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# Estimation of constant elasticity of substitution (CES) production function with capital and labour inputs of agri-food firms in Tanzania

**Joel Chongela*, Vasudev Nandala and Suhasini Korabandi**

Department of Agricultural Economics, Acharya N. G. Ranga Agricultural University, Hyderabad 500-030, Andhra Pradesh, India.

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**This paper focused on estimation of the impact of capital and labour inputs to the gross output of agri-food products using constant elasticity of substitution (CES) production function in Tanzanian context. The CES production functional form was estimated by maximum likelihood estimator (MLE). The empirical results revealed that capital and labour inputs have impacted the gross output of the agri-food products by 97, 96, 94, 78 and 61%, for bakery, grain mill, fruits and vegetables, oil seeds and fats, meat and dairy products, respectively. However, interventions in terms of economic policy instruments such as capital goods and skilled labour are highly encouraged for the worthwhile economies of the agri-food firms' in the country.**

**Key words:** Agri-food, capital, constant elasticity of substitution (CES), gross output, labour, maximum likelihood estimator (MLE), Tanzania.

## INTRODUCTION

In Tanzania agri-food industry can be divided into the processing and retail sectors. The processing sector includes manufacturing and packaging of food. The retail sector includes distribution and selling of food to final consumers. However, food retail is dominated by mini-supermarkets, small groceries and fresh markets, although the number of supermarkets is on the raise. While in 2002, there were only four supermarkets in the entire country, accounting for 21% of the vegetable and 7% of the fruit supply. The sector is dominated by the South African firm Shoprite, along with the local Imalaseko and Shopper's Plaza. The largest agri-food industries in Tanzania are brewing, milling, baking, confectionery (sweets), animal and vegetable oil, sugar, dairy products, fruits and vegetables, soft drinks, fish and meat processing. Ethyl alcohol distillation, sprit blending, wines, bottling of natural spring and mineral waters.

A couple of studies have estimated constant elasticity of substitution (CES) production function with capital and labour inputs in USA, European Union, Asian and African economies. Some of these studies include: Antras (2004) examined the econometric estimation of production function using data stemming from the US economy. Papageorgiou and Saam (2005) estimated Two-Level CES Production Technology using the Solow and Diamond Growth Models in USA and Germany economies. Klump et al. (2007) examined the econometric estimation of production function using data stemming for the US economy. Sato and Morita (2009) estimated the growth rates of capital and labour efficiencies for the Japanese and US economies. Batisani and Yarnal (2010), estimated elasticity of capital

*Corresponding author. E-mail: chongela@yahoo.com.

land substitution in housing construction: Implications for smart growth policy and affordable housing in Gaborone, Botswana using regression analyses.

Noda and Kyo (2011) examined the technical changes in Taiwan and South Korea at the macroeconomic level using Bayesian Estimation of the CES Production Function with Labour and Capital-Augmenting Technical Change.

However, few studies have employed the CES production function approach to study the impact of capital and labour inputs on the gross output of agri-food products using time series data, particularly in the Tanzanian economy. Hence, this paper estimated impact of capital and labour inputs to gross output of agri-food products by employing CES production function. The CES was estimated by Maximum Likelihood method. The agri-food products included in the study were meat and dairy, fruits and vegetables, oil seeds and fats, grain mill, and bakery products. These agri-food products were studied due to availability of its time series data on gross output, capital and labour investments for 30 years.

The paper focused on bridging the knowledge gap in the scientific research of agri-food production using CES functional form. On the other hand the policy implications pointed out by this paper are useful for agri-food producing firms' particularly in Tanzanian economy.

## MATERIALS AND METHODS

### Types of data

This paper used time series data set collected annually from Annual Survey of Industrial Production and Performance conducted by National Bureau of Statistics (NBS) in collaboration with Ministry of Industry, Trade and Marketing (MITM), and Confederation of Tanzania Industries (CTI) from 1981 to 2010. The time series data collected were gross output, capital and labour investments of agri-food products include: Meat and dairy, fruits and vegetables, oil seeds and fats, grain mill, and bakery products, respectively.

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### Sampling method

A total of 210 agri-food industries involved in this paper were sampled by systematic sampling technique from 730 industrial establishments operating in Tanzania Mainland from 1981 to 2010.

