Study of the properties of nuclear isomers is a current research focus. We have studied the systematic $8^+$ isomeric levels, half-lives, deformation parameters, and reduced transition probabilities between $8^+ \rightarrow 6^+$ state of even-even $^{76}\text{Ni}$ to $^{94}\text{Pd}$ nuclei for $N = 48$ neutrons. The calculated half-lives and quadrupole moments are compared with the experimental values. Moreover, we have studied the systematic B (E2) values, intrinsic quadrupole moments and values of $B/\beta_{zz}$ as a function of atomic number (Z) for N = 48 neutrons.

**Key words:** Half-life, quadrupole moment, deformation parameter, reduced transition probability.

**INTRODUCTION**

The nuclear structure studies are at the heart of understanding the formation of nuclear isomers with applications to many aspects in nuclear physics. The study is particularly interesting and important for unstable nuclei, such as those in neutron-rich, proton-rich, and super heavy mass regions. Usually ground state is more stable than the excited states. However, the lifetime of ground state of unstable nuclei is short, which makes the laboratory study extremely difficult (Sun, 2008).

Octupole electric transition in even-even nuclei for $N = 48$ have recently been of much interest both theoretically and experimentally. From a theoretical point of view, the yrast states up to $I^\pi = 8^+$ in N= 48 isotones can be ascribed to the two-hole states $\nu g_{9/2}$ for the N=50 closed shell. Moreover, their $8^+$ states were confirmed to become an isomer from even-even nuclei $^{76}\text{Ni}$ to $^{94}\text{Pd}$ (Gorska et al., 1995, Mrginean et al., 2003; Chakraborty et al., 2004; Sawicka et al., 2003; Grzywacz, 2005). These isomers occur because of large spin differences in the configuration of the initial and final states or a reduction in transition energies as one approaches the highest spin possible for the given seniority multiplet. However, the details calculation of quadrupole moments, deformation parameter, moment of inertia $\beta$ of the $8^+$ state of even-even nuclei for N=48 are not been calculated yet. At present, we have reported E2 transitions energy from $8^+ \rightarrow 6^+$, deformation...
parameter \( \beta \), existence of correlations between half-lives, reduced transition probabilities, \( Q_0 \), and \( \beta_{2\text{E2}} \) values as a function of atomic number, and other nuclear spectroscopic properties of the \( N=48 \) nuclei, \(^{76}\text{Ni} \), \(^{78}\text{Zn} \), \(^{80}\text{Ge} \), \(^{82}\text{Se} \), \(^{84}\text{Kr} \), \(^{86}\text{Sr} \), \(^{88}\text{Zr} \), \(^{90}\text{Mo} \), \(^{92}\text{Ru} \), and \(^{94}\text{Pd} \) by theoretical investigations.

**THEORETICAL SURVEY**

**Half-life**

The low-lying levels of even-even nuclei (\( J_i \), \( J_i \), \( I \), \( J_i \) = 2 in this case, the \( \gamma \)-ray half-life \( T_{1/2}^\gamma \) of the E2 transition is (Venkova and Andrejtscheff, 1981).

\[
T_{1/2}^\gamma = T_{1/2}^{\text{(exp)}} \left( 1 + \alpha_{\text{cor}} \right)
\]

(1)

where \( \alpha_{\text{cor}} \) is the total internal conversion coefficient of gamma transition and \( T_{1/2}^{\text{(exp)}} \) is experimental half-life, is related to downward reduced transition probability \( B(E2) \) in units of \( e^2b^2 \) (Venkova and Andrejtscheff, 1981).

\[
T_{1/2}^\gamma (\text{second}) = \frac{e^{6.87}}{B(E2)_{\text{expt}}} \quad (2)
\]

where \( E_\gamma \) is adopted \( \gamma \)-ray energies. The upward transition probability \( B(E2) \uparrow \) is related to this value (Venkova and Andrejtscheff, 1981).

\[
B(E2; J_i \rightarrow J_F) \uparrow = B(E2; J_F \rightarrow J_i) \downarrow \times g
\]

(3)

with

\[
g = \frac{(2J_F+1)}{(2J_i+1)}
\]

(4)

