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Synchronous Nyquist folding receiver using dual harmonics local oscillator signals

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A synchronous Nyquist folding receiver (S NYFR) could intercept wideband signals in multi-Nyquist zones with two analog-to-digital converters. The instantaneous Nyquist zone (INZ) of the SNYFR may not exist or is not unique, and the dual local harmonics was presented to improve the stability of the SNYFR. It was found that the existence and unique conditions of the INZ determined that the frequency of the local oscillator signal (LOS) in one zone was constant. A symmetrical low-pass filter and dual harmonics LOSs were adopted to facilitate the detection of the INZ. Simulations showed that the proposed structure is valid for the interception of wideband radar signal.

Key words: Electronic countermeasures, parameter estimation, linear frequency modulation, synchronous Nyquist folding receiver.

INTRODUCTION

The ideal electronic support measurement (ESM) receiver should be able to intercept the whole radar frequency range, namely, about 18 GHz or even 30 GHz. The ESM may find the location of the transmitter (Chen et al., 2008; Gui et al., 2011). However, the sampling rate of commercially available off-the-shelf analog-to-digital converter (ADC) is often less than or equal to 5 gigasamples per second (GSPS). Therefore, it is difficult to directly sample the signals in such a wide frequency range according to the Nyquist-Shannon sampling theorem. Donoho (2006) shows that compressive sensing (CS) can reconstruct the signal from the received signal with a high probability using a low rate ADC by introducing a signal independent observation matrix and an optimal solution algorithm. Typical CS receivers, which could be used in ESM receiver, include the receiver using random demodulation (Laska et al., 2007) and the one with random filters (Tropp et al., 2006). However, how to quickly determine the sparse domains of the received signals in a non-cooperative case and the optimal solution still needs to be studied.

A Nyquist folding receiver (NYFR) modulates the received analog signal in the front-end of the receiver, and demodulates the signal in digital signal processor (DSP) (Fudge et al., 2008). By changing the modulation type and the number of zones, the whole interception frequency could be intercepted using single or dual ADCs without frequency sweeping. However, the NYFR is easily affected by noise when using zero crossing rising voltage time to control the radio frequency sample clock, and would lose the initial phase of the received signal. A synchronous NyFR (SNYFR) digitally compensates for these shortcomings of the NYFR (Zeng et al., 2011). Unfortunately, none of the NYFR or SNYFR has considered the existence and unique condition of instantaneous Nyquist zone (INZ).

This paper takes the classical linear frequency modulated (LFM) signal as an example, and obtains the existence and unique conditions of the INZ. Moreover, an algorithm to the parameter estimation of a LFM signal under this condition is proposed.

SNYFR

Structure of SNYFR

The structure of SNYFR is as shown in Figure 1. Assume the input analog signal has been preprocessed into I/Q

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signals. Firstly, the input signal is filtered by a ultra wideband (UWB) filter (LPF₁) whose bandwidth is \( B_i \) to remove the out-of-band noise to get the complex signal \( x(t) \). Secondly, \( x(t) \) is mixed by the UWB complex local oscillator signal (LOS) \( \rho(t) \) to obtain the modulated signal \( r(t) = x(t)p^*(t) \), where the mark * stands for complex conjugation, then \( r(t) \) is filtered by the second complex low-pass filter (LPF₂) with the passband \([-f_s/2, f_s/2]\) to get the signal \( s(t) \), where \( f_s \) is the sampling rate for digital signal processing. Finally, sample \( s(t) \) at the rate \( f_s \) to obtain \( s(n) \). \( \rho(t) \) is generated by the digital analog converter (DAC) and direct digital synthesizer (DDS), where DDS is synthesized by the digital signal \( \rho(n) \). \( B_i \) could be 18 GHz or even 30 GHz. Then, this structure uses dual ADCs to cover the whole interception frequency range.

