Full Length Research Paper

Study of effect of rotor vanes to rotor-casing dimensions on performance of a zero pollution vane type novel air turbine

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Accepted 28 April, 2010.

This article describes a new concept of compressed air energy storage system using atmospheric air at ambient temperature as a power source for running zero pollution vehicle (ZPV) such as a motorcycle. The proposed air turbine transforms the energy of the compressed air into shaft work. The mathematical modeling and performance evaluation of a small capacity compressed air driven vane type novel air turbine is presented here. The effect of isobaric admission and adiabatic expansion of high pressure air for different vane numbers to rotor-casing diameter ratios have been considered and analyzed. It is found that the shaft work output is optimum for some typical values of number of vanes at particular rotor-casing diameter ratios. In this study, the casing diameter is considered 50, 100 and 150 mm and rotor to casing diameter ratios are varied from 0.95 to 0.55 and vane numbers from 4 - 12 and the air turbine is found to develop maximum power to the order of 1.3, 5.3 (7.1 HP) and 11.9 kW respectively where 5.3 kW power developed with casing diameter 100 mm is sufficient to run motorcycles.

Key words: Zero pollution, compressed air, vane turbine, vane angle, rotor / casing diameter ratios.

INTRODUCTION

Marion King Hubbert, a noted US based geophysicist was the first person to apply the principles of geology, physics and mathematics in 1956 to predict that the reserves of conventional crude-oil production will peak at around 1976 and thereafter will start depleting and that within 40 years (that is, by 1995), it would cause a serious threat to mankind (Hubbert, 1956). He also noted that worldwide faster consumptions of fossil fuel in transport vehicles will cause a rapid depletion in energy resources whilst simultaneously releasing huge quantity of pollutants into the atmosphere. Aleklett and Campbell (2003) indicated that with the current rate of consumptions, resources of oil and gas production will peak and begin to decline by around 2010 (Aleklett, 2003). These concerns raised the need to seek environment friendly alternative to fossil fuel oil, or some method of conserving natural resources using non-conventional options; such as bio-diesel, wind power, photo voltaic cells etc. and energy conversion systems like battery storage, hydrogen cell, compressed air etc in order providing driving force for vehicle engines (Honton, 2004; Rose, 2004; Singh and Singh, 2006a, b; 2007 a, b; 2008a).

The pioneer work in the area of compressed air engines has been carried out by a French technologist and also by a Canadian inventor of quasi turbines (Guy Negre, 2004; Saint Hilaire, 2004). They used highly compressed air in an energy storage system where the air pressure reaches 20 bars within 15 - 20 min and reused for running compressed air engines. Considering attractive features such as almost zero pollution and especially when air compression is generated using non-conventional resources, the compressed air engine appears to be a highly attractive technology for sustainable transportation.

In this paper the parametric analysis of a small capacity...
air turbine having vane type rotor has been carried out and presented for investigating the effect of rotor to casing diameter ratios with respect to vane angles. Results obtained using the mathematical modeling are presented and analyzed here.

VANE TYPE NOVEL AIR TURBINE

In this study the air turbine is considered to operate on the reverse working principle of a vane type compressor. The casing and rotor diameters of small multi-vane type air turbine are assumed as 100 and 75 mm respectively as shown in Figures 1a and b. In this arrangement total shaft work is cumulative effect of isobaric admission of compressed air jet on vanes and the adiabatic expansion of high pressure air. In an earlier study conducted by the authors a prototype of air turbine was developed and its functionality was ensured (Singh and Singh, 2008 b). A cylinder for the storage of compressed air with a capacity of storing air for the requirement of 30 min running at initial stage and maximum pressure of 20 bar is used as a source of compressed air. The compressed air storage cylinder is designed to produce constant pressure for the minimum variation of torque at low volumes of compressed air and attached with filter, regulator and lubricator.

The clean air then admits into air turbine through inlet nozzle. The vanes of the novel air turbine are placed under spring loading to maintain their regular contact with the casing wall and minimize leakage and offer a substantial improvement over the currently available vane turbine. A study on a high efficiency energy conversion system for liquid nitrogen (Knowlen, 1998), design and verification of airfoil and its tests, influence of tip speed ratios for small wind turbine and parabolic heat transfer and structural analysis was also carried out, in order to conceptualize the energy conversion system and for the design of the air turbine (Fuglsang et al., 2004; Gorla et al., 2005; Schreck et al., 2004; Selig et al., 2004). These studies have also confirmed the feasibility of the novel, vane type air turbine (Singh and Singh, 2008c, d, e, f; 2009a, b). The present objective is to investigate the performance of an air turbine with the variation of injection angle, that is, angle at which air should be admitted into the turbine between first two consecutive vanes. The air turbine considered here has capability to yield output of 5.25 to 6.50 HP at 4 - 6 bar air pressure for speed of 2000 - 2500 rpm, which is suitable for a motorcycle.