Mean while the value of \( B(E2) \) in units of \( e^2b^2 \), is related to \( B(E2) \) in units of Weisskopf single particle transition (W.u) (Schreckenbach et al., 1982).

\[
B(E2)_{\text{W.u}} = 5.94 \times 10^{-6} \times A^{4/3} \times B(E2) \quad e^2b^2
\]

(5)

For the low-lying levels of even-even nuclei decay with more than one gamma transition, \( T_{1/2}^\gamma \) is related to half-life, \( T_{1/2} \) by the following equation (Firestone et al., 1999).

\[
T_{1/2}^\gamma (k) = T_{1/2} \sum_{i=1}^{n} \frac{4(1+\alpha_{\text{cor}} km^2)}{h^2}
\]

(6)

where the summation is taken over the intensity \((I)\) of all gamma transition from the exciting level, \( k \), is the intensity of \( k_{\text{ex}} \) (E2) transition.

**Quadrupole moments**

The intrinsic quadrupole moments of nuclei can be derived from the transition rate \( B(E2; J_i \rightarrow J - 2) \) values according to Equation (7) (Venkova and Andrejtscheff, 1981).

\[
B(E2) = \frac{16}{22\pi} \frac{(J-2)}{(2J-1)} \frac{J}{2(J+1)} e^2 \quad Q_0^2 \quad (J \rightarrow J - 2)
\]

(7)

the quadrupole moment \( Q(J) \) is related to \( Q_0 \) by (El-Khosh, 1993).

\[
Q(J) = \frac{3(2J+1)}{5(2J-1)} \quad Q_0
\]

(8)

And in the considered of the ground state \( J=\kappa \),

\[
Q(J) = \frac{J(J+1)}{5(J-1)(2J+3)} \quad Q_0
\]

(9)

The quantity \( \hbar \omega \) and moment of inertia \( \mathcal{I} \) have been derived by means of the familiar relation (Venkova and Andrejtscheff, 1981).

\[
\hbar \omega^2 = (J^2 + 1) \left[ \frac{\mathcal{E}(J \rightarrow J - 2)}{2J-2} \right]^2
\]

(10)

and

\[
\frac{23}{h^2} = \frac{4J-2}{\mathcal{E}(J \rightarrow J - 2)}
\]

(11)

Here, \( \mathcal{E}(J \rightarrow J - 2) \) is the level spacing between states with spin \( J \) and \( J-2 \) and corresponds to the \( \gamma \)-ray transition energy \( E_{\gamma} \).

**Deformation parameters**

If one assumes a uniform charge distribution out to the distance \( R \) \( (\theta, \varphi) \) and zero charge beyond that, then he finds that in the limit of small deformation, the quadrupole deformation parameters \( \beta \) is related to the formula (Chandan et al., 2004).

\[
\beta = \left[ B(E2) \uparrow \right]^{1/2} \left[ 32R_0^3 / 4\pi \right]^{-1}
\]

(12)

where \( R_0 \) is the average radius of the nucleus, and (Raman et al., 2001)

\[
R_0^2 = 0.0144 A^{2/3} b
\]

(13)

In the mean time, the single particle deformation \( \beta_{2\text{np}} \) is given by (Raman et al., 1987).
\[ \beta_{2sp} = 1.59/Z \]

where, \( Z \) is atomic number.

The main deformation parameters (El-Khosht, 1993).

\[ \delta = 0.895\beta^2 \]

RESULTS AND DISCUSSION

Half-life \( (T_{1/2}) \)

The results obtained in this study is the most simple and powerful for performing the nuclear model in calculating parameters specified by nuclear structure. In the present work, the value of \( E_i \) were arising from nuclear level \( \beta^+ \rightarrow 6^\pi \) with corresponding value of \( B(E2) \) in units of \( \text{W.u} \), as well as \( \alpha_{\text{vol}} \) and \( I_2 \) values for all gamma transition were accumulated from (Mazzocchi et al., 2005; Makishima et al., 1999; Abriola et al., 2009; Singh, 2001; Mukherjee and Sonzogni, 2005; Baglin, 1992; Rsel et al., 1978; Band et al., 1976; Browne, 1997).