### Model specification

### Constant Elasticity of Substitution (CES)

The Constant Elasticity of Substitution (CES) production functional form pioneered by Solow (1956) and later made popular by Arrow et al. (1961) consists of capital and labour input cost deliveries employed to measure output / input relationships of gross output of agri-food products using capital and labour inputs invested in the production process of agri-food products to meet the consumers' demand of agri-food in Tanzania. However, CES production function was chosen because of its flexibility coming from the substitution parameters and the inclusion of an additional input which makes it an attractive choice for many applications in economic theory and empirical studies.

The CES production function can be expressed as:

$$Y_t = A\left[\delta K_t^{-\beta} + (1-\delta)L_t^{-\beta}\right]^{\frac{-1}{\beta}} + \mu_t \quad (1)$$

Where: $Y_t$ = Gross output (TZS), $K_t$ = Capital cost (TZS), $L_t$ = Labour cost (TZS); A = Scale parameter, δ = Capital share parameter to gross output (0 < δ < 1), 1 – δ = Labour share parameter to gross output, β = Substitution parameter (β ≥ -1); $\mu_t$ = Stochastic error term, and TZS = Tanzanian Shilling.

Equation (1) can be re-written as Equation (2) ass follows:

$$y_t = a_t x_{1t} + b_t x_{2t} + \mu_t \quad (2)$$

Where: yt = gross output (TZS), $x_{1t}$ = capital cost (TZS), $x_{2t}$ = labour cost (TZS), $a_t$ and $b_t$ = coefficients for capital and labour. A = 1, $y_t = (-1/\beta)^{-\beta}$, $x_{1t} = -1/\beta K_i^{-\beta}$, $x_{2t} = -1/\beta\ L^{-\beta}$, $a_t = \delta^{-\beta}$, $b_t = (1-\delta)^{-\beta}$.

## RESULTS

### Estimation of Constant Elasticity of Substitution (CES)

The empirical results for estimation of CES using Maximum Likelihood method for meat and dairy, fruits and vegetables, oil seeds and fats, grain mill, and bakery products are presented in Table 1.

### Trends of growth of gross output, capital and labour costs of agri-food producing firms from 1981 to 2010

Trends of growth of gross output, capital and labour costs of meat and dairy, fruits and vegetables, oil seeds and fats, grain mill, and bakery products from 1981 to 2010 are presented in Figures 1 to 5.

## DISCUSSION

### Agri-food products maximum likelihood estimates

#### *Model summary*

The empirical results showed that 97, 96, 94, 78 and 61% of share of variation in the gross output of bakery, grain mills, fruits and vegetables, oil seeds and fats, meat and dairy products, respectively are explained by variation in the amount of capital and labour invested on the production process of agri-food products. This implies that investment on capital and labour is expected to increase significantly well the gross output of bakery, grain mills, fruits and vegetables, oil seeds and fats, meat and dairy products by 97, 96, 94, 78 and 61%, respectively under *ceteris paribus* assumption (Table 1).

