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Fuzzy investment decision based on economic and strategic factors: A case of air logistics service provider

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In this paper, an algorithm is proposed to perform investment decision in a fuzzy environment. This algorithm considers both economic and strategic factors to select the optimal investment project. By using this algorithm, the ambiguities involved in the assessment data can be effectively represented and processed to assure a more convincing and effective decision-making. Finally, the air logistics service provider (Air LSP) is used to assess the optimal investment decision.

Key words: Multi-criteria decision making, fuzzy set theory, investment project, air logistics service provider (Air LSP).

INTRODUCTION

In conventional precision-based investment decision, the total revenue, the cost and economic considerations are all expressed in crisp values (Au and Au, 1983; Targuin and Blank, 1989; Dubois and Prade, 1978; Liou and Wang, 1992; Mansfield, 1985; Park et al., 1990). However, owing to incomplete or uncertain information, it is often difficult to obtain the exact assessment data such as total revenue, gross income, expenses, depreciation, salvage value, inflation rate, etc. Hence, the precision-based evaluation may not be practical.

In fact, when decision-makers engage in evaluating investment alternatives, they tend to give assessments based on their knowledge, experience and subjective judgment. Linguistic terms, for example, “approximately between $410,000 and $420,000”, “about $6,000”, are frequently used to convey their estimations. Thus, fuzzy set theory can play a significant role in this kind of decision-making environment.

Fuzzy set theory was introduced by Zadeh (1965). It can be utilized to deal with problems in which the uncertainties and ambiguities exist. Let $X$ be a collection of objects, called the universe, a fuzzy set $A$ in $X$ is a set of ordered pairs $A = \{(x, f_A(x)) \mid x \in X \}$, where $f_A(x)$ is the membership function of $x$ in $A$ which associates with each element $x$ in $X$ a real number in the interval $[0, 1]$. The larger the $f_A(x)$, the stronger the degree of belongingness for $x$ in $A$.

FUZZY SET THEORY

In this section, the notation and concepts of the fuzzy set theory will be briefly introduced. The definitions of relevant terms used in this study are stated.

Fuzzy set

Fuzzy set theory can be used to deal with problems in which the uncertainties and ambiguities exist. Let $X$ be a collection of objects, called the universe, a fuzzy set $A$ in $X$ is a set of ordered pairs $A = \{(x, f_A(x)) \mid x \in X \}$, where $f_A(x)$ is the membership function of $x$ in $A$ which associates with each element $x$ in $X$ a real number in the interval $[0, 1]$. The larger the $f_A(x)$, the stronger the degree of belongingness for $x$ in $A$.

Fuzzy numbers

A fuzzy number is described as a subset of the real
numbers whose membership function \( f \) is a continuous mapping from \( \mathbb{R} \) (real line) to a closed interval \([0, 1]\), which has the following characteristics: (1) \( f_A(x) = 0 \) for all \( x \in (-\infty, c] \cup [d, \infty) \), (2) \( f_A(x) \) is strictly increasing on \([c, a]\) and strictly decreasing on \([b, d]\), and (3) \( f_A(x) = 1 \) for all \( x \in [a, b] \). The left membership function of \( f_A \), that denoted it by \( A^L \), defines \( f_A^L(x) = f(x), \forall x \in [c, a] \). And the right membership function of \( f_A \), that denoted it by \( A^R \), defines \( f_A^R(x) = f(x), \forall x \in [b, d] \). A fuzzy number \( A \) in \( \mathbb{R} \) (real line) is a trapezoidal fuzzy number, if its membership function \( f_A: \mathbb{R} \rightarrow [0, 1] \) is equal to

\[
f_A(x) = \begin{cases} 
(x - c) / (a - c), & c \leq x < a, \\
1, & a \leq x \leq b, \\
(x - d) / (b - d), & b < x \leq d, \\
0, & \text{otherwise} 
\end{cases}
\]

with \(-\infty < c \leq a \leq b \leq d < \infty \). The trapezoidal fuzzy number can be denoted by \( A = (c, a, b, d) \). The maximal grade of \( f_A(x) \) is 1, that is \( f_A(a) = 1 \), and parameter \( \alpha \) is the most possible value of the assessment data. \( c \), and \( d \), are the lower and upper bounds of assessment data, and it can be used to reflect the fuzziness of the assessment data. If \( c = a \) and \( b = d \), \([a, b]\) becomes the tolerance interval of the measurement. If \( a = b \) \((a, a, a, a)\) becomes the triangular fuzzy number which is a special case of trapezoidal fuzzy number. The reasons for using the trapezoidal fuzzy numbers or triangular fuzzy numbers are because they are easy to use and easy to interpret for the decision-makers to perform the evaluation. For example, ‘Approximately $300’ can be represented by (295, 300, 300, 302). ‘Approximately between $360 and $380” can be denoted by (357, 360, 380, 382). The non-fuzzy number ‘\( \alpha \)’ can be expressed as \((a, a, a)\).