\[ B(E2) \]

in units of \( e^2b^2 \) for \( 8^\pi \) state were calculated for 10 even-even nuclei in the mass range \( A=76 \rightarrow 94 \), using Equation (5). However, the values \( T_{1/2} \) of \( 8^\pi \) states were evaluated using Equations (2 and 6). Figure 1 shows the systematic isomeric levels of experimental low-lying yrast \( 8^\pi \) state as a function of \( Z \) for even-even nuclei \(^{76}\text{Ni} \) to \(^{94}\text{Pd} \). The energy of \( 8^\pi \) level increases towards higher proton up to \( Z=34 \), and then decreases up to \( Z=46 \). The maximum isomeric level is 3236 KeV for \(^{68}\text{Se} \) nucleus and minimum value is 2440 KeV for \(^{76}\text{Ni} \) nucleus.

Figure 2 shows reduced transition probability \( B(E2) \) in \( e^2b^2 \) as a function of \( Z \) number for \( N = 48 \) Isotones from \(^{76}\text{Ni} \) to \(^{94}\text{Pd} \). The maximum value of \( B(E2) \) is 0.0070 \( e^2b^2 \) for \(^{90}\text{Mo} \) nucleus, while the minimum value is 0.0008 \( e^2b^2 \) for \(^{80}\text{Ge} \) nucleus. The \( B(E2) \) values as well as isomeric level do not show the similar tendency as a function of atomic number \( Z \). The \( B(E2) \) values of the \( N = 48 \) Isotones with \( Z \leq 38 \) differ significantly from those with \( Z \geq 38 \). The difference probably originates from the orbital occupied by valence proton; in the former nuclei the valence proton mainly occupy; the \( f_p \) orbitals while in the
Figure 2. Reduced transition probabilities $B(E2)$ as function of atomic number (Z) for even-even $^{76}$Ni to $^{94}$Pd nuclei.

Figure 3. Half-lives of $8^+$ levels as a function of atomic number (Z) for even-even $^{76}$Ni to $^{94}$Pd nuclei. *(Mazzocchi et al., 2005; Makishima et al., 1999; Abriola et al., 2009; Singh, 2001; Mukherjee and Sonzogni, 2005; Baglin, 1992; Rsel et al., 1978; Band et al., 1976; Browne, 1997).*

letter they occupy the $g_{9/2}$ orbital.

Figure 3 shows the calculated and experimental values of half-lives in ns as a function of Z number for N=48 isotones from $^{76}$Ni to $^{94}$Pd nuclei. It was found that the calculated data overlap to experimental values except Z= 42.
We have calculated the values of $Q_0$, $Q(j)$, $\hbar^2 \omega_j^2$ and $2\hbar^2 / \hbar^2$ for even-even nuclei $N = 48$. One can conclude that the evaluated intrinsic quadrupole moment are in good agreement with corresponding values of $Q_0$ which have been experimentally done already by the other authors (Abriola et al., 2009; Mukherjee and Sonzogni, 2005; Browne, 1997). The maximum difference in $Q_0$ values, however, is found to be 0.12 eb, according to Equation 7. Figure 4 shows that the $Q_0$ values increase with proton number from $Z = 32$ to 38, then decrease on semi-magic number $Z = 40$ and then increases again at $Z = 42$, after that they are continually decreases until $Z = 46$. The shape of the changing $Q_0$, $B(E2)$ and $\beta / \beta_{25P}$ as a function of atomic number $Z$ for even-even nuclei $^{76}$Ni to $^{94}$Pd are similar. Furthermore, the information on quantities $\hbar^2 \omega_j^2$ and $2\hbar^2 / \hbar^2$ were calculated, so far in the $8^+$ levels of doubly even nuclei from $^{76}$Ni to $^{94}$Pd are represented in Table 1.