**Output of SNYFR for LFM**

We take the wideband LFM signal as an example, which is given by:

\[
x(t) = A \exp\{j[2\pi(f_s t + K_0 t^2/2) + \phi_0]\} + \nu(t)
\]

where \( A, f_0, K_0 \) and \( \phi_0 \) are amplitude, frequency, chirp rate and initial phase, respectively. \( \nu(t) \) is white Gaussian noise with zero mean and variance \( \sigma^2 \).

The \( m \)th Nyquist zone covers frequency \([m f_s, (m+1)f_s)\), \( m \in [0, M-1] \), where \( M \) is the number of zones. In the SNYFR, the LOS is:

\[
\rho(t) = \sum_{m=0}^{M-1} \exp\{j[2\pi f_s t/2 + m(2\pi f_s t + \theta(t))]\}
\]

where \( \theta(t) \) is the instantaneous phase of the zone 1, and the spectrum of \( \theta(t) \) is almost symmetric about \( f_s \) (Zeng et al., 2011), \( B_i = M f_s \). Then,

\[
s(t) = A \exp\{j[2\pi(f_s t + 0.5K_0 t^2 - f_s t^2/2) + \phi_0 - m(t)[2\pi f_s (t + \theta(t))]\}] + \nu'(t)
\]

where the variance of \( \nu'(t) \) is equal to \( \sigma^2 \), while, the power spectrum density of \( \nu'(t) \) is \( M \) times of \( \nu(t) \) (Zeng et al., 2011). \( m(t) \) is the INZ.

**Existence and unique condition**

In the SNYFR, let \( \theta'(t) \) be the instantaneous frequency of the zone 1, then \( m(t) \in \{0, 1, ..., M-1\} \), \( f_0 + K_0 t - f_s/2 < m(t) [f_s + \theta'(t)] \in [-f_s/2, f_s/2] \). That is, \( m_{up}(t) = (f_0 + K_0 t - f_s/2) / (f_s + \theta'(t)) < m(t) \leq (f_0 + K_0 t) / (f_s + \theta'(t)) = m_{sub}(t) \), where \( m_{up}(t) \) and \( m_{sub}(t) \) are the upper and lower bounds of \( m(t) \), respectively.

**Theorem 1**

The existence and unique conditions of the INZ of the SNYFR determine that the frequency of LOS in one zone is constant.

**Proof:** If \( m(t) \) does exist, then \( m_{up}(t) - m_{sub}(t) \geq 1 \). If \( m(t) \) is unique, then \( m_{up}(t) - m_{sub}(t) \leq 1 \). Then, if \( m(t) \) does exist and is unique, we have \( m_{up}(t) - m_{sub}(t) = 1 \), and \( \theta'(t) = 0 \). Equation 3 is the same as the undersampling signal after sampling and lose the INZ information. To expand the interval between \( m_{sub}(t) \) and \( m_{up}(t) \), we widen the passband LPF₂ to \([-f_s/2 - a f_s/2 + b \), where \( a \geq 0 \) and \( b \geq 0 \) are constants. Then,

\[
f_0 + K_0 t - f_s/2 - m(t) [f_s + \theta'(t)] \in [-f_s/2 - a f_s/2 + b, \]

and

\[
m_{sub}(t) = [(f_0 + K_0 t + b) / (f_s + \theta'(t)) < m(t) \leq (f_0 + K_0 t + a) / (f_s + \theta'(t)) = m_{up}(t)]
\]

Then, \( (a + f_s + b) / (f_s + \theta'(t)) \equiv 1 \), that is \( \theta'(t) = a + b \). Since \( a \) and \( b \) are constants, if \( \theta'(t) \) varies along time, there would be no solution, that is, \( \theta'(t) \) should be a constant. In conclusion, under the conditions of existence and uniqueness, the frequency of the LOS in one zone is constant.