**Fuzzy operation with \( \alpha \)-cut**

The \( \alpha \)-cut of fuzzy number \( A \) is an ordinary interval \( A^\alpha = \{x \in \mathbb{R} | f_i(x) \geq \alpha, 0 \leq \alpha \leq 1\} \). \( A^\alpha \) and \( A^\alpha \) are the lower and upper bounds of the assessment data with ‘grade of membership’ \( \alpha \). Thus, fuzzy number \( A \) can be constructed by a series of \( \alpha \) nested intervals such that \( A^\alpha \subseteq A^{\alpha^\prime} \subseteq \cdots \subseteq A^1 \), where \( \alpha_i > \alpha_{i+1} > \cdots > \alpha, \alpha \in [0, 1], i=1, \cdots, t \) and \( t \geq 2 \). According to the definition stated above, the operations of changing sign, addition, subtraction, multiplication, division and inverse of fuzzy numbers can be tackled by using the \( \alpha \)-cut and the operations of closed intervals.

A fuzzy number \( A = (c, a, b, d) \) is called a positive fuzzy number if \( c > 0 \) or \( A^\alpha > 0 \), for all \( \alpha \in [0, 1] \). Let \( A \) and \( B \) be positive fuzzy numbers, such that \( A^\alpha = [A^\alpha, A^\alpha] \) and \( B^\alpha = [B^\alpha, B^\alpha] \). By the vertex method (Choobineh and Li, 1990), the following operations are true:

- **Changing sign**
  \( -A^\alpha = [-A^\alpha, -A^\alpha] \)

- **Addition**
  \( (A + B)^\alpha = [A^\alpha + B^\alpha, A^\alpha + B^\alpha] \)

- **Subtraction**
  \( (A - B)^\alpha = [A^\alpha - B^\alpha, A^\alpha - B^\alpha] \)

- **Multiplication**
  \( (A \times B)^\alpha = [A^\alpha B^\alpha, A^\alpha B^\alpha] \)

- **Inversion**
  \( (A^{-1})^\alpha = [(A^\alpha)^{-1}, (A^\alpha)^{-1}] \)

- **Division**
  \( (A \div B)^\alpha = [A^\alpha / B^\alpha, A^\alpha / B^\alpha] \)

**Ranking fuzzy numbers**

In investment decision analysis, ranking the investment projects under consideration is essential. Many methods of ranking fuzzy numbers have been proposed (Bortolani and Deani, 1985; Buckley, 1984; Campbell, 2002; Choobineh and Behrens, 1992; Choobineh and Li, 1990; Frank, 1991). However, certain shortcomings of some of the methods have been reported in papers (Bortolani and Deani, 1985; Campbell, 2002). For effectiveness in problem solving, a method based on the concepts (Choobineh and Li, 1990; Frank, 1991) is used.

Let \( A_i = (c_i, a_i, b_i, d_i), \ i = 0, 1, \ldots, n \), be \( n \) fuzzy numbers with membership functions \( f_{A_i}(x) \). Define the left and right membership functions of \( f_{A_i}(x) \) by
fuzzy numbers or trapezoidal numbers, respectively. Suppose \( G^R_A \) is the inverse function of \( f^R_A \), and \( G^L_A \) is the inverse function of \( f^L_A \), then the right integral value of fuzzy number \( A_i \) is defined as

\[
RI (A_i) = \int_0^1 d - g^R_{A_i}(y) \, dy,
\]

and the left integral value of \( A_i \) is defined as

\[
LI (A_i) = \int_0^1 g^L_{A_i}(y) \, dy - c\dy.
\]

The ranking value of fuzzy number \( A_i \), denoted by \( R(A_i) \), is defined as

\[
R(A_i) = \frac{[1 - RI (A_i) - LI (A_i)](d-c)}{2}
\]

Where \( c = \min\{c_1, c_2, \ldots, c_n\} \) and \( d = \max\{d_1, d_2, \ldots, d_n\} \).

