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Distribution of earth pressure behind retaining walls considering different approaches

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In this paper, the earth pressure distribution generated behind a retaining wall is estimated by the finite element method and compared with that obtained from classical earth pressure theories. In this analysis, the behavior of the soil is assumed to be elasto-plastic with Mohr-Coulomb failure criterion. The concrete retaining wall is represented by linear elastic model. The thin layer interface element proposed by Desai et al. (1984) is used to represent the interface between the wall and the surrounding soil. A two-dimensional plane-strain finite element computer program CRISP is utilized after some modification. The results show that the distribution of earth pressure depends on the mode of wall movement (whether translation, rotation or both), and the active earth pressure obtained from the finite element analysis is close to Coulomb's solution in case of bottom wall rotation. Whereas, in case of top wall rotation, the obtained earth pressure is greater than that of Coulomb solution. This is mainly because Coulomb equation is originally based on wall rotating about its base (failure mechanism).

Key words: Earth pressure, Coulomb’s solution, Dubrova’s method, FEM, soil-structure interaction, retaining wall.

INTRODUCTION

The conventional design techniques provide little information about the distribution and magnitude of lateral earth pressures and wall deformations. The soil mass is assumed to be in the limiting state condition ($K_a$ and $K_p$), which is actually not correct (Potts and Fourie, 1984). The limiting pressures are not mobilized unless sufficient ground and wall movements are developed.

A flexible wall is very likely to deform sufficiently in the active pressure case prior to failure. However, a very rigid wall might shear off suddenly without the active pressure being allowed to develop (Bowles, 1988).

In this paper, a comparison is made between the conventional theories of earth pressure and the finite element method in predicting the distribution of earth pressure behind cantilever retaining walls.

PREVIOUS STUDIES

The purpose of this section is to present a review of previous work, in which the finite element method was used to analyze the soil-structure interaction of earth retaining structures. The finite element method of analysis has been applied to a variety of earth retaining structures and used to calculate stresses and movements for problems involving a wide variety of boundary and loading conditions. Some of the modeling features to be considered in a successful soil-structure interaction analysis are summarized in this section, along with the results from selected soil-structure interaction analyses.

Procedures for the finite element analysis of conventional, stable earth retaining structures are well established. They have been successfully applied to the evaluation of the soil-structure interaction for a variety of earth retaining structures during the past decades, including U-frame locks, gravity walls, and basement structures.

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walls (Ebeling, 1990).

The earliest study was performed by Clough and Duncan (1969), in their analysis of two reinforced concrete U-frame locks at Port Allen and Old River that had been extensively instrumented. The soil backfill was represented by finite element mesh. During preliminary analysis, it was found that a gravity turn-on analysis was insufficient for the analysis of soil-structure interaction problems. It was recognized that the analytical procedure used must take into account the nonlinear stress-strain response of soils during loading. In addition, it was shown that the best agreement is obtained when the actual construction process was simulated as closely as possible. The use of incremental finite element analysis with nonlinear, stress-dependent, stress-strain behavior of the soil was adopted. Linear elastic behavior was assumed for the concrete lock wall.

An additional analytical feature used in the Port Allen and Old River study was the inclusion of the Goodman et al. (1968), which interface elements between the concrete lock walls and the soil backfill. The interface between the backfill and the wall is constrained in previous work so that both move in the same direction by equal magnitude. In actuality, there is no such constraint on the backfill and wall. This constraint influences both the resulting displacements and computed stresses within the wall and the backfill. The presence of interface elements between the backfill and the wall allows the backfill to move somehow independent of the wall.

Clough and Duncan (1969) found that their developed procedures gave results in good agreement with the results of the extensive instrumentation program for Port Allen lock and Old River lock.

In a study in 1971, Clough and Duncan showed that nonlinear incremental finite element procedures could be used to predict lateral earth pressures for conditions ranging from an unmoving wall to limit conditions where the wall is being displaced enough to generate active or passive earth pressures. A 20-ft (6.096 m) high wall retaining a sand backfill and founded on rock was used in this analysis.

Interface elements were placed between the wall and soil and between the rock and soil. The computed relationships between wall movements and the resultant horizontal earth pressure force were found to be in good agreement with the classical earth pressure theories and the computed deformations were in agreement with those measured by Terzaghi (1934) in his retaining wall tests. The use of interface elements along the soil-to-wall interface was shown to influence the computed earth pressures.