MATHEMATICAL MODELING

The mathematical model shown here has been reported in earlier publications but is included for the clarity in the present paper (Singh and Singh, 2009c, d, e, f, g, 2010). The high pressure jet of air at ambient temperature drives the rotor in novel air turbine due to isobaric admission and adiabatic expansion. The high pressure air when it enters through the inlet passage pushes the vane to produce rotational movement through the vane and thereafter air collected between two consecutive vanes of the rotor is gradually expanded up to exit passage.

This isobaric admission and adiabatic expansion of high pressure air contribute in producing the shaft work from air turbine. Compressed air leaving the air turbine after expansion is sent out from the exit passage. It is assumed that the scavenging of the rotor is perfect and the work involved in recompression of the residual air is absent as seen from Figure 1b.

From Figure 2, it is seen that work output is due to isobaric admission (E to 1), and adiabatic expansion (1 to 4) and reference
Figure 2. Thermodynamic Processes (Isobaric, adiabatic and isochoric expansion).

Figure 3. Variable length BG and IH and injection angle Ø.

2, 3 in the figure shows the intermediate position of vanes. Thus, the total work output due to thermodynamic process can be written as:

\[
\text{Total work output} = \text{[Thermodynamic expansion work} (W_1)] + \text{[Exit steady flow work} (W_2)]
\]

Considering air turbine has \( n \) number of vanes and applying values of \( W_1 \) and \( W_2 \) from thermodynamic and fluid flow equations, the shaft output can be written as (Singh Onkar, 2009):

\[
w_i = n \left( \frac{\gamma}{\gamma - 1} \right) p_A v_i \left[ 1 - \left( \frac{p_A}{p_i} \right)^{\frac{\gamma-1}{\gamma}} \right] + n (p_A - p_3) v_3
\]

where \( W_i \) is work output (in Nm), for complete one cycle.

Therefore, the total power output or work done per unit time \( (W_e) \), for speed of rotation \( N \) rpm, will be:

\[
W_e = n(N/60) \left( \frac{\gamma}{\gamma - 1} \right) p_A v_i \left[ 1 - \left( \frac{p_A}{p_i} \right)^{\frac{\gamma-1}{\gamma}} \right] + n(N/60)(p_A - p_3) v_3
\]

Figure 1a shows that if vanes are at angular spacing of \( \theta \), then total number of vanes will be \( n = 360/\theta \). The variation in volume during expansion from inlet to exit (that is, \( v_1 \) to \( v_4 \)) can be derived by the variable extended length of vane as shown in Figure 3 at every point of movement between two consecutive vanes. Figure 3, shows that when two consecutive vanes at OK and OL move to position OH and OB, the extended vane lengths varies from SK to IH and LM to BG, thus the variable length BG at variable \( \alpha \) is assumed as \( X_{\text{variable} \alpha} \) can be written from the geometry:

\[
BG = X_{\text{variable} \alpha} = R \cos \left[ \sin^{-1} \left( \frac{R - r}{R} \sin \alpha \right) \right] + (R - r) \cos \alpha - r
\]

where \( 2R = D \) is diameter of casing and \( 2r = d \) is diameter of rotor, \( \alpha \) is angle \( \angle BOF \), \( \beta \) is angle \( \angle BAF \) and \( \theta \) is angle \( \angle HOB \) or \( \angle H'OB \) or \( \angle KOL \), between two consecutive vanes and \( \phi \) is angle \( \angle KOJ \) at which injection pressure admits to the air turbine.

Variable volume of cuboid B-G-I-H-B can be written as:

\[
v_{\text{cuboid}} = L \left( \frac{X_{\text{1min}} + X_{\text{2max}} + 2r + X_{\text{1min}}}{4} \right) \sin \theta
\]

where \( BG = X_{\text{1min}} \) and \( IH = X_{\text{2max}} \) variable length of vanes when rotated into turbine as shown in Figure 3.

The volume at inlet \( v_1 \) or \( v_{\text{min}} \) will fall between angle \( \angle LOF = \alpha_{\text{1min}} = (180 - \theta - \phi) \) and angle \( \angle KOF = \alpha_{\text{2max}} = (\alpha_{\text{1min}} + \theta) = (180 - \phi) \), when air is admitted into turbine at an angle \( \phi \). Applying above conditions into equation (3) and the values of LM = \( X_{\text{1min}} \) and SK = \( X_{\text{2max}} \) to equation (4):

\[
v_1 = v_{\text{min}} = L \left( \frac{X_{\text{1min}} + X_{\text{2max}} + 2r + X_{\text{1min}}}{4} \right) \sin \theta
\]

The Volume at exit \( v_4 \) or \( v_{\text{max}} \) will fall between angle \( \angle BOF \).
Table 1. Input parameters.