Let \( A = (G_i, a_i, b_i, d_i) \), \( i = 0, 1, \ldots, n \), be \( n \) trapezoidal fuzzy numbers. By using equations (1), (2), (3), and (4), the ranking value \( R(A_i) \) of trapezoidal fuzzy number \( A_i \) can be obtained:

\[
R(A_i) = (c_i + a_i + b_i + d_i - 4c)/[4(d-c)],
\]

Define the fuzzy ranking of \( A_i \) and \( A_j \) as:

\[
A_i > A_j \iff R(A_i) > R(A_j),
\]

\[
A_i = A_j \iff R(A_i) = R(A_j),
\]

\[
A_i < A_j \iff R(A_i) < R(A_j).
\]

By using Equation (4) or (5), one can easily calculate the ranking values of the \( n \) fuzzy numbers or trapezoidal fuzzy numbers. Then based on the ranking rule described above, the ranking of the \( n \) fuzzy numbers or trapezoidal fuzzy numbers can be effectively determined.

**METHOD AND PROCEDURE OF INVESTMENT DECISION**

In this section, a systematic approach for Air LSP to make investment decision by using the concepts of fuzzy set theory is presented. Owing to this algorithm considers both economic factors and strategic factors, so it is very suitable for making investment decision under fuzzy environment.

**Algorithm:** A stepwise description of investment project selection procedure is given in the following.

1. Determine the Air LSP’s goals and objectives.
2. Select all investment projects suitable to the Air LSP’s goals and objectives.
3. Identify the required economic factors associated with all investment projects.
4. Evaluate the economic factors associated with each investment project.
5. Find the fuzzy net present value \( (FNPV) \) for each investment project.
6. Calculate the economic factors ranking values \( (EFRV_i) \) of all the investment projects.
7. Assign importance weights to the strategic factors and the fuzzy scores to the investment projects versus various strategic factors.
8. Calculate the strategic factors ranking values \( (SFRV_i) \) of all the investment projects.
9. Calculate the final ranking values \( (FRV_i) \) of all the investment projects.
10. Select the investment project with the maximal final ranking value.

**Calculating the FNPV and economic factors ranking**

In general, the economic model contains various elements: total revenue, initial investment cost, salvage value, depreciation and economic consideration parameters (e.g. income tax, inflation free discount rate, and inflation rate), etc. Thus, the fuzzy net present value can be calculated by utilizing the four elements.

The fuzzy net present value after tax, \( FNPV \), can be computed by the following equation:

\[
FNPV = \sum_{j=0}^{d_j} \left( FC_{ij} \cdot F_{ij} \right) + FS_j + FD_j + FTRV_j \cdot (1 - i_j) \cdot (1 + f_j)^{t+j}
\]

Where

\[
j = 0, 1, \ldots, n \text{ (project planning horizon in years), } i_j = \text{annual fuzzy income tax rate at period } j, \quad d_j = \text{annual fuzzy inflation free discount rate at period } j, \quad f_j = \text{annual fuzzy inflation rate at period } j, \quad FCI_j = \text{fuzzy initial investment cost at period } j, \quad FE_j = \text{fuzzy expense at period } j, \quad FS_j = \text{fuzzy salvage value at period } j, \quad FD_j = \text{fuzzy depreciation at period } j, \quad FTRV_j = \text{fuzzy total revenue at period } j.
\]

To effectively represent the \( \alpha \)-cuts of \( FNPV \), define the \( \alpha \)-cuts of \( i_j, d_j, f_j, FCI_j, FE_j, FS_j, FD_j, \) and \( FTRV_j \) as follows:

\[
i^\alpha_j = [i^\alpha_j, i^\alpha_j], \quad d^\alpha_j = [d^\alpha_j, d^\alpha_j], \quad f^\alpha_j = [f^\alpha_j, f^\alpha_j],
\]

\[
FC^\alpha_I = [FCI^\alpha_j, FCI^\alpha_j], \quad FE^\alpha_I = [FE^\alpha_j, FE^\alpha_j], \quad FS^\alpha_j = [FS^\alpha_j, FS^\alpha_j], \quad FD^\alpha_j = [FD^\alpha_j, FD^\alpha_j], \quad FTRV^\alpha_j = [FTRV^\alpha_j, FTRV^\alpha_j].
\]
By the extension principle [16], $G$ is also a trapezoidal fuzzy number, that is

$$G_i = (\sum_{j=1}^{p} w_j q_j, \sum_{j=1}^{p} w_j o_j, \sum_{j=1}^{p} w_j s_j, \sum_{j=1}^{p} w_j r_j)$$