It was found that the earth pressure forces from the finite element analysis were greater than those computed using the classical earth pressure theory for an active stress state, but less than at-rest values. Two contributing factors are the incorporation of the compressibility of the foundation in the analysis and the non-uniform loading of the foundation sands.

Matsuo et al. (1978) investigated the characteristics of the earth pressure acting on a retaining wall on the basis of the large scale prototype tests in a field. They built a 10 m high concrete wall with silty sand and slags as backfill materials, in order to study the influence of displacement of the wall on the magnitude and distribution of earth pressure in the vertical direction. Based on the information obtained from the tests, they proposed that a general retaining wall should be designed against the earth pressure at rest.

They also compared the measured earth pressures with the analyzed results obtained by the finite element method. They represented the soil as a linear elastic material with triangular elements. They found that the influence of the unit weight (γ) and Young’s modulus (E) on the calculated results are very small. That is, it is proper from the engineering point of view, to use the rough values of γ and E in the calculation of earth pressure at rest, but the calculation is very sensitive to variation in the value of Poisson’s ratio.

Roth et al. (1979) described the backfill placement analysis of an instrument (deep basement wall) using the same finite element procedure by Clough and Duncan (1969). The instrumentation measurements after completion of backfilling were compared to the computed results. Good agreement was found between the calculated and the measured lateral earth pressures when interface elements were included along the backfill-to-wall interface. By using interface elements in the finite element analyses of a rigid wall, they were able to simulate the settlement of the backfill adjacent to the wall, resulting in the mobilization of a shear force along the back of the wall. In the parametric analyses, they found that the value of Poisson’s ratio assigned to the backfill was the most important parameter affecting the calculated lateral earth pressure, and the stiffness assigned to the backfill had little influence on the calculated lateral pressures.

10-story basement walls of a high-rise office building in Los-Angeles city were constructed. Roth and Crandall (1981) used the hyperbolic elastic finite element techniques for the prediction of elastic earth pressures against these walls. They used silty sand as a backfill material, whereas, a corrugated bentonite-filled cardboard and fiberboard was used as an interface material (because bentonite is well known for its low shear strength and high swelling potential). They carried out triaxial test and direct shear test in order to define the strength and load-deformation behavior of both backfill and interface materials.

They concluded the following points:

1. Poisson’s ratio is the single important parameter affecting the calculated lateral earth pressure.
2. Changing the modulus of elasticity (E) in the finite element analysis did not significantly change the calculated horizontal wall pressures.
3. The interface material is one of the possible important
factors in governing the earth pressure against the wall. 

4. The effect of soaking the interface material eliminated the cohesion intercept, but did not appear to alter the angle of sliding friction.

Potts and Fourie (1984) carried out a numerical study about the behavior of a propped retaining wall. In their study, the finite element is used to investigate the influence of type of construction (excavation or backfilling) and the initial stress in the soil on the behavior of single propped retaining walls. A linearly elastic-perfectly plastic with a Mohr-Coulomb yield surface is used to model the soil behavior, while the wall is assumed to be linearly elastic and a rigid propped is assumed to act at the top of the wall. The problem was solved as a plane strain condition with eight-noded isoparametric elements. The following conclusions were drawn from this work:

1. The limit equilibrium method used in current design procedures produces reliable estimates for the depth of wall embankment required to maintain stability.
2. For excavation of walls in soils with a high value of coefficient of earth pressure at rest condition ($K_o$), prop force and wall bending moments greatly exceed those calculated by using the simple limit equilibrium approach. In addition, large soil and wall movements are experienced even at shallow depths of excavation. The behavior is dominated by the vertical unloading caused by excavation process and large movement still occurs even if the wall is fully restrained from horizontal movement.
3. For backfilled and excavated walls in soils with a low ($K_o$) values, the analyses indicate that the displacements are much smaller in magnitude and that the approximate limit equilibrium calculations produce conservative values of prop force and bending moments.
4. Large zones of failed soils, especially in the front of the wall, are predicted for excavation walls in high ($K_o$) soils and the lateral wall pressures behind the wall differ substantially from the classic active distribution. Passive conditions in front of the wall are completely mobilized at small excavation depths and before active conditions are approached down the back of the wall. In contrast, excavated walls and backfilled walls in low ($K_o$) soils show lateral pressures, which are in agreement with the classical distributions.

