$\mu$</td>
<td>0.30</td>
<td>Indraratna et al. (2006)</td>
</tr>
<tr>
<td>$\theta_l$</td>
<td>0.1</td>
<td>Rees et al. (2006)</td>
</tr>
<tr>
<td>$\theta_s$</td>
<td>0.4</td>
<td>Rees et al. (2006)</td>
</tr>
<tr>
<td>$a$</td>
<td>0.560</td>
<td>Rees et al. (2006)</td>
</tr>
<tr>
<td>$m$</td>
<td>0.29</td>
<td>Rees et al. (2006)</td>
</tr>
<tr>
<td>$n$</td>
<td>1.4</td>
<td>Rees et al. (2006)</td>
</tr>
<tr>
<td>$l$</td>
<td>0.5</td>
<td>Rees et al. (2006)</td>
</tr>
</tbody>
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Figure 2. Variation of ground movement with depth near the tree for the current research.

Figure 3. Variation of ground movement with depth near the tree after Fredlund and Hung (2001).

Figure 4. Simulated and site ground displacements profiles after 190 days at radial distance of 1.4 m from lime tree.

and 270 days as shown in Figure 5 at 3.0 m away from the trunk of the lime tree. The effect of elapsed time for full cycle was simulated for a period that covers a spring/summer soil-drying phase of 9 months. The results of this evaluation show that the soil ground movement increases with an increase in elapsed time and the ground displacement decreases as the lateral distance from the tree trunk increases. This might be attributed to the time and space dependent nature of abstraction of ground water by vegetation.

RESULTS AND DISCUSSIONS: PARAMETERS AFFECTING GROUND DISPLACEMENT

Effect of elapsed time on capillary potential

The capillary potential, as shown in Figure 6 is also affected by elapsed time as well as spatial distance from the tree trunk for 190 days. The capillary potential decreases as the lateral distance increases from the tree trunk.
of the tree to check the convergence problems are shown in Figure 8.

The results of the capillary potential as shown in Figure 8 at 1.4 m away from the tree and elapse time of 30 days, 90 days, 190 days and 270 days differ with not more than ±5%. The tested initial time step size in the context of spatial variation and elapse time was found to differ in all respect with not more than ±5% which is considered satisfactory.

Effect of actual transpiration rate ($T_a$)

Five different numerical simulations were carried out to demonstrate the effect of actual transpiration rate, using actual transpiration rate $T_a$ from 3 to 7 mm/days. Figure 9 shows the variation of generated soil displacement with various actual transpiration rates for 3.0 m away from the tree trunk after 190 days. A higher rate of actual transpiration rate $T_a$ leads to a higher ground settlement.

The analysis shows that an increase in rate of actual transpiration increases ground settlement. The analysis confirms that the most sensitive parameter is transpiration rate this is in agreement with Fatahi et al. (2009). Its accurate determination or measurement cannot be therefore, over emphasized for an acceptable prediction of ground conditions in the vicinity of trees.

Effect of unit weight of the soils (UW)

The effect of unit weight of soil the on the numerical simulation was evaluated via five different numerical simulations using 14 to 22 kN/m$^3$ range of soil unit weight. Figure 10 shows the effects of soil unit weight on this simulation work at 4.9 m away from trunk of the tree for elapse time of 190 days. The results indicate decrease in ground displacement with an increase in soil unit weight.

In Figure 10 the ground displacement decreases from 20.34 mm to 12.94 mm from unit weight of 14 to 22 kN/m$^3$ respectively. The ground displacement is inversely proportional to soil unit weight. The soil unit weight strongly affects the ground settlement due to the fact that soil unit weight is ratio of weight of the soil to its total volume. When the soil weight is much higher compare to its volume, soil matrix is more compact with minimal pores to contain air and water.

Effect of soil initial void ratios (IVR)

Figure 11 shows the effect of soil initial void ratio at 1.4 m away from trunk of the tree after 30 days. There is gradual decrease in soil settlement with increase in soil initial void ratio. The effect of IVR on ground displacement may be due to the complex relationship between capillarity, volume of void and soil permeability.
Figure 7. Variations of volumetric moisture content with depth at various lateral distance from lime tree after 270 days.

Figure 8. Variations of time step size with capillary potential and depth at 1.4 m from the lime tree after 30, 90, 190 and 270 days.

Figure 9. Variation of ground settlement with depth at various potential transpiration rates at 3.0m away from lime tree after 190 days.

The generated ground displacement shows inverse proportionality with an increase in soil initial void ratio.

Effect of soil re-compression index (RCI)

The effect of soil re-compression index on ground displacement, five simulations were carried out. Soil re-compression index of 0.11 to 0.15 and elapse time of 30, 90, 190 and 270 days were used. The ground settlement increased with an increased in soils re-compression index as shown in Figure 12 for 190 days. This result also seems to agree with the fact that soil re-compression index greatly influences the shrinkage and heave of the soil.

Conclusion

Parametric analysis was done to check the effect of
elapse time on matric suction and ground displacement, effect of elapse time on capillary potential, effect of elapse time on volumetric moisture content, initial time step sizes, effect of actual transpiration rate, effect of unit weight of the soils, effect of soil initial void ratios and effect of soil re-compression indices. The results of the analysis show that the predicted ground displacement is sensitive to all the parameters tested with potential transpiration rate as the most sensitive. This suggests that accurate measurement of these parameters is necessary for the study of ground displacement due to root water-uptake. For the effect of initial time step sizes used in the analysis, it was found that the results differ from each other with not more than ±5%, which is satisfactory an indication that the results converged.

REFERENCES


Figure 12. Variation of ground settlement with depth at various soils re-compression index at 3.0 m away from tree after 190 days.