<table>
<thead>
<tr>
<th>Symbols</th>
<th>Parameters</th>
</tr>
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<tbody>
<tr>
<td>$(d / D) = (2r / 2R)$</td>
<td>0.95, 0.9, 0.85, 0.80, 0.75, 0.70, 0.65, 0.60 and 0.55 when casing diameter is kept $D = 50$ mm, 100 mm, and 150 mm</td>
</tr>
<tr>
<td>$p_1$</td>
<td>6 bar (=90 psi)</td>
</tr>
<tr>
<td>$p_4$</td>
<td>$(v_1 / v_4) \gamma, p_1 &gt; p_3$ assuming adiabatic expansion</td>
</tr>
<tr>
<td>$p_3$</td>
<td>$(p_4 / 1.2) &gt; 1.0132$ bar (atmospheric pressure)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>30, 36, 45, 60, 90° (that is, rotor contains correspondingly 12, 10, 8, 6, 4 number of vanes)</td>
</tr>
<tr>
<td>$N$</td>
<td>2500 rpm (as total power is directly proportion to rpm)</td>
</tr>
<tr>
<td>$L$</td>
<td>45 mm length of rotor</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>1.4 for air</td>
</tr>
<tr>
<td>$n$</td>
<td>Number of vanes = $(360 / \theta)$</td>
</tr>
<tr>
<td>$\phi$</td>
<td>60° angle at which compressed air enters through nozzle into rotor</td>
</tr>
</tbody>
</table>

$X_{1\text{max}} = \alpha = 0$ and angle $\angle \text{HOF}$ $\alpha_{2\text{max}} = (\alpha_{1\text{max}} + \theta) = \theta$

and applying above conditions into equation (3), the values of $FE = X_{1\text{max}} = \text{Corresponding to BG at } \alpha = 0^\circ$ and $\text{IH} = X_{2\text{max}} = \text{Corresponding IH at } (\alpha + \theta) = \theta$ degree to equation (4):

$v_4 = v_{\text{max}} = L \left( \frac{(X_{\text{1max}} + X_{\text{2max}}) (2r + X_{\text{1max}})}{4} \right) \sin \theta$  \hspace{1cm} (6)

Applying values of $v_1$ and $v_4$ from equations (5) and (6) to equation (2), the total power output available $W_{\text{total}}$ can be written as:

$W_{\text{total}} = \text{rt}(N \delta) \left[ \frac{r_2}{r_1} - 1 \right] \left[ p_1 \left( \frac{L \left( X_{\text{max}} + X_{\text{min}} \right) (2r + X_{\text{min}})}{4} \right) \sin \theta \right]$

$+ \text{rt}(N \delta) \left[ p_1 - p_3 \right] \left[ L \left( X_{\text{max}} + X_{\text{min}} \right) (2r + X_{\text{min}}) \sin \theta \right]$  \hspace{1cm} (7)

where

$X_{\text{max}} = R \cos \left[ \sin^{-1} \left( \frac{\left( \frac{R-r}{R} \right) \sin (180^\circ - \theta - \phi)}{\sin (180^\circ - \theta - \phi)} \right) \right] + \left[ (R-r) \cos (180^\circ - \theta - \phi) - r \right]$

$X_{\text{min}} = R \cos \left[ \sin^{-1} \left( \frac{\left( \frac{R-r}{R} \right) \sin (180^\circ - \phi)}{\sin (180^\circ - \phi)} \right) \right] + \left[ (R-r) \cos (180^\circ - \phi) - r \right]$

$X_{1\text{max}} = (D-d) = 2(R-r)$, and

$X_{2\text{max}} = R \cos \left[ \sin^{-1} \left( \frac{\left( \frac{R-r}{R} \right) \sin (180^\circ - \theta)}{\sin (180^\circ - \theta)} \right) \right] + \left[ (R-r) \cos (180^\circ - \theta) - r \right]$

**ASSUMPTIONS OF INPUT PARAMETER**

Detailed analysis of varying injection angles was carried out in earlier publications for expansion work, flow work, percentage contribution of expansion and flow work and total works at different injection pressure 2 - 6 bar and different speed of rotation 500 - 2500 rpm. The contribution of total expansion work was found large when injection angle of air turbine is kept above 30° and found maximum when it is 60°, at constant vane angle 45° (that is, 8 vanes), injection pressure 6 bar and speed of rotation 2500 rpm.

