Vegetative induced ground displacements: a comparison of numerical and experimental study

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A volume change was modeled as a result of matric suction change caused by vegetative induced moisture transfer. Theory of unsaturated soil mechanics and two stress state variables principles approach was used in the stress-deformation model. The negative pore-water pressures are estimated through governing partial differential equations for unsaturated soils. The results of the root water-uptake analysis are then used as an input for the prediction of ground displacements in a stress-deformation analysis in an uncoupled manner. A mature Leyland Cypress tree located on Gault clay was analyzed, covering a full spring/summer drying period for the first case study. The second and third cases were analysis on Mature Lime tree located on a Boulder clay sub-soil for period covering a full spring/summer drying period; while the third case considered field capacity in winter and extends through a full spring/summer drying period and subsequent autumn recharge. Time varying boundary conditions have also been considered in the third case and the sink term was activated to represent water uptake by transpiration, during spring/summer soil-drying phase and deactivated during the autumn/winter recharge phase. The stress-deformation model has been validated by direct comparison to field measurements recorded for three cases. A good overall correlation between field data and simulated results has been achieved where the difference between the two set of results was less than 5% for all cases considered.

Key words: Unsaturated soils, numerical, water-uptake, simulation, stress-deformation, experimental.

INTRODUCTION

In general soil mechanics, the deformation of soil matrix is analyzed as a result of increase in applied load, whereas in groundwater field, the soil deformation is studied due to extraction of groundwater. In both cases, deformation takes place, because pore volume decreased (Ali et al., 2010). In the study of groundwater extraction it is assumed that there is no change in the applied load field, and changes occur in the effective stress due to changes in the pore pressure. Deformation in the soil mass takes place due to changes in the effective stress and these changes occur due to two reasons. Firstly, the effective stress may increase because of increase in the externally applied load accompanied with free drainage of water. Secondly, the effective stress might increase due to reduction in the pore pressure as a result of extraction of water from the soil mass (Mu’azu et al., 2010).

Therefore, a relatively straightforward simulation was considered to provide an assessment of the model for a spring/summer drying period for Leyland Cypress tree and the corresponding displacements were simulated. The second simulation considers also spring/summer drying period in more details for Lime tree; also, the corresponding ground displacements were also simulated. The third simulation on Lime tree was
extended and developed to explore the application of the model to simulate a full annual cycle starting from field capacity in winter, extending through a full spring/summer drying period and including the subsequent autumn recharge. Data from the nearest Bureau of Meteorological Station (2006) to the site under consideration was used to estimate behavior at the soil surface. The study aimed to validate the developed numerical model using field measurements.

WATER-UPTAKE CONCEPTS

In prediction of soil movement, two fundamental stages are generally involved; an assessment of the changes in moisture conditions and the knowledge of the volumetric strain induced by these changes. The first step is dealt with through the use of modified Richard (1931) equation; two-dimensional axi-symmetric governing equation for unsaturated soils with sinks term:

\[
\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial r} \left( K \frac{\partial \psi}{\partial r} \right) + \frac{1}{r} \frac{\partial}{\partial r} \left( r K \frac{\partial \psi}{\partial r} \right) + \frac{\partial}{\partial z} \left( K \frac{\partial \psi}{\partial z} \right) - S(\psi, r, z) = 0
\]

(1)

Where \( K(\psi) \) is the unsaturated hydraulic conductivity, \( t \) is the time, \( r \) and \( z \) are the coordinate, \( \theta \) is the volumetric moisture content and \( \psi \) is the capillary potential, \( S(\psi, r, z) \) is the root water extraction function and \( r \) is the radial coordinate.