### Parameter estimates

The empirical results showed that there is a positive

**Table 1.** Maximum Likelihood Parameter estimates of gross output, capital and labour costs of agri-food products.

| Product name | Variable | Parameter | Adj $R^2$ | Coefficient | Std error | t-Value | P-value |
|---|---|---|---|---|---|---|---|
| | Gross output | $\tilde{\sigma}_t^{\,2}$ | | -2.08E+12 | 2.562E+12 | -0.811 | 0.425 |
| Meat and dairy | Capital cost | $\tilde{a}_t$ | 0.61 | 9.000 | 3.621 | 2.482 | 0.0001***** |
| | Labour cost | $\tilde{b}_t$ | | 182.128 | 66.897 | 2.723 | 0.0001***** |
| | Gross output | $\tilde{\sigma}_t^{\,2}$ | | 5.371E+13 | 5.264E+13 | 1.020 | 0.316 |
| Fruits and vegetables | Capital cost | $\tilde{a}_t$ | 0.94 | 1.742 | 0.350 | 4.970 | 0.0001* |
| | Labour cost | $\tilde{b}_t$ | | 238.406 | 55.799 | 4.273 | 0.0001* |
| | Gross output | $\tilde{\sigma}_t^{\,2}$ | | 1.7668E+14 | 2.7292E+14 | 0.64735 | 0.522 |
| Oil seeds and fats | Capital cost | $\tilde{a}_t$ | 0.78 | 0.55 | 0.827 | 0.664 | 0.512 |
| | Labour cost | $\tilde{b}_t$ | | 521.362 | 73.501 | 7.093 | 0.0001* |
| | Gross output | $\tilde{\sigma}_t^{\,2}$ | | 3.9607E+13 | 4.03E+13 | 0.9829 | 0.334 |
| Grain mill | Capital cost | $\tilde{a}_t$ | 0.96 | 1.663 | 0.494 | 3.367 | 0 .002* |
| | Labour cost | $\tilde{b}_t$ | | 310.418 | 37.898 | 8.191 | 0.0001* |
| | Gross output | $\tilde{\sigma}_t^{\,2}$ | | -2.641E+20 | 1.9845E+20 | -1.331 | 0.194 |
| Bakery | Capital cost | $\tilde{a}_t$ | 0.97 | 1.179 | 0.1001 | 11.778 | 0.0001* |
| | Labour cost | $\tilde{b}_t$ | | 53.363 | 6.456 | 8.265 | 0.0001* |

*****Implies statistically significant at 5% level of significance.

relationship between gross output, capital and labour invested on various agri-food products. If investment on capital in meat and dairy products rose by 1 TZS, a gross output is expected to increase by 9 TZS per year under *ceteris paribus* assumption (Table 1). This implies that for 1 TZS invested per year as capital in the production process of meat and dairy products would pay a gross output of 9 TZS per year (Table 1). Conversely, empirical results showed that if investment on labour increases by 1 TZS the gross output is expected to increase by 182 TZS per year (Table 1) under *ceteris paribus* assumption. The empirical results imply that labour contributed significantly well to the gross output of meat and dairy products producing firms as compared to capital due to availability of cheap labour in the country. The empirical results of fruits and vegetable products showed that if the capital invested on fruit and vegetable products rose up by 1 TZS, the gross output is expected to increase by 2 TZS per year under *ceteris paribus* assumption. On the other hand if investment on labour increases by 1 TZS per year the gross output is expected to increase by 238 TZS per year under *ceteris paribus* assumption (Table 1).

The empirical findings of oil seeds and fats products showed that if the capital invested on oil seeds and fats increases by 1 TZS per year; the gross output is expected to increase by 0.55 TZS per year. This implies that 1 TZS invested as capital in the production process of oil seed and fat products is expected to yield a gross output of 55% per each TZS invested in the production process of oil seeds and fats annually under *ceteris paribus* (Table 1). On the other hand, if investment on labour increases by 1 TZS per year, the gross output is expected to increase by 521 TZS per year. The empirical results imply that labour contributed significantly well to the gross output of oil seeds and fats products producing firms at 5% level of significance (Table 1).