Potts and Fourie (1986) employed the finite element method to examine the influence of wall movement on the generation of earth pressure. The effects of wall translation, rotation about the top and rotation about the bottom of the wall have been investigated. An elasto-plastic constitutive law using Mohr-Coulomb yield surface has been used to model the soil behavior. The following conclusions arise from their investigation:

1. The nature of the wall movement, whether translation or rotation, has an effect on the equivalent values of $K_a$ and $K_p$ for both rough and smooth walls.
2. The final values of $K_a$ and $K_p$ are essentially unaffected by the value of $K_o$ or the distribution of Young's modulus in the soil.
3. The relative displacements necessary to mobilize active and passive conditions depend on the wall, initial $K_p$ value and distribution of Young's modulus.
4. The mode of wall movement has a considerable effect on the distribution of earth pressure.

Bhatia and Bakeer (1989) performed a finite element analysis of 10 m high instrumented experimental wall, resting on a hinged base that was tested by Matsuo et al. (1978) in order to discuss some factors that influence the results of a finite element idealization of the problem of earth pressure behind a gravity wall with dry, cohesionless backfill. The problem was modeled by two dimensional, isoparametric, quadratic and quadrilateral eight-noded elements. The material model used for the soil elements is a nonlinear elastic-perfectly plastic model with a Von-Mises yield criterion, where a yield stress is inputted at different strain levels. A series of analyses, similar to that of Clough and Duncan (1969) analyses were conducted for the boundary conditions ranging from a wall with zero displacement to the case where the crest of the wall was displaced.

They found that a finite element mesh with a backfill extending horizontally, four times its height, and having a free lateral boundary, or six times its height and having a restrained lateral boundary is required to model a gravity wall retaining a dry, cohesionless backfill. Fine elements should be used in the backfill behind the wall-back in region extending horizontally from a distance of at least the height of a wall for the active case.

Hazarika and Matsuzawa (1996) proposed a new numerical method, based on a smeared shear band technique, for the analysis of earth pressure that incorporates two shear bands for a localized element. The method, which is valid for plane strain condition, is applied to explain the generation of the active earth pressure against a rigid retaining wall for different modes of the displacement that the wall is likely to undergo. It exposes the deficiency of the conventional methods of analysis based on continuity of stress throughout the entire deformation process of the backfill. The proposed methodology can adequately capture the progressive deformation characteristics of the backfill. The wall displacement modes are seen to govern the progressive failure pattern, as a result, it depends on the modes of displacement, the active state distribution of the earth pressure and the related parameters at magnitudes.

Al-Shikhany (2000) investigated the earth pressure distribution on a flexible (propped and cantilever) wall for both excavation and backfilling construction methods.
Eight-noded quadrilateral elements were used to represent the soil and the wall, whereas, the relative displacement between the wall and the soil was simulated by a thin layer interface element. The behavior of the soil and the interface was assumed to be elasto-plastic with Mohr-Coulomb criterion, while the wall was assumed to be an elastic material.

Al-Shikhany found that the earth pressure depends on the deformation and the movement of the wall and on the initial stresses at rest, $K_0$, and existence of prop. It was also found that the results for low $K_0$ values are almost the same for both methods of constructions (excavation and backfilling), whereas, for high $K_0$ values, the results will be different.

**ACTIVE EARTH PRESSURE FOR TRANSLATION OF RETAINING WALL**

Under certain circumstances, retaining walls may undergo lateral translation, as shown in Figure (1). A solution to the distribution of active pressure for this case was provided by Dubrova (1963) and was also described by Harr (1966) and Das (2007). The solution of Dubrova assumes the validity of Coulomb's solution. In order to understand this procedure, let us consider a vertical wall with a horizontal granular failure as shown in Figure (2).

For rotation about the top of the wall, the resultant $R$ of the normal and shear forces along the rupture line $AC$ is inclined at an angle $\phi$ to the normal drawn to $AC$. According to Dubrova, there exists infinite number of quasi-rupture lines such as $A'C'$, $A''C''$, ... for which the resultant force $R$ is inclined at an angle $\psi$, where:

$$\psi = \frac{\phi}{2}$$  \hspace{1cm} (1)

Now, refer to the Coulomb's active earth pressure equation. For $\beta = 90^\circ$ and $\alpha = 0$, the relationship for Coulomb's active force can be rewritten as:

$$P_a = \frac{\gamma}{2 \cos \phi} \left[ \frac{H}{\cos \phi} + (\tan^2 \phi + \tan \phi \tan \delta)^{\alpha \beta} \right]$$  \hspace{1cm} (2)