The numerical solution of Equation 1 was achieved via the finite element spatial discretization procedure and a finite-difference time-stepping scheme. Adopting particular Galerkin weighted residual approach which yields the discretized matrix form for full detail see (Rees and Ali, 2006):

\[
K \frac{\psi^{n+1}}{\Delta r} + C \frac{\psi^{n+1}}{\Delta z} + J + S^{n+1} = 0
\]

(3)

The parabolic shape functions and eight-node isoperimetric elements are employed (Zienkiewicz and Taylor, 1989). The time-dependent nature of Equation 3 is dealt with via a mid-interval backward difference technique, yielding:

\[
K \frac{\psi^{n+1}}{\Delta r} \psi^{n+1} + C \frac{\psi^{n+1}}{\Delta z} \left( \frac{\psi^{n+1} - \psi^n}{\Delta r} \right) + J + S^{n+1} = 0
\]

(4)

GROUND MOVEMENT CONCEPTS

The second step is tackled through stress-deformation formulation considering unsaturated soil mechanics concept using ground water field concept. Fredlund and Hung (2001) stated that the volume change constitutive relations for the unsaturated soils are formulated using the two stress state variables, namely, net normal stress \( (\sigma - u_a) \) and matric suction \( (u_a - u_w) \). Constitutive relationships are to compliment governing flow equation, thus, providing additional relationship between deformation and stress state variables. It is also assumed that the pore-air pressure is the same with atmospheric pressure, so that the distribution of pore-water pressure is equivalent to the matric suction distributions. A change in the negative pore-water pressure occurs as a result of root water-uptake and can be related to changes in soil volume through the use of constitutive relations. Swelling in the field occurs along the rebound curve at an overburden pressure of \( (\sigma - u_a) \) and matric suction \( (u_a - u_w) \). Shrinkage occurs along either a recompression curve or the virgin compression curve. The mathematical expression for the recompression curve can be expressed as:

\[
de = c_1^\prime d(\sigma - u_a) + c_2^\prime d(u_a - u_w)
\]

(5)

Where \( de \) is change in void ratio, \( c_1^\prime \) is coefficient of compressibility with respect to a change in \( (\sigma - u_a) \) and \( c_2^\prime \) is coefficient of compressibility with respect to a change in \( (u_a - u_w) \).

\( \sigma = \) total effective stress, \( u_w = \) pore water pressure and \( u_a = \) pore air pressure.

The form of the constitutive equation for the rebound curve is similar in expression to Equation 5, except that the moduli are from the rebound curve. The net normal stress state within the soil mass can be computed using Equation 6, while horizontal net normal stresses can be estimated from the vertical stresses and \( K_0 \) using Equation 7:

\[
\sigma_z = \int_0^H \rho g dy
\]

(6)
\[ \sigma_r = K_0 \sigma_z \]  \hspace{1cm} (7)

Where \( \sigma_z \) is vertical net normal stress, \( \sigma_r \) is horizontal stress, \( K_0 \) is coefficient of earth pressure at rest and \( H \) is depth of soil under consideration. While the soil is a normally consolidated clay with a consolidation behavior that can be described by:

\[ de = C_r \ln \left( \frac{\sigma_v + \Delta \sigma_v - u_{wf}}{(\sigma_v - u_a) + (u_a - u_w)_c} \right) \]  \hspace{1cm} (8)

Where \( de \) is the change of void ratio in the element, \( C_r \) is the re-compression index, \( \sigma_v \) is the vertical total stress, \( \Delta \sigma_v \) is the change in the total vertical stresses, \( u_{wf} \) is the final pore water pressure, and \( (u_a - u_w)_c \) is the matric suction equivalent (Fredlund and Rahardjo, 1993).

The elasticity parameters are functions of the stress state, net normal stress and the matric suction. These elastic moduli parameters \( E \) and \( H \) could be estimated from volume change indices, initial void ratio and Poisson's ratio (Fredlund and Hung, 2001). The capillary potential \( (\psi) \) was estimated from Equation 1 which was used as an input for the stress-deformation analysis. Where \( E \) = elasticity parameter for the soil structure with respect to a change in the net normal stress and \( H \) = elasticity parameter for the soil structure with respect to a change in matric suction. This relationship was established to perform the necessary ground displacements estimation:

\[ \frac{\partial \varepsilon}{\partial \psi} = \frac{1}{H} \frac{\partial V_v}{\partial \psi} \]  \hspace{1cm} (9)