Let \( \theta'(t) = \theta_{LOS} \), then \( \theta_{LOS} = a + b \). To facilitate the design of LPF₂, we assume that the filter is symmetric
about zero frequency, then \( a = b = t_{\text{LOS}} / 2 \). The LOS should be: \( \phi(t) = \sum_{m=0}^{M-1} \exp[j(2\pi m f t / 2 + m [2\pi f t + t_{\text{LOS}}])] \), and \( [t_{s} + K_{0} t - t_{s}/2]/(t_{s} + t_{\text{LOS}})] < m(t) < [t_{s} + K_{0} t + t_{\text{LOS}} / 2]/(t_{s} + t_{\text{LOS}}) \). Then,

\[
m(t) = \left[ (t_{s} + K_{0} t + t_{\text{LOS}} / 2) / (t_{s} + t_{\text{LOS}}) \right]
\]

where \( \left[ \right] \) stands for rounding towards zero.

In this condition, the LOS consists of harmonics and we define this as the dual local harmonics.

### Dual harmonics LOSs and parameter estimation

If we just use a single channel, the detection of the INZ is a complex detection problem with unknown parameters, such as the initial frequency and chirp rate of the received signal. To simplify the following processing, we add one more channel to transform this to be a detection problem with known parameters. The signals before sampling the outputs of LPF1 are:

\[
\begin{align*}
\left[ s_{1}(t) = & \exp[j(2\pi f t + 0.5K\phi_{f}^{2} - t_{s}/2) + \phi_{f} - 2m(0)(t + t_{\text{LOS}}))] \right] \\
\left[ s_{2}(t) = & \exp[j(2\pi f t + 0.5K\phi_{f}^{2} - t_{s}/2) + \phi_{f} - 2m(0)(t + t_{\text{LOS}}))] \right]
\end{align*}
(4)
\]

The signals after sampling the outputs of LPF1 are:

\[
\begin{align*}
\left[ s_{1}(n) = & \exp[j(2\pi f nT + 0.5K\phi_{f}^{2} - n/2) + \phi_{f} - m_{0}(n)(n + t_{\text{LOS}}))] \right] \\
\left[ s_{2}(n) = & \exp[j(2\pi f nT + 0.5K\phi_{f}^{2} - n/2) + \phi_{f} - m_{0}(n)(n + t_{\text{LOS}}))] \right]
\end{align*}
(5)
\]

where \( t_{\text{LOS}} \geq 0 \) and \( f_{\text{LOS2}} \geq 0 \), and

\[
\begin{align*}
m_{1}(n) &= \left[ (t_{s} + K_{0} n T_{s} + t_{\text{LOS1}} / 2) / (t_{s} + t_{\text{LOS1}}) \right] \\
m_{2}(n) &= \left[ (t_{s} + K_{0} n T_{s} + t_{\text{LOS2}} / 2) / (t_{s} + t_{\text{LOS2}}) \right]
\end{align*}
(7)
\]

To eliminate the unknown parameters of the received signal, define

\[
s_{s}(n) = s_{1}(n) s_{2}^{*}(n) = A^{2} \exp[j(2\pi m_{s}(n) f_{\text{LOS2}} - m_{s}(n)(n + t_{\text{LOS1}}) n T_{s})]
\]

where \( m_{s}(n) \) has two INZs, namely \( m_{1}(n) \) and \( m_{2}(n) \). Let \( t_{\text{LOS1}} = 0 \), then \( s_{s}(n) = A^{2} \exp[j(2\pi m_{s}(t) f_{\text{LOS2}} n T_{s})] \), and the detection of two INZs is reduced to the detection of one INZ. The estimation of \( m_{s}(n) \) in \( s_{s}(n) \) is a simple detection problem with known parameters. Meanwhile, the estimation of \( m_{s}(n) \) is a demodulation problem of a frequency shift keying (FSK) signal. We need to design a filter bank for demodulation. Since \( m_{s}(n) \in \{0, \ldots, M-1\} \), the minimum and maximum frequencies in of Equation 8 are zero and \((M-1)f_{\text{LOS2}} \), respectively. The demodulation performance would increase with the increase of the frequency hop size, and then we could get the maximum hop size by \( M f_{\text{LOS2}} = f_{s} \). Therefore, We can define \( M \) filters, where the cutoff frequencies of the \( m \)th bandpass filter \( h_{m}(n) \) are \((m-1/2)f_{\text{LOS2}} \) and \((m+1/2)f_{\text{LOS2}} \), respectively. The estimation of the INZ is:

\[
\hat{m}(n) = \arg\max_{m} \left| s_{s}(n) \otimes h_{m}(n) \right|
\]