The force against the wall at any $z$ is then given as:

$$P_a = \frac{\gamma}{2 \cos \phi} \left[ \frac{z}{\cos \phi} (\tan^2 \phi + \tan \phi \tan \delta)^{\alpha \beta} \right]$$  \hspace{1cm} (3)

The active pressure at any depth $z$ for wall rotation about the top is:

$$\sigma_a'(z) = \frac{\partial P_a}{\partial z} = \frac{\gamma}{\cos \phi} \left[ \frac{z \cos \phi}{(\tan^2 \phi + \tan \phi \tan \delta)} + \frac{\tan \phi \tan \delta}{H(\tan^2 \phi + \tan \phi \tan \delta)} (\tan \psi + m) \right]$$  \hspace{1cm} (4)
Figure 3. Horizontal earth-pressure distribution behind a model rigid retaining wall (Note: sand backfill, \( \phi' \approx 34^\circ \), \( \delta' = 2/3 \phi' \), \( \gamma = 15.4 \ \text{kN/m}^3 \)), (based on Matsuzawa and Hazazika, 1996).

For frictionless walls, \( \delta' = 0 \) and Equation (3) is simplified as:

\[
\sigma_a'(z) = \gamma \tan^2 \left( 45 - \frac{\phi'}{2} \right) \left( z - \frac{z^2 \tan \phi'}{2 \sin \phi'} \right)
\]

(5)

For wall rotation about the bottom, a similar expression can be found as:

\[
\sigma_a'(z) = \frac{\gamma z}{\cos \delta'} \left( \frac{\cos \delta'}{1 + \tan \phi' \sin \delta'} \right)^2
\]

(6)

For translation of the wall, the active pressure can then:

\[
\sigma_a'(z)_{\text{translation}} = \frac{1}{2} \sigma_a'(z)_{\text{rotation about top}} + \frac{1}{2} \sigma_a'(z)_{\text{rotation about bottom}}
\]

(7)

An experimental verification of this procedure was provided by Matsuzawa and Hazazika (1996). The results were obtained from large-scale model tests and are shown in Figure (3). The theory and experimental results show good agreement.

DESCRIPTION OF THE PROBLEM

In order to give a meaningful study about the earth pressure distribution, it was decided to analyze the problem adopted by Potts and Fourie (1984), and by Fourie and Potts (1989). The problem geometry and finite element mesh are as shown in Figures 4 and 5, while the material properties are given in Table 1. The earth pressure generated behind a retaining wall was studied and compared with that of classical earth pressure theories of Coulomb and Dubrova for three types of wall movements (rotation about the top and about the bottom of the wall and free wall translation).

ANALYSES

Effect of the wall movement on the lateral earth pressure

The effect of mode of wall movement on the stress distribution behind a retaining wall has been investigated using the finite element computer program CRISP.
Table 1. Material properties (Al-Shikhany, 2000).

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Soil</th>
<th>Interface</th>
<th>Wall</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>kN/m²</td>
<td>$5.5 \times 10^4$</td>
<td>$5.5 \times 10^4$</td>
<td>$28 \times 10^6$</td>
</tr>
<tr>
<td>$v$</td>
<td>-</td>
<td>0.2</td>
<td>0.2</td>
<td>0.15</td>
</tr>
<tr>
<td>$c$</td>
<td>kN/m²</td>
<td>0</td>
<td>0</td>
<td>-</td>
</tr>
<tr>
<td>$\phi$</td>
<td>degree</td>
<td>25</td>
<td>25</td>
<td>-</td>
</tr>
<tr>
<td>$G$</td>
<td>kN/m²</td>
<td>-</td>
<td>250</td>
<td>-</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>kN/m³</td>
<td>20</td>
<td>20</td>
<td>24</td>
</tr>
</tbody>
</table>

| Interface | M      | -          | 0.05      | -          |

Active stress, kPa

![Active stress graph](image)

Figure 6. Active earth pressure against retaining wall for $\phi = 25^\circ$ and $\delta = 0.0^\circ$.