(10)

By using equation (5), the ranking value $R(G)$ of the weighted fuzzy score $G$ can be obtained, that is

$$R(G_i) = \frac{\sum_{j=1}^{p} w_j q_j + \sum_{j=1}^{p} w_j o_j + \sum_{j=1}^{p} w_j s_j + \sum_{j=1}^{p} w_j r_j - 4c}{[4(d-c)]}$$

(11)

where

$$c = \min\{\sum_{j=1}^{p} w_j q_j, \sum_{j=1}^{p} w_j o_j, \sum_{j=1}^{p} w_j s_j, \sum_{j=1}^{p} w_j r_j\}$$

and

$$d = \max\{\sum_{j=1}^{p} w_j q_j, \sum_{j=1}^{p} w_j o_j, \sum_{j=1}^{p} w_j s_j, \sum_{j=1}^{p} w_j r_j\}$$

To make the ranking values comparable, the ranking value of the strategic factors for each investment project is normalized:

$$SFRV_i = \frac{R(G_i)}{\sum_{i=1}^{m} R(G_i)}$$

(12)

**Calculating the final ranking values**

If the economic and strategic factors are not equally important, then a weighting factor $(\beta)$ is assigned to the economic factors, and $1 - \beta$ is assigned to strategic factors. Thus, the final ranking value $(FRV_i)$ of the $i$-th investment project can be defined as

$$FRV_i = \beta \times EFRV_i + (1 - \beta) \times SFRV_i, \ 0 \leq \beta \leq 1$$

(13)

By (13), the final ranking values of the $m$ investment projects can easily be obtained. Based on these ranking values, the decision-maker can determine the most suitable investment project.

**NUMERICAL EXAMPLE**

The characteristics of air logistics service provider

(Air LSP)

Owing to the development of globalization and deregulation in the corporate world, many Air Logistics Service Providers (Air LSP) view the entire world as a single marketplace. Furthermore, companies set up their base in many locations all over the world. Certain factors such as the globalization of corporate purchasing, the division of labor and production, and the marketing involved in the process of the global exchange of cargo have created an urgent need for Air Logistics Service Provider. In order to cope with the high-value-added distributing service and to maintain an effort to
accumulate sufficient commodities, many Air LSPs strategize bases of operation to stockpile their cargo. As a result, Air LSPs have set up logistics centers in the Airport districts and the surrounding areas. This is done in an effort to strengthen competitive advantage in the vigorous environment.

**Investment decision is the source of Air LSP’s competitive advantages**

Air Logistics Service Provider (Air LSP) is normally the rapid expansion of corporate scale operations, to provide globalization, all-dimensional international logistics, and supply chain management service. And, Air LSP could obtain competitive advantage in the vigorous environment. Air LSP competitive advantage originated in expanding investment scale, purchasing or combining the British Excel and MSAS; UPS combined Fritz flag and the new logistics Air LSP is UPS Supply Chain Solutions (UPSSCS), thus integrated physical logistics services, and through logistics and information flow control immediately close connection with the goods to the multi customer satisfaction with the global logistics supply chain management requirements and to reduce operating costs.

Therefore, decision-makers engage in evaluating investment alternatives; they tend to give assessments based on their knowledge, experience and subjective judgment. In fact, the decision-makers of Air LSP processing investment programs have to face uncertain and complex environment.

According to this analysis, the competitive advantages of the LSP are closely related to the investment decision. Thus, it is important for decision-makers utilized a systematic approach to evaluate the investment decision problem. In this paper, a model is considered to construct an algorithm of measuring the investment project of fuzzy net present value and fuzzy scores with respect to strategic factors are proposed to facilitate the decision making process to select the best investment project. Then, a hypothetical investment project selection problem was designed to demonstrate the computational process of this investment project selection algorithm. The exact steps are shown below.

**Steps 1 and 2**

Suppose the Air LSP needs to select the optimal investment project. After preliminary screening, three investment projects $P_1$, $P_2$ and $P_3$ remain for further evaluation.