**DISCRETIZATION, BOUNDARY AND INITIAL CONDITIONS**

The mesh consists of 8-noded isoperimetric linear strain quadrilateral elements. The entire finite element mesh consists of 1281 nodes and 400 elements; the axisymmetric domain is shown in Figure 1. The mesh was configured to offer some refinement within the root zone area, since this is the region where the most significant moisture content variations were expected to occur. The simulation employs a time-step size of 21600 seconds, which was held constant for the entire period considered. The lower boundary of the domain and the far-field vertical boundary remained unconstrained (natural) throughout the simulation. The soil parameters are shown in Table 1, which is a typical value for Gault clay, while Table 2 is for Boulder clay. Based on the field observations provided by Biddle (1998), the root zone is assumed to extend to a depth of 1.0 m and a radial distance of 5.0 m, for Leyland Cypress while Lime tree extend to a depth of 2.0 m and a radial distance of 5.0 m. Spatial discretisation has been achieved via the finite element mesh as shown in Figure 1.

Where \( K_s \) is saturated hydraulic conductivity, \( T_a \) is actual transpiration rate, \( \Psi_w \) is suction at wilting point, \( \gamma \) is unit weight of soil, \( e_0 \) is initial void ratio, \( C_r \) is re-compression index, \( \mu \) is Poisson's ratio, \( \theta_r \) is residual water content, \( \theta_s \) is saturated water content, \( \alpha \) is water stress, \( m \) is empirical shape fitting parameters, \( n \) is empirical shape fitting parameters and \( l \) is soil specific parameter generally assumed to be 0.5.

The required soil moisture retention characteristics and unsaturated hydraulic conductivity was simulated from the closed form equation developed by vanGenuchten (1980).

**CASE STUDY I LEYLAND CYPRRESS TREE ON GAULT CLAY**

The particular experimental data is based on the field measurements undertaken at a site at Corpus Christi College, Cambridge, England. The data was recorded in the vicinity of a single Leyland Cypress tree (a conifer) 10 m in height, located on a Gault clay sub-soil. Field observations were available from February 1981 to October 1981. The initial conditions employed for the simulation were based on the experimental moisture
profile at the start of the period considered. The simulated period covered a spring/summer soil-drying phase of 8 months (7th February 1981 to 8th October 1981). The drying phase was represented via the application of the transpiration rate described as earlier mentioned.

LEYLAND CYPRUS TREE SIMULATION RESULTS FOR SPRING/SUMMER DRYING PERIOD

Figure 2 showed the simulated and site ground displacements profiles after 240 days at radial distance of 1.5 m from Leyland Cypress tree and Figure 3 shows simulated contours of ground displacements generated at the end of simulation (240 days).

### Table 1. Parameters used in the analysis for case I.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Values</th>
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<tr>
<td>$k_s$</td>
<td>$10^6$ m/s</td>
<td>Biddle (1998)</td>
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<tr>
<td>$T_a$</td>
<td>5 mm/day</td>
<td>Biddle (1998)</td>
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<tr>
<td>$\psi_d$</td>
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<td>Feddes et al. (1976)</td>
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<td>$\gamma$</td>
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<td>Rees and Ali (2006)</td>
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<td>$m$</td>
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<td>Rees and Ali (2006)</td>
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<td>$n$</td>
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<tr>
<td>$l$</td>
<td>0.5</td>
<td>Rees and Ali (2006)</td>
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### Table 2. Parameters used in the analysis for Case II and III.

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<td>$l$</td>
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DISCUSSIONS OF LEYLAND CYPRUS TREE SIMULATION FOR SPRING/SUMMER DRYING PERIOD

Based on the field observations provided by Biddle (1998), the root zone is assumed to extend up to a depth of 1.0 m and a radial distance of 5.0 m. Radial distance of 1.5 m is chosen in order to have a more representative scenario. The predicted and measured results of ground displacement at the end of drying period (240 days) at a radial distance of 1.5 m from the centre-line of the tree are also presented. The simulated results and field measured results are plotted as shown in Figure 2. It is clear that the majority of the ground displacement at 240 days has occurred near the surface with the ground displacement reducing to 24 mm at approximately 0.3 m.
depth. It was found that the difference between the two set of results was generally less than 5 %. This shows that the model is capable of estimating ground displacements since the difference between the simulated and measured results is within the standard limit of ±5 %. The ground displacements reduced with an increase in radial distance as shown in Figure 3.