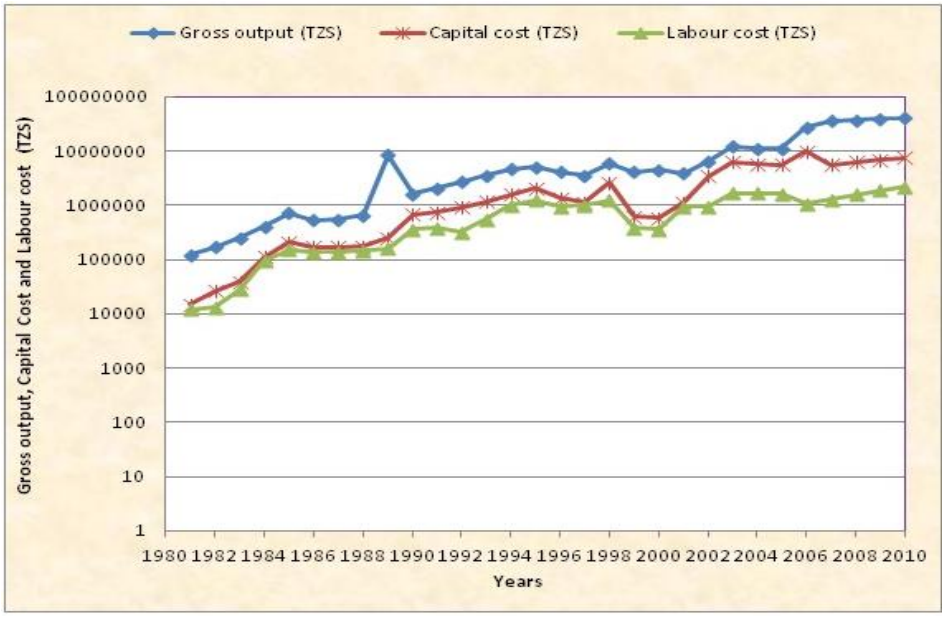


**Figure 1.** Tanzania mainland: Trends of growth output, capital and labour costs of meat and dairy products from 1981 to 2010 (Million, TZS).

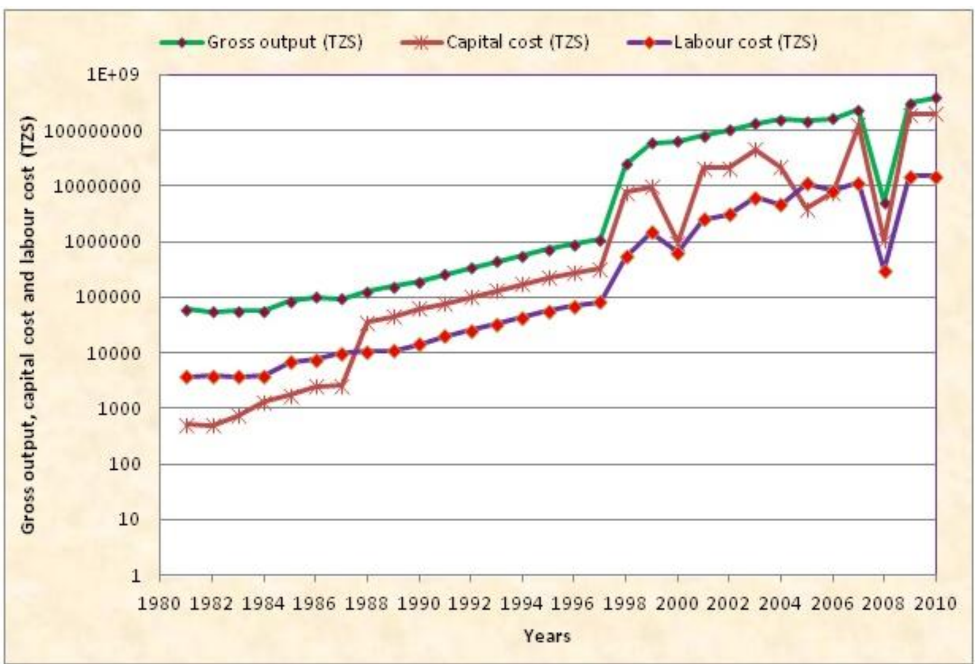


**Figure 2.** Tanzania mainland: Trends of growth output, capital and labour costs of fruits and vegetable products from 1981 to 2010 (Million, TZS).

The empirical results of grain mill products showed that if investment on capital increases by one TZS per year, the gross output is expected to increase by 2 TZS per year. This implies that for one TZS invested as capital per year in the production process of grain mill products is expected to yield a gross output of 2 TZS per year under ceteris paribus assumption. On the other hand if investment on labour increases by one TZS per year, the gross output is expected to increase by 310 TZS and 418 cents per year under *ceteris paribus assumption* (Table 1).