**Quadrupole moments**

**Deformation parameter ($\beta$)**

In Table 2, the values $B(E2)$, $\beta$ and $\beta_{25P}$ were calculated from Equations 3, 12 and 14 respectively. The values of ratio $\beta$ to $\beta_{25P}$ and $\beta$ were also computed. From Table 2, one can observe that the maximum values of $\beta$ and $\beta / \beta_{25P}$ are 0.03 and 0.67 for $^{86}$Sr and $^{90}$Mo nuclei respectively, while the minimum values are 0.01 and 0.25 for $^{80}$Ge nucleus. The values of $\beta / \beta_{25P}$ as a function of atomic number ($Z$) were graphically presented in Figure 5. It is shown that the ratio $\beta / \beta_{25P}$ have strong dependence on the atomic number $Z$. The values increase with increase of proton number from $Z=32$ to 38, then decrease on semi-magic number $Z=40$ and then increases again at $Z=42$. After that they are decreasing until $Z=46$. The pattern of the $\beta / \beta_{25P}$, $B$ (E2) and $Q_0$ as a function of atomic number $Z$ for even-even nuclei $^{76}$Ni to $^{94}$Pd is similar. Usually deformation...
Table 1. Reduced transition probabilities B(E2) and half-lives $T_{1/2}$ of the 8$^+$ state.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>Isomeric levels* (keV)</th>
<th>$E_T$ (E2) KeV</th>
<th>B.R (%) **</th>
<th>$B(E2)$ in units</th>
<th>$T_{1/2}$ (present)</th>
<th>$T_{1/2}$ (exp) *</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{78}$Ni</td>
<td>2440(1)</td>
<td>144(2)</td>
<td>100</td>
<td>0.7(2)</td>
<td>0.0013</td>
<td>608.90 ns</td>
</tr>
<tr>
<td>$^{78}$Zn</td>
<td>2637.7(10)</td>
<td>144.7(5)</td>
<td>100</td>
<td>1.21</td>
<td>0.0024</td>
<td>319.56 ns</td>
</tr>
<tr>
<td>$^{80}$Ge</td>
<td>3445.11(8)</td>
<td>466.76(4)</td>
<td>100</td>
<td>0.422(9)</td>
<td>0.0009</td>
<td>2.95 ns</td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>3518.5(5)</td>
<td>347(7)</td>
<td>100</td>
<td>0.56(3)</td>
<td>0.0012</td>
<td>8.60 ns</td>
</tr>
<tr>
<td>$^{84}$Kr</td>
<td>3236.07(18)</td>
<td>63.5(1)</td>
<td>100</td>
<td>2.33(6)</td>
<td>0.0051</td>
<td>1.83 µs</td>
</tr>
<tr>
<td>$^{86}$Sr</td>
<td>2955.68(21)</td>
<td>96.68(3)</td>
<td>100</td>
<td>2.83(10)</td>
<td>0.0064</td>
<td>0.46 µs</td>
</tr>
<tr>
<td>$^{90}$Zr</td>
<td>2887.79(6)</td>
<td>76.99(1)</td>
<td>100</td>
<td>1.75(3)</td>
<td>0.0041</td>
<td>1.30 µs</td>
</tr>
<tr>
<td>$^{92}$Mo</td>
<td>2874.73(15)</td>
<td>63.15(1)</td>
<td>100</td>
<td>2.92(13)</td>
<td>0.0070</td>
<td>1.95 µs</td>
</tr>
<tr>
<td>$^{92}$Ru</td>
<td>2834.6(20)</td>
<td>161.9(4)</td>
<td>100</td>
<td>1.672(24)</td>
<td>0.0041</td>
<td>100.21 ns</td>
</tr>
<tr>
<td>$^{94}$Pd</td>
<td>2702.13(20)</td>
<td>324(1)</td>
<td>100</td>
<td>≥ 1.2</td>
<td>0.0030</td>
<td>5.08 ns</td>
</tr>
</tbody>
</table>

* (Mazzocchi et al., 2005; Makishima et al., 1999; Abriola et al., 2009; Singh, 2001; Mukherjee and Sonzogni, 2005; Baglin, 1992; Rsel et al., 1978; Band et al., 1976; Browne, 1997). ** (Chakraborty et al., 2004; Firestone et al., 1999).

Table 2. The Intrinsic quadrupole moment $(Q_0)$, quadrupole moment $Q(J)$, quantity $h^2 \omega^2$, moment of inertia $2\Sigma/h^2$, reduced transition probability $B(E2)$ and deformation parameter $(\beta)$ of the 8$^+$ of even-even nuclei for N=48.