CASE STUDY II MATURE LIME TREE ON BOULDER CLAY

The particular experimental data is based on the field measurements undertaken at a site located at Stacey Hall, Wolverton, England. The case considered here relates to a single mature Lime tree, 15 m in height, located on a Boulder clay sub-soil. A uniform initial value of capillary potential of -17 cm was applied throughout the domain; representing an initial volumetric water content of 37.5 % which corresponds to a degree of saturation of approximately 93.75 % was used as an initial value of capillary potential. The initial value of capillary potential would be in steady state and the subsequent steady state value is applied to the simulation. The simulated period covered a spring/summer soil-drying phase of 9 months (23rd January 1979 to 24th October 1979). The drying phase was represented via the application of the transpiration rate.

LIME TREE SIMULATION RESULTS FOR SPRING/SUMMER DRYING PERIOD

Figures 4, 5 and 6 shows predicted and site ground displacements profiles at a radial distance of 1.4 m from
Figure 4. Simulated and site ground displacements profiles after 190 days at radial distance of 1.4 m from Lime tree.

Figure 5. Simulated and site ground displacements profiles after 238 days at radial distance of 1.4 m from Lime tree.
**DISCUSSIONS OF LIME TREE SIMULATION FOR SPRING/SUMMER DRYING PERIOD**

These times relate to the specific dates of 2\textsuperscript{nd} August, 21\textsuperscript{st} September and 24\textsuperscript{th} October, 1979 for Figure 4, 5 and 6 respectively. The two set of results were plotted and an estimate of the difference between them was calculated. It was found that the difference between the two set of results was less than 5%. The sink term is active within the depth of the root zone of the upper 2.0 m
of the soil profile and to a radial extent of 5 m. Majority of the ground displacement has occurred near the surface with the ground displacement reducing to 37.61 mm at approximately 0.3 m depth. The figure indicates that a reasonable agreement between the simulated results and field measured ground displacement profiles. Looking at Figures 7, 8 and 9 at a radial distance of 4.9 m, it is clear that the tree roots have relatively little effect on the
seasonal moisture depletion as the maximum ground displacement is about 7.54 mm at the ground surface. A good agreement between simulated and measured ground displacements profiles is also achieved at this distance, with errors ranging between 0.9% and 1.7%. Figures 10 and 11 showed the simulated contour for
190 and 270 days respectively. These indicate the spatial and time dependent nature of root water uptake as shown by the corresponding ground displacement. The ground displacements increase with an increase of elapse and decreases at the radial distance increases.

**CASE STUDY III MATURE LIME TREE ON BOULDER CLAY WITH TIME DEPENDENT BOUNDARY CONDITIONS**

The particular period now considered for simulation is from 23rd January 1979 to 21st April 1980. A non-hysteretic behavior is assumed, this assumption implies that the same moisture retention curve is used to described soil drying and soil wetting behavior (Thomas and Rees, 1990). Rainfall data provided by the meteorological office (2006) has been acquired for the nearest weather station to the site. Simulation employs time varying boundary conditions based on the rainfall data. In this case, the simulated period now covers a spring/summer soil-drying phase of 9 months followed by an autumn/winter 6 month recharge phase. The daily rainfall data has been used to specify a variation in fixed (Dirichlet) boundary conditions at the soil surface. The sink term was activated to represent water uptake by transpiration during spring/summer soil-drying phase and deactivated during the autumn/winter recharge phase. During the simulation, it has been assumed that the soil surface was saturated when the daily rainfall exceeded evaporation demand from the grass cover while at all other times, the surface boundary was unconstrained. During the wetting periods, saturation of the soil surface was simulated via application of a fixed boundary condition of zero capillary potential prescribed at the surface nodes. The saturated boundary condition was only applied to the surface at times when the rate of the rainfall was found to be more than the cut-off line. Blight (2003) suggested that the maximum evaporation (cut-off line) from an open grassy site is approximately 2.7 mm/day. Therefore, the surface boundary is switching between a fixed saturated state and a free state (un-prescribed).