In this study various input parameters are listed in Table 1 for investigation of larger shaft output at different rotor to casing diameter ratios (d/D) with respect to different vane angles when casing diameter (D) is kept 100 mm, injection pressure 6 bar (90 psi).

**RESULTS AND DISCUSSION**

Based on the various input parameters listed in Table 1 and using mathematical model, the effects of different rotor to casing diameters ratios with respect to different vane angles at speed of rotation 2500 rpm and injection pressure 6 bar, the total power outputs obtained from air turbine are studied and compared considering casing diameter as $D = 50$, 100 and 150 mm. Here the injection angle $\phi$ of the air turbine is considered to be constant at 60° as it was found to develop optimum shaft output in earlier studies. The results obtained have been plotted in Figures 4 to 9, for the rotor to casing diameter ratio (d/D), varying as 0.95, 0.90, 0.85, 0.80, 0.75, 0.65, 0.60 and
0.55, at different vane angles of 30, 36, 45, 60, 90° and at constant injection angle of 60° and injection pressures of 6 bar (90 psi) at the speed of rotation 2500 rpm.

**Case 1: Total power out when D = 50 mm**

Figures 4a, b, c, d, and e shows the variations of expansion power, flow power, percentage contribution of expansion power, percentage contribution of flow power and total output power from the air turbine. It is seen from Figure 4e that the total power becomes large, for a particular vane angle and for different range of rotor/casing diameter ratio when injection pressure is 6 bar, speed of rotation is 2500 rpm, and it ranges from:

a. 1.1 to 1.3 kW, when rotor to casing diameter ratios are 0.65 to 0.55, and vane angle is kept 45° (vanes numbers 8),
b. 0.87 - 1.0 kW, when rotor to casing diameter ratios are of 0.75 - 0.70 and vane angle is kept 36° (vanes numbers10), and
c. 0.19 - 0.72 kW, when rotor to casing diameter ratios are of 0.95 - 0.80 and vane angle is kept 30° (vanes numbers 12).

**Case 2: Total power out when D = 100 mm**

Figure 5 shows that the total output power from the air turbine becomes maximum for a particular vane angle and for different range of rotor/casing diameter ratio when injection pressure is 6 bar, speed of rotation is 2500 rpm, and it ranges from:
Figure 4(e). Total power output versus vane angles with respect to rotor / casing diameter (d/D) ratio when D = 50 mm.

Figure 5. Total power output versus vane angles with respect to rotor / casing diameter (d/D) ratio when D = 100 mm.

a. 4.5 to 5.3 kW, when rotor to casing diameter ratios are 0.65 to 0.55 and vane angle is kept 45° (vanes numbers 8).
b. 3.5 - 4.0 kW, when rotor to casing diameter ratios are of 0.75 - 0.70 and vane angle is kept 36° (vanes numbers 10) and
c. 0.8 - 2.9 kW, when rotor to casing diameter ratios are of 0.95-0.80 and vane angle is kept 30° (vanes numbers 12).

Case 3: Total power out when D = 150 mm

Figure 6 shows that the total output power from the air turbine becomes maximum for a particular vane angle and for different range of rotor/casing diameter ratio when injection pressure is 6 bar, speed of rotation is 2500 rpm, and it ranges from:

a. 10.1 to 11.9 kW, when rotor to casing diameter ratios
are 0.65 to 0.55 and vane angle is kept 45° (vanes numbers 8).

b. 7.8 - 9.0 kW, when rotor to casing diameter ratios are of 0.75 - 0.70 and vane angle is kept 36° (vanes numbers 10), and

c. 1.9 - 6.5 kW, when rotor to casing diameter ratios are of 0.95 - 0.80 and vane angle is kept 30° (vanes numbers 12).

**Case 4: Comparison of Total Power output at different d/D ratio 0.55, 0.70 and 0.80 when D = 50, 100 and 150 mm.**

Figure 7 shows that the total power output is seen maximum to the order of 11.9, 5.3 and 1.3 kW at (d/D) = 0.55 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 45° (vane number 8).

Figure 8 shows that the total power output is seen maximum to the order of 9.0, 4.0 and 1.0 kW at (d/D) = 0.55 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 36° (vane number 10).

Figure 9 shows that the total power output is seen maximum to the order of 6.5, 2.9 and 0.72 kW at (d/D) = 0.80 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 30 - 36° (vane number 12 - 10).