In this study, the computer program CRISP has been developed to perform two-dimensional analysis of soil-wall interaction. The program is primarily based on a program provided by Britto and Grunn (1987), named CRISP (CRitical State Program). The program uses the finite element technique and allows predictions to be made of ground deformations using critical state theories. Some modifications are made on the main finite element computer program (CRISP) to obtain the present computer program (Mod-CRISP) in order to achieve the computations needed in the present study. These include the addition of eight-noded quadrilateral isoparametric consolidation element with 16-d.o.f. and additional 4-d.o.f. on corner nodes, namely for excess pore water pressure and the addition of thin-layer interface element developed by Desai et al. (1984).

The results of modes of motion, such as rotation about the bottom and about the top and free translation are compared with the method proposed by Dubrova in 1963 (Harr, 1966; Das, 2007), and both results with that of Coulomb method. The comparison is carried out using different values of soil friction angle ($\phi$) and friction angle between the wall and the backfill soil ($\delta$).

Figures 6 - 9 show the earth pressure distribution behind the retaining wall for the case when $\delta = 0$ (free relative movements between the wall and the soil). It is seen that Dubrova’s method gives greater values than Coulomb equation, for all modes of wall movements. Whereas, the results obtained from the finite element analysis indicate that the stress distribution is more or less equal to Coulomb equation and ranging at about
Figure 7. Active earth pressure against retaining wall for $\phi = 30^\circ$ and $\delta = 0.0^\circ$.

Figure 8. Active earth pressure against retaining wall for $\phi = 35^\circ$ and $\delta = 0.0^\circ$.

90% of the depth for $\phi = 25^\circ$ and 60% for $\phi = 40^\circ$. Below that depth, the pressure distribution becomes much greater than that obtained by Coulomb equation.

When $\delta > 0$, Figures 10 - 17, the pressure distribution behind the retaining wall (for the case of bottom rotation and free translation) obtained from the finite element analysis is less than that of Coulomb equation to a certain depth ranging from about 85% for $\phi = 25^\circ$ to about 50% for $\phi = 40^\circ$. Below this depth, the pressures are much greater than Coulomb's, whereas, the pressure distribution for the wall rotating about its top is always greater than that by Coulomb equation.

From these figures, the following points can be
Figure 9. Active earth pressure against retaining wall for $\phi = 40^\circ$ and $\delta = 0.0^\circ$.

Figure 10. Active earth pressure against retaining wall for $\phi = 25^\circ$ and $\delta = 0.5\phi$.

recorded:

1. The pressure distribution obtained from Dubrova’s method is greater than that of Coulomb equation, while, the finite element analysis gives results that are closer to Coulomb equation.
2. The pressure distribution due to top rotation is greater than that by Coulomb equation, whereas, for bottom
rotation, it is very close to that by Coulomb equation. This is because Coulomb’s equation is originally based on the assumption of walls rotating about their toe (Harr, 1966).

3. The wall friction angle, $\delta$, (when it is greater than zero) will not affect considerably the values of pressure distribution behind the retaining wall.

4. The coefficient of the active earth pressure and the point of application of the resultant active thrust depend on the modes of the wall movement. Dubrova’s analytical solutions are able to express the different nonlinear distributions of the active stress for various modes. However, the resultant active thrust given by that method
Figure 13. Active earth pressure against retaining wall for $\phi = 40^\circ$ and $\delta = 0.67\phi$.

Figure 14. Active earth pressure against retaining wall for $\phi = 25^\circ$ and $\delta = 0.75\phi$.

coincides with Coulomb’s solution (that is, irrespective of the wall displacement modes), but differs from that predicted by the finite element method. This conclusion is similar to that found by Hazarika and Matsuzawa (1996). 5. The pressure distribution at lower portions of the wall obtained from the finite element analysis is always greater than that of Coulomb’s equation. This may be related to the incorporation of the compressibility of the foundation in the analysis and the non-uniform loading of the wall to the underlying soil. This conclusion is similar
to that found by Clough and Duncan (1971).

CONCLUSIONS

1. The value of wall friction angle (when greater than zero) will not affect the values of pressure distribution behind the retaining wall as the wall friction angle affects mainly the shear stresses between the retained soil and the wall.
2. The active earth pressure obtained by the finite element method is close to Coulomb's solution in case of bottom wall rotation, whereas, in case of top wall rotation, the obtained earth pressure is greater than that of
Coulomb solution. This is mainly because Coulomb’s equation is originally based on wall rotating about its base (failure mechanism).

3. The nature of wall movements, whether translation or rotation, has a considerable effect on the distribution of earth pressure. Greater movement in any part of the wall away from the backfill reduces the earth pressure, and movement towards the soil increases the earth pressure.

REFERENCES


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