**Step 3**

The planning horizon is a three-year period. The economic factors include fuzzy initial investment cost, fuzzy expense and fuzzy total revenue. The strategic factors contain the ability to integrate with other investment projects, response to market change, management control, competitive advantage and image of Air LSP.

**Step 4**

Table 1 shows the economic factors. The non-fuzzy initial investment costs ($10^2$) are 300, 285 and 290, respectively. The Air LSP’s annual fuzzy income tax rate is approximately 35%, that is $i_1 = i_2 = i_3 = (0.32, 0.35, 0.35, 0.37)$. The Air LSP’s annual fuzzy inflation free discount rate on investment is assumed to be 15%, that is $d_1 = d_2 = d_3 = (0.15, 0.15, 0.15, 0.15)$. And the annual fuzzy inflation rate is assumed to be approximately between 4 and 6%, that is $f_1 = f_2 = f_3 = (0.37, 0.04, 0.06, 0.062)$. Besides, assume that investment projects are with no salvage value and depreciation.

**Step 5**

The $\alpha$-cuts, at $\alpha = 0, 0.5, 1$ from each of the three membership functions of annual fuzzy income tax rate, annual fuzzy inflation free discount rate and annual fuzzy inflation rate are shown in Table 2. And the $\alpha$-cuts, at $\alpha = 0, 0.5, 1$ from each of the two membership functions of various economic factors, listed in Table 1, at different period are shown in Tables 3 and 4, respectively. For a given $\alpha$-cut and by the equations (7) and (8), the $\alpha$-cuts of fuzzy net present value $FNPV$ of various investment projects can be obtained as shown in Table 5.

The membership function of fuzzy net present value $FNPV_i$ of investment project 1, constructed by assembling the three intervals as shown in Table 5, is depicted in Figure 1.

**Step 6**

According to the $\alpha$-cuts of $FNPV_i$ of various investment projects shown in Table 5, we can obtain $c = \text{min} \{54.00, 47.84, 43.46\} = 43.46$, and $d = \text{max} \{117.48, 106.93, 99.42\} = 117.48$. By using Equations (2) and (3), the left integral and right integral values of the three fuzzy net present values can be obtained:

$$L_i(FNPV_1) = 19.965, \quad R_i(FNPV_1) = 15.033$$
$$L_i(FNPV_2) = 13.143, \quad R_i(FNPV_2) = 23.710$$
$$L_i(FNPV_3) = 8.765, \quad R_i(FNPV_3) = 30.405$$
Table 1. The fuzzy expenses and fuzzy total revenues of various projects at different period ($10^5$).

<table>
<thead>
<tr>
<th>Economic factor</th>
<th>$j$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fuzzy expense at period $j$</td>
<td>1</td>
<td>(23.0, 24.0, 24.0, 25.0)</td>
<td>(19.5, 20.0, 20.0, 20.5)</td>
<td>(20.5, 21.0, 21.0, 21.5)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(23.3, 24.3, 24.5, 26.0)</td>
<td>(19.8, 20.5, 20.5, 20.8)</td>
<td>(21.5, 22.0, 22.0, 22.5)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>(24.0, 25.0, 25.0, 27.0)</td>
<td>(20.0, 21.0, 22.0, 23.5)</td>
<td>(21.7, 22.5, 23.0, 23.5)</td>
</tr>
<tr>
<td>Fuzzy total revenue at period $j$</td>
<td>1</td>
<td>(299, 300, 300, 308)</td>
<td>(274, 275, 275, 280)</td>
<td>(278, 280, 280, 281.5)</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>(304, 305, 307, 309)</td>
<td>(287, 290, 292, 293)</td>
<td>(290, 293, 293, 295)</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>(310, 312, 312, 313)</td>
<td>(289, 291, 294, 296)</td>
<td>(285, 286, 288, 290)</td>
</tr>
</tbody>
</table>

Table 2. The $\alpha$-cuts of economic consideration parameters.