The empirical findings revealed that if capital invested on bakery products rose by one TZS, on average the

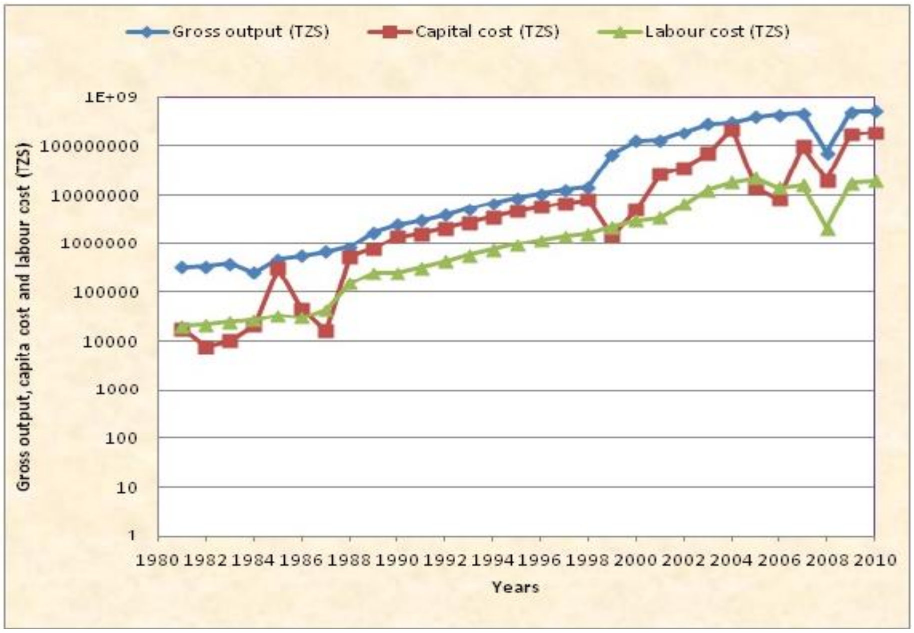


**Figure 3.** Tanzania mainland: Trends of growth output, capital and labour costs of oil seed and fat food products from 1981 to 2010 (Million, TZS).

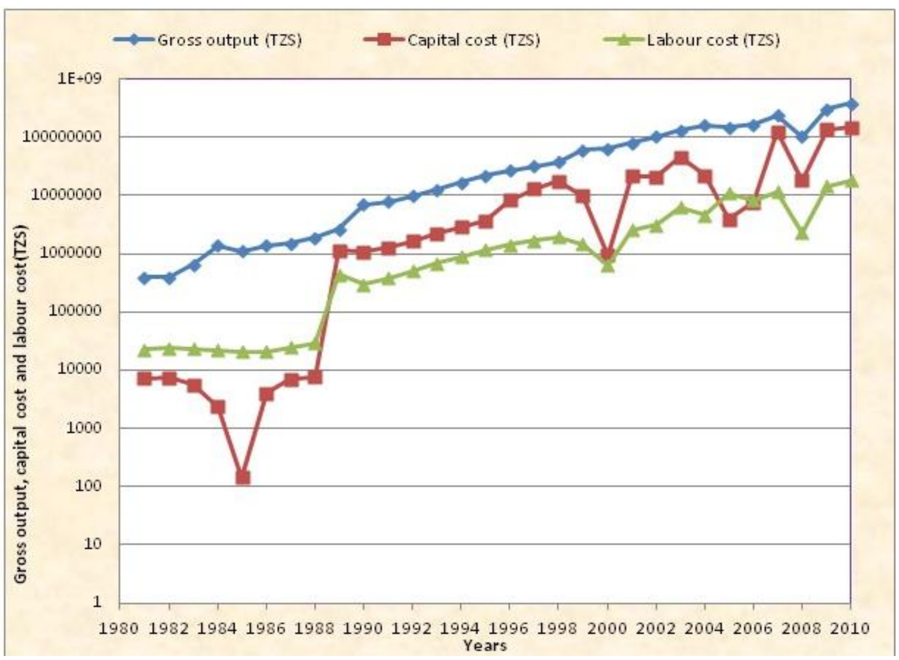


**Figure 4.** Tanzania mainland: Trends of growth output, capital and labour costs of grain mill products products from 1981 to 2010 (Million, TZS).

gross output would increase by 1.179 TZS per year under *ceteris paribus* assumption. Also, the empirical results showed that if investment on labour for bakery products increases by one TZS per year, on average gross output would increase by 53 TZS per year under *ceteris paribus* (Table 1).