Thus from results it is critically seen that for different range of rotor to casing dimensions at D = 150 and d/D = 0.55, injection pressure 6 bar, injection angle 60°, and at speed of rotation 2500 rpm the total power becomes optimum to the order of 11.9 kW at vane angle 45° (vane number 8) after considering all input parameters such as different rotor/casing diameter ratio of 0.95, 0.9, 0.85, 0.80, 0.75, 0.70, 0.65, 0.60 and 0.55 at casing diameter D = 50, 100 and 150 mm. Thus it is obvious that the expansion power output as well as total power output is found optimum when rotor/casing diameter ratio lies at 0.55 at lowest rotor to casing diameter ratio and is a deciding factor for desired shaft power output.

**Conclusion**

The results obtained from the above investigation based on input parameters such as injection pressure, injection angle and speed of rotation are kept 6 bar, 60° and 2500 rpm respectively, following conclusions are drawn:

1. There exists an optimal value of rotor/casing diameter ratio (approx. 0.65 to 0.55) for the considered air turbine for vane angle 45° (8 vanes).
2. The exit flow power due to steady flow is seen to be large at rotor / casing ratio 0.55 for all casing diameter 50, 100 and 150 mm.
3. Total output power from the air turbine is seen to be maximum for the higher injection air pressure and there exists an optimum value of rotor/casing diameter ratio for injection pressure 6 bar, speed of rotation 2500 rpm and at particular vane angles and it ranges from:

   a. 11.9, 5.3 and 1.3 kW at (d/D) = 0.55 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 45° (vane number 8).
Figure 7. Total power output versus vane angles when rotor / casing diameter (d/D) ratio is 0.55 and D = 50, 100 and 150 mm.

Figure 8. Total power output versus vane angles when rotor / casing diameter (d/D) ratio is 0.70 and D = 50, 100 and 150 mm.

Figure 9. Total power output versus vane angles when rotor / casing diameter (d/D) ratio is 0.80 and D = 50, 100 and 150 mm.
b. 9.0, 4.0 and 1.0 kW at (d/D) = 0.70 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 36° (vane number 10) and
c. 6.5, 2.9 and 0.72 kW at (d/D) = 0.80 and D = 150, 100 and 50 mm, respectively when vane angle (θ) is kept 30 - 36° (vane number 12 - 10).

Thus optimum shaft power output of a novel vaned type air turbine is obtained when the design parameters for rotor diameter to casing diameter (d/D) ratios are kept between 0.65 to 0.55 and vane angle is 45° (that is, rotor vane numbers 8), which plays an important role to develop desired shaft output for a particular size of vane turbine. As such the use of compressed air turbine (Singh BR and Singh Onkar, 2008b), for running light vehicle is found cost effective, develops 70-80% efficiency and also releases nearly zero pollution as compared to the fossil fuel or other alternative energy sources, (e.g. batteries, fuel cells, hydrogen, bio-fuels, etc).

NOMENCLATURE

\[
\begin{align*}
  d & : \text{Diameter of rotor (2r) in meter} \\
  D & : \text{Diameter of outer (2R) cylinder in meter} \\
  L & : \text{Length of rotor having vanes in meter} \\
  n & : \text{Number of vanes} = (360/\theta) \\
  N & : \text{Number of revolution per minute} \\
  P & : \text{Pressure in bar} \\
  p_1, v_1 & : \text{Pressure and volume respectively at which air strike the Turbine} \\
  p_5 & : \text{Pressure at which turbine releases the air to atmosphere} \\
  v & : \text{Volume in cum} \\
  W & : \text{Theoretical work output in (J) Joules} \\
  W & : \text{Theoretical power output (W) Watts} \\
  X_{1i} & : \text{Variable extended lengths of vane at point 1} \\
  X_{2i} & : \text{Variable extended lengths of vane at point 2}
\end{align*}
\]

Subscripts

\[
\begin{align*}
  r, 2...5 & : \text{subscripts – indicates the positions of vanes in casing} \\
  \exp & : \text{expansion} \\
  \min & : \text{minimum} \\
  \max & : \text{maximum}
\end{align*}
\]

Greek symbols

\[
\begin{align*}
  \alpha & : \text{Angle BOF (Figure 3)} \\
  \alpha_i & : \text{Angle LOF (=180 - } \phi \text{) (Figure 3)} \\
  \beta & : \text{Angle BAF (Figure 3)} \\
  \gamma & : 1.4 \text{ for air} \\
  \theta & : \text{Angle between 2-vanes (BOH) (Figure 3)} \\
  \phi & : \text{Angle at which compressed air enters into rotor through nozzle} \\
  \varepsilon_d & : \text{Eccentricity (R-r)}
\end{align*}
\]

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