**LIME TREE SIMULATION RESULTS WITH TIME DEPENDENT BOUNDARY CONDITIONS**

Figures 12, 13 and 14 show predicted and site ground displacements profiles at a radial distance of 1.4 m from the centre-line of the tree while Figures 15, 16 and 17 show that of distance of 4.9 m from the tree.

**DISCUSSIONS OF LIME TREE SIMULATION WITH TIME DEPENDENT BOUNDARY CONDITIONS**

As shown in Figures 12, 13 and 14, the site data is simply plotted as discrete data points at the times when measurements were recorded on site. Since there was no field data available at intermediate times, no extrapolation has been undertaken. Numerical results
can be output at any number of time intervals; hence, a continuous variation has been plotted for comparison. Specifically, Figure 14 indicates relatively little ground displacements variation occurred at a depth of 2.0 m. These figures also show that a full drying period and full wetting period are now simulated. Transient variations results presented more fully below reveal the overall predicted patterns over the full annual cycle. The problem simulated above clearly involves both drying and wetting phases.
The drying phase was represented via the application of the actual transpiration rate $T_a$ of 5 mm/day as shown in Table 2. Although approximate, this approach is supported by the available soil moisture deficit (SMD)
which showed a continuous decrease over this period at 1.4 m and 4.9 m distance from tree. These SMD values have been calculated by Biddle (1998) by reference to a spring average profile. The development of a deficit during the season is shown to occur at depths of 0.3 m, 1.0 m and 2.0 m below ground level. The problem simulated above clearly involves both drying and wetting phases. Therefore, it is likely that some hysteresis may arise in the water retention characteristics of the soil. The site of the field experiment is likely to have been subjected to repeated wetting and drying over many years. Therefore, the effect of the time varying boundary conditions is clearly visible in these results. It is reasonable to assume that any such hysteresis is likely to be of a closed-loop form (Yong and Warkentin, 1974).

The overall predicted trend appears to be in fair agreement with the site data as shown in Figures 15, 16 and 17. The difference between simulated ground displacements and the field data is within the standard limit of ±5 %.

**CONCLUSIONS**

The approach proposed utilizes radial symmetry and a linear distribution of water extraction rate with both depth and radius. The deformation model has been validated by direct comparison to field data measurements recorded for three cases. The results of the root water-uptake analysis are then used as an input for the prediction of displacements in a stress-deformation analysis in an uncoupled manner. A mature Leyland Cypress tree located on Gault clay covering a full spring/summer drying period for the first case study. The second and third cases are on Mature Lime tree located on Boulder clay sub-soil covering a full spring/summer drying period and the third case starts from field capacity in winter and extends through a full spring/summer drying period and subsequent autumn recharge. Time varying boundary conditions have also been considered in the third case and the sink term was activated to represent water uptake by transpiration, during spring/summer soil-drying phase and deactivated during the autumn/winter recharge phase. The stress-deformation model has been validated by direct comparison to field data measurements recorded for three cases. A good overall correlation between field data and simulated results has been achieved where the difference between the two set of results was less than 5 % for all cases considered. The majority of the moisture extraction occurred near the surface; likewise, the ground displacement occurred mostly near the ground surface. Ground displacement reduced significantly when the distance from the tree increased. Simulated contours of ground displacement

![Figure 17. Simulated and transient site ground displacements profiles variation at depth 2 m and radial distance of 4.9 m from Lime tree.](image)
generated by the simulation were presented that produced an overall ground displacement pattern similar to that observed in the field.

REFERENCES


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