(6)

Since the filter is not ideal, the INZ nearby zone changes would be wrongly judged. Assuming the times of zone changes are \([l_{1}, l_{2}, \ldots, l_{D}] \), we need to modify these changes by introducing a maximum modification factor \( L \). The \( d \)th modified change is:

\[
l_{d} \rightarrow l_{d} + \arg\max_{j} \left\{ \sum_{n} s_{s}(n) e^{j2\pi m_{d}(n) f_{\text{LOS1}} n T_{s}} \right\}, n \in [l_{d} - L, l_{d} + L]
\]

(7)

where \( m_{d}(n) \) is the new INZ series with time change \( l_{d} + l, d \in \{1, D\} \), \( |l| \leq L \).

After estimating \( l_{d} \) for all the changes, we could get the modified INZ \( \hat{m}(n) \). When \( f_{\text{LOS1}} = 0 \), \( s_{s}(n) \) is equivalent to a under-sampled LFM signal. After estimating \( \hat{m}(n) \), we have \( \hat{m}(n) = \hat{m}(n) \), then we could demodulate \( s_{s}(n) \) using \( \hat{m}(n) \). Moreover, \( s_{s}(n) \) and \( \hat{m}(n) \) could be averaged to increase 3 dB signal to noise ratio (SNR). The parameter estimation algorithm for LFM is detailed in Liu (1999) work.

### SIMULATION RESULTS

Simulations have been done to verify the performances of the proposed structure and method: \( f_{s} = 1\text{GHz} \), \( M = 18 \), \( A = 1 \), \( f_{0} = 1.2\text{GHz} \), \( K_{0} = 1.5\text{GHz}/1.024\mu s \) and \( \phi_{f} = \pi / 6 \). The pulse width was 1.024 µs. We run 2000 simulations for each condition.

#### Probability of correct decision (PCD) of INZ

PCD is defined as:

\[
\text{PCD} = \frac{N_{D}}{N_{A}}
\]

(8)

where \( N_{D} \) and \( N_{A} \) are the number of correct decision samples of the INZ and the total number of samples, respectively.
Figure 2 shows the comparison between the original and modified PCD of the INZ, respectively. When the SNR was greater than 3 dB, the PCD of the original one was greater than 99%, and could not be converged to 100% with the increase of SNR. When the SNR was greater than 2 and 5 dB, the PCDs of the modified INZs were greater than 99% and equal to 100%, respectively. In conclusion, the performance of the modified one was better than the original one.

**Parameter estimation performance**

After zone detection, we estimated the parameters using the algorithm of Liu (1999). Figure 3 shows the normalized mean squared error (NMSE) of all the parameters...
of LFM. When the SNR was greater than 2 dB, the estimation performance of the initial frequency was close to the Cramer-Rao lower bound (CRLB) (Peleg and Porat, 1991), where the CRLB of two channels was about 3 dB lower than the one of single channel. The other parameters were all close to the CRLBs in the testing SNRs. The reason why the performance of chirp rate was better than the one of the initial frequency was that the chirp rate was independent of the INZ. Moreover, the estimation performance of dual LOSs was better than the one of single LOS.

When the signal does not belong to LFM, that is, the signal may be monopulse or phase shift keying, then the bandwidth may be less than the one of LFM, and the possibility of locating in multi-INZ is much less than the one of LFM. The dual local harmonics should also be useful for these kinds of signals.

Conclusion

The proposed SNYFR using dual local harmonics ensured the existence and uniqueness of the INZ, and we transformed the detection of the INZ into the demodulation of FSK. We used dual channels to simplify the detection of the INZ and improved the performance of single LOS. The performance was close to CRLB when SNR was greater than 2 dB.

The further work is to detect the INZs in a multi-signal environment. Moreover, how to extrapolate the INZ in the wideband LOS is also interesting.

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