**Trends of growth of gross output, capital and labour costs of agri-food products industries from 1981 to 2010**

In Figures 1 to 5 empirical results showed that the trend of growth of gross output increases as time goes on due

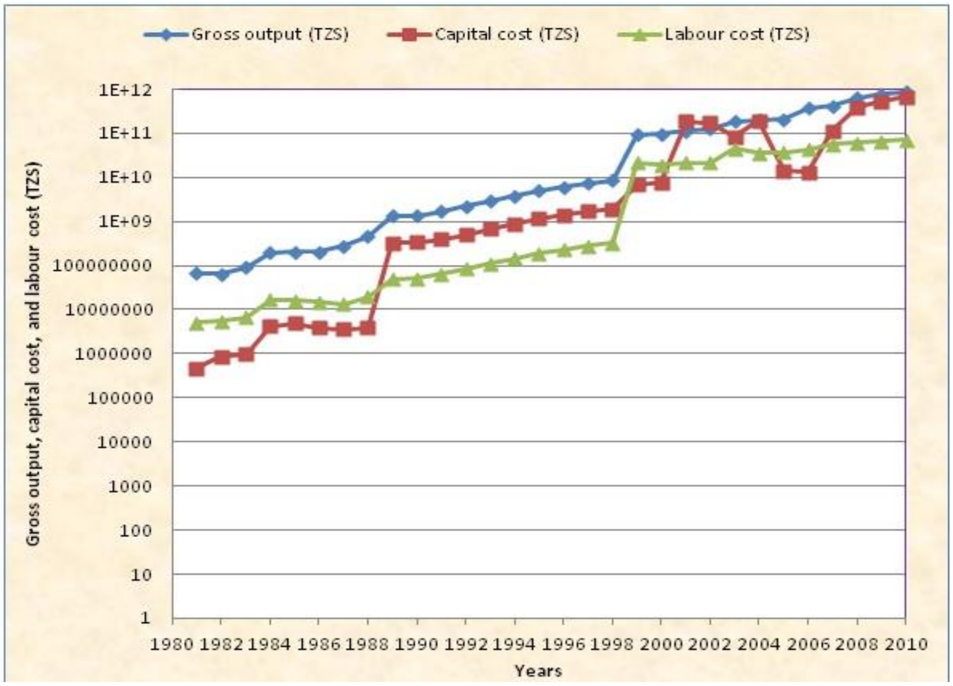


**Figure 5.** Tanzania mainland: Trends of growth output, capital and labour costs of bakery products from 1981 to 2010 (Million, TZS).

to increase in investment on capital followed by labour. The empirical results showed that the investment on capital and labour in the 1980's was less due to infant stage of agri-food firms. However, in the 1990's investment on capital and labour started to increase due to increase in the economies of scale of agri-food producing firms resulted into much more returns. In 2000's investment on capital increased much more due to increase in consumer demand of agri-food products attributed to increase in disposable incomes of consumers, population and urbanization. This led to speed up the inflow of FDI of Multinational food firms like Shoprite to start operating in the country. The investment on labour in 1990's and 2000's was increasing but not as much as capital increase; less investment in labour was driven by availability of cheap labour in developing countries like Tanzania.

In 2008 there was a depression in gross output which was driven by less capital investment due to financial and economic recessions in US and Europe this affected the inflow of Foreign Direct Investment (FDI) to developing countries like Tanzania. However, in 2009 to 2010 the gross output started to rise due to increase in capital and labour investments in agri-food products attributed by recovery of the economy of agri-food firms (Figures 1 to 5).

## CONCLUSSION AND POLICY IMPLICATIONS

According to the empirical findings of the study capital and labour inputs have impacted the gross output of agri-food producing firms due to the fact that capital and labour inputs are the main driving forces of the gross output of agri-food producing firms. Capital input has impacted much more on the gross output of meat and dairy, fruits and vegetables, grain mill and bakery products, respectively. The labour input has impacted much more on the gross output of oil seeds and fats, grain mill, fruits and vegetables, meat and dairy products, respectively. However, the study suggested that policy instruments should focus much more on skilled labour and capital goods for the worth while economies of the agri-food producing firms'. Moreover, skilled labour policies should focus much more on capacity building of labour as the major contributors to the agri-food firms' economies. Furthermore, capital goods policies should focus much more on manufacturing of food processing machineries within the country rather than importing from abroad so that to lower investment costs of agri-food producing firms.