<table>
<thead>
<tr>
<th>$\alpha$</th>
<th>$i^\alpha = [i_l^\alpha, i_u^\alpha]$</th>
<th>$d^\alpha = [d_l^\alpha, d_u^\alpha]$</th>
<th>$f^\alpha = [f_l^\alpha, f_u^\alpha]$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>[0.32, 0.37]</td>
<td>[0.15, 0.15]</td>
<td>[0.037, 0.062]</td>
</tr>
<tr>
<td>0.5</td>
<td>[0.335, 0.36]</td>
<td>[0.15, 0.15]</td>
<td>[0.0385, 0.061]</td>
</tr>
<tr>
<td>1</td>
<td>[0.35, 0.35]</td>
<td>[0.15, 0.15]</td>
<td>[0.04, 0.06]</td>
</tr>
</tbody>
</table>

Table 3. The $\alpha$-cuts, at $\alpha=0$, 0.5, and 1 from the membership functions of various investment project of fuzzy expense $FE_j$, $j = 1, 2, 3$.

<table>
<thead>
<tr>
<th>$\alpha$-cuts of $FE_j$</th>
<th>$\alpha$</th>
<th>$j$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>[23.00, 25.00]</td>
<td>[19.50, 20.50]</td>
<td>[20.50, 21.50]</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>[23.30, 26.00]</td>
<td>[19.80, 20.80]</td>
<td>[21.50, 22.50]</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td>[24.00, 27.00]</td>
<td>[20.00, 23.50]</td>
<td>[21.70, 23.50]</td>
</tr>
<tr>
<td>$FE_j^\alpha = [FE_j^\alpha_1, FE_j^\alpha_2]$</td>
<td>0.5</td>
<td>1</td>
<td>[23.50, 24.50]</td>
<td>[19.75, 20.25]</td>
<td>[20.75, 21.25]</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2</td>
<td>[23.80, 25.25]</td>
<td>[20.15, 20.65]</td>
<td>[21.75, 22.25]</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3</td>
<td>[24.50, 26.00]</td>
<td>[20.50, 22.75]</td>
<td>[22.10, 23.25]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>[24.00, 24.00]</td>
<td>[20.00, 20.00]</td>
<td>[21.00, 21.00]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>[24.30, 24.50]</td>
<td>[20.50, 20.50]</td>
<td>[22.00, 22.00]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>[25.00, 25.00]</td>
<td>[21.00, 22.00]</td>
<td>[22.50, 23.00]</td>
</tr>
</tbody>
</table>

Table 4. The $\alpha$-cuts, at $\alpha=0$, 0.5, and 1 from the membership functions of various investment project of fuzzy total revenues $FTRV_j$, $j = 1, 2, 3$.

<table>
<thead>
<tr>
<th>$\alpha$-cuts of $FTRV_j$</th>
<th>$\alpha$</th>
<th>$j$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>1</td>
<td>[299, 308]</td>
<td>[274, 280]</td>
<td>[278, 281.5]</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>2</td>
<td>[304, 309]</td>
<td>[287, 293]</td>
<td>[290, 295]</td>
</tr>
<tr>
<td></td>
<td>0</td>
<td>3</td>
<td>[310, 313]</td>
<td>[289, 296]</td>
<td>[285, 290]</td>
</tr>
<tr>
<td>$FTRV_j^\alpha = [FTRV_j^\alpha_1, FTRV_j^\alpha_2]$</td>
<td>0.5</td>
<td>1</td>
<td>[299.5, 304]</td>
<td>[274.5, 277.5]</td>
<td>[279, 280.75]</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>2</td>
<td>[304.5, 308]</td>
<td>[288.5, 292.5]</td>
<td>[291.5, 294]</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>3</td>
<td>[311, 312.5]</td>
<td>[290, 295]</td>
<td>[285.5, 289]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>1</td>
<td>[300, 300]</td>
<td>[275, 275]</td>
<td>[280, 280]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>2</td>
<td>[305, 307]</td>
<td>[290, 292]</td>
<td>[293, 293]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>3</td>
<td>[312, 312]</td>
<td>[291, 294]</td>
<td>[286, 288]</td>
</tr>
</tbody>
</table>
Table 5. The $\alpha$ -cuts, at $\alpha$ = 0, 0.5, and 1 of $FNPV_i$ of various investment projects $P_1$, $P_2$ and $P_3$.