<table>
<thead>
<tr>
<th>Nuclei</th>
<th>$Q_0$ (eb)***</th>
<th>$Q_0$ (eb)</th>
<th>$Q(J)$ (eb)</th>
<th>$h^2 \omega^2$ (MeV$^2$)</th>
<th>$2\Sigma/h^2$ (MeV$^{-1}$)</th>
<th>$B(E2)$ (e$^2$b$^2$)</th>
<th>$\beta$</th>
<th>$\beta_{2SP}$</th>
<th>$\beta/\beta_{2SP}$</th>
<th>$\delta_{10^3}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$^{76}$Ni</td>
<td>0.20</td>
<td>0.14</td>
<td>5.25</td>
<td>208.33</td>
<td>0.0010</td>
<td>0.0190</td>
<td>0.0567</td>
<td>0.353</td>
<td>0.324</td>
<td></td>
</tr>
<tr>
<td>$^{78}$Zn</td>
<td>0.27</td>
<td>0.18</td>
<td>5.30</td>
<td>207.32</td>
<td>0.0018</td>
<td>0.0227</td>
<td>0.0530</td>
<td>0.429</td>
<td>0.464</td>
<td></td>
</tr>
<tr>
<td>$^{80}$Ge</td>
<td>0.16</td>
<td>0.11</td>
<td>55.19</td>
<td>64.27</td>
<td>0.0007</td>
<td>0.0125</td>
<td>0.0496</td>
<td>0.253</td>
<td>0.141</td>
<td></td>
</tr>
<tr>
<td>$^{82}$Se</td>
<td>0.19</td>
<td>0.13</td>
<td>30.50</td>
<td>86.45</td>
<td>0.0009</td>
<td>0.0136</td>
<td>0.0467</td>
<td>0.291</td>
<td>0.166</td>
<td></td>
</tr>
<tr>
<td>$^{84}$Kr</td>
<td>0.36(4)</td>
<td>0.39</td>
<td>0.27</td>
<td>1.02</td>
<td>472.44</td>
<td>0.0039</td>
<td>0.0263</td>
<td>0.0441</td>
<td>0.596</td>
<td>0.619</td>
</tr>
<tr>
<td>$^{86}$Sr</td>
<td>0.44</td>
<td>0.30</td>
<td>2.40</td>
<td>304.01</td>
<td>0.0049</td>
<td>0.0274</td>
<td>0.0418</td>
<td>0.656</td>
<td>0.674</td>
<td></td>
</tr>
<tr>
<td>$^{88}$Zr</td>
<td>0.51(3)</td>
<td>0.39</td>
<td>0.24</td>
<td>1.50</td>
<td>389.66</td>
<td>0.0031</td>
<td>0.0205</td>
<td>0.0397</td>
<td>0.515</td>
<td>0.376</td>
</tr>
<tr>
<td>$^{90}$Mo</td>
<td>0.58(3)</td>
<td>0.46</td>
<td>0.32</td>
<td>1.01</td>
<td>475.05</td>
<td>0.0053</td>
<td>0.0252</td>
<td>0.0378</td>
<td>0.668</td>
<td>0.571</td>
</tr>
<tr>
<td>$^{92}$Ru</td>
<td>0.35</td>
<td>0.24</td>
<td>6.56</td>
<td>185.29</td>
<td>0.0031</td>
<td>0.0187</td>
<td>0.0361</td>
<td>0.518</td>
<td>0.314</td>
<td></td>
</tr>
<tr>
<td>$^{94}$Pd</td>
<td>0.30</td>
<td>0.21</td>
<td>26.59</td>
<td>92.59</td>
<td>0.0024</td>
<td>0.0148</td>
<td>0.0345</td>
<td>0.429</td>
<td>0.196</td>
<td></td>
</tr>
</tbody>
</table>

*** (Abriola et al., 2009; Mukherjee and Sonzogni, 2005; Browne, 1997).

Conclusion

We have presented the calculated half-lives of the 8$^+$ levels, reduced transition probabilities, quadrupole moments and deformation parameters for even-even $^{78}$Ni to $^{94}$Pd nuclei. The calculated half-lives and $Q_0$ as well as experimental data are in good agreement. These results are quite useful for compiling to nuclear data table, which makes it a good reference containing $T_{1/2}$, $Q_0$, $Q(J)$,

ACKNOWLEDGMENTS

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\[ p^2 \alpha^2, \frac{2\gamma}{p^2}, \beta, \text{ and } \beta_{2\gamma}. \]

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