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**Parameterization of CES**

The CES production function was estimated using Maximum Likelihood method. The likelihood function (lf) equation formed is as shown in Equation 3. Assuming that the model $y_t = a_t x_{1t} + b_t x_{2t} + \mu_t$, whereby $y_t$ is normally and independent distributed $y_t \sim N[a_t x_{1t} + b_t x_{2t}, \sigma^2]$ with mean = $a_t x_{1t} + b_t x_{2t}$ and variance = $\sigma^2$.

$$lf = f\left(\frac{y_t}{a_t x_{1t} + b_t x_{2t}, \sigma^2}\right). \quad (1)$$

$$f(y_t) = \frac{1}{\delta^n\left(\sqrt{2\pi}\right)^n} \exp\left[-\frac{1}{2}\Sigma \frac{(y_t - a_t x_{1t} - b_t x_{2t})^2}{\sigma^2}\right] \quad (2)$$

$$lf\left(a_t, b_t, \sigma^2\right) = \frac{1}{\sigma^n\left(\sqrt{2\pi}\right)^n} \exp\left[-\frac{1}{2}\Sigma \frac{(y_t - a_t x_{1t} - b_t x_{2t})^2}{\sigma^2}\right] \quad (3)$$

$$Inlf = -nIn\sigma - \frac{n}{2} In(2\pi) - \frac{1}{2}\Sigma \frac{(y_t - a_t x_{1t} - b_t x_{2t})^2}{\sigma^2} \quad (4)$$

$$Inlf = -\frac{n}{2} In\sigma^2 - \frac{n}{2} In(2\pi) - \frac{1}{2}\Sigma \frac{(y_t - a_t x_{1t} - b_t x_{2t})^2}{\sigma^2} \quad (5)$$

Differentiate equation (5) with respect to $a_t$, $b_t$, $\sigma^2$

$$\frac{\delta Inlf}{\delta a_t} = -\frac{1}{\sigma^2}\Sigma(y_t - a_t x_{1t} - b_t x_{2t})(-x_{1t}). \quad (6)$$

$$\frac{\delta inlf}{\delta b_t} = -\frac{1}{\sigma^2}\Sigma(y_t - a_t x_{1t} - b_t x_{2t})(-x_{2t}) \quad (7)$$

$$\frac{\delta Inlf}{\delta \sigma^2} = -\frac{n}{2\sigma^2} + \frac{1}{2\sigma^4}\Sigma(y_t - a_t x_{1t} - b_t x_{2t})^2 \quad (8)$$

Let Equations (6) to (8) equal to zero (1st order partial derivatives) and letting $\tilde{a}_t$, $\tilde{b}_t$ and $\tilde{\sigma}^2$ indicate maximum likelihood estimators to obtain the following equations:

$$-\frac{1}{\tilde{\sigma}^2}\Sigma(y_t - \tilde{a}_t x_{1t} - \tilde{b}_t x_{2t})(-x_{1t}) = 0 \quad (9)$$

$$-\frac{1}{\tilde{\sigma}^2}\Sigma(y_t - \tilde{a}_t x_{1t} - \tilde{b}_t x_{2t})(-x_{2t}) = 0. \quad (10)$$

$$-\frac{n}{2\tilde{\sigma}^2} + \frac{1}{2\tilde{\sigma}^4}\Sigma(y_t - \tilde{a}_t x_{1t} - \tilde{b}_t x_{2t})^2 = 0 \quad (11)$$

After simplification Equations (9) and (10) become:

$$\sum y_t = n\tilde{a}_t x_{1t} + \tilde{b}_t \sum x_{2t} \quad (12)$$

$$\sum y_t x_{1t} x_{2t} = \tilde{a}_t \sum x_{1t} + \tilde{b}_t \sum x^2{}_{2t} \quad (13)$$

By substituting maximum likelihood estimators into Equation (11) and after simplification the following equation was obtained:

$$\tilde{\sigma}^2 = \frac{1}{n}\Sigma(y_t - \tilde{a}_t x_{1t} - \tilde{b}_t x_{2t})^2 \quad (14)$$

Where: $\tilde{a}_t$, $\tilde{b}_t$ and $\tilde{\sigma}^2$ are maximum likelihood estimators.