<table>
<thead>
<tr>
<th>$\alpha$ -cuts of $FNPV_i$</th>
<th>$\alpha$</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$FNPV_i^\alpha = [FNPV_{il}^\alpha, FNPV_{iu}^\alpha]$</td>
<td>0</td>
<td>[54.00, 117.48]</td>
<td>[47.84, 106.93]</td>
<td>[43.46, 99.42]</td>
</tr>
<tr>
<td></td>
<td>0.5</td>
<td>[63.42, 102.40]</td>
<td>[56.59, 93.73]</td>
<td>[52.21, 87.05]</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>[72.86, 87.51]</td>
<td>[65.39, 80.69]</td>
<td>[61.02, 74.78]</td>
</tr>
</tbody>
</table>

By using Equation (4), the ranking value of each investment project’s fuzzy net present value can be obtained:

$R(FNPV_1) = 0.5333$, $R(FNPV_2) = 0.4286$, $R(FNPV_3) = 0.3538$.

By using Equation (9), the economic factors ranking values are

$EFRV_1 = 0.4053$, $EFRV_2 = 0.3258$, $EFRV_3 = 0.2689$.

Step 7

The real value weights of the strategic factors and the fuzzy scores of the investment projects under the various strategic factors are shown in Table 6.

Step 8

By using equation (10), the weighted fuzzy scores ($G$) of investment project $P$ for strategic factors are as follows:

$G_1 = (76.30, 77.40, 79.25, 80.70)$,

$G_2 = (77.05, 77.85, 80.30, 81.80)$,

$G_3 = (75.10, 77.40, 78.80, 81.20)$.

By using Equation (12), the ranking value $R(G)$ of weighted fuzzy score $G$ can be obtained. The results are:

$R(G_1) = 0.4944$, $R(G_2) = 0.6194$, $R(G_3) = 0.4515$.

By using Equation (13), the strategic factors ranking values are

$SFRV_1 = 0.3158$, $SFRV_2 = 0.3957$, $SFRV_3 = 0.2885$.

Step 9

By using Equation (14) and taking $\beta = 0.6$, the final ratings can be obtained:

$FRV_1 = 0.3695$, $FRV_2 = 0.3538$, $FRV_3 = 0.2767$.

Step 10

The ranking order of three investment projects is $P_1$, $P_2$ and $P_3$. Therefore, it is obvious that the best selection is investment project $P_1$. 

Figure 1. The membership function of fuzzy net present value $FNPV_1$ of investment project 1.
Table 6. The weights and fuzzy (or non-fuzzy) scores of the three investment projects $p_1$, $p_2$, and $p_3$.

<table>
<thead>
<tr>
<th>Strategic factors</th>
<th>Weight</th>
<th>$P_1$</th>
<th>$P_2$</th>
<th>$P_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ability to integrate with other investment projects</td>
<td>0.15</td>
<td>80(80,80,80,80)</td>
<td>approximately 78</td>
<td>approximately between 75 and 79 (74,75,79,82)</td>
</tr>
<tr>
<td>Response to market change</td>
<td>0.20</td>
<td>approximately 65 (64,65,65,68)</td>
<td>approximately 72 (70,72,72,73)</td>
<td>approximately 75 (70,75,75,77)</td>
</tr>
<tr>
<td>Management control</td>
<td>0.20</td>
<td>approximately between 79 and 82 (77,79,82,85)</td>
<td>86(86,86,86,86)</td>
<td>81 (81,81,81,81)</td>
</tr>
<tr>
<td>Competitive advantage</td>
<td>0.25</td>
<td>approximately between 80 and 85 (78,80,85,86)</td>
<td>approximately between 78 and 84 (77,78,84,85)</td>
<td>approximately 79 (76,79,79,82)</td>
</tr>
<tr>
<td>Air LSP image</td>
<td>0.20</td>
<td>85(83,83,83,83)</td>
<td>approximately 80</td>
<td>approximately between 76 and 80 (74,76,80,84)</td>
</tr>
</tbody>
</table>

Conclusion

In this study, an algorithm is proposed to solve the project investment selection problem under fuzzy environment.

In project or alternative selection, very often, information available for making decision is incomplete and uncertain. So it is difficult to exactly evaluate the relative data such as investment total revenues, expenses, salvage value, depreciation, etc. Hence the conventional precision-based investment decision methods tend to be less effective in conveying the imprecision or vagueness nature of the decision environment. The concepts of fuzzy numbers and linguistic values are used to assess the economic and strategic factors in such a manner that the viewpoints of an entire decision-making body can be expressed without any constraints.

The investment project selection algorithm can not only tackle the conventional precision-based (non-fuzzy) problem but also help decision-makers to make suitable decision under fuzzy environment. This method in here can be computerized. Thus by conducting fuzzy or non-fuzzy assessments, the decision-makers can obtain the suitable investment project automatically.

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