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Analytical approaches for modeling tree crown volume in black wattle (*Acacia mearnsii* De Wild.) stands

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In this paper, four strategies were proposed for modeling tree crown volume using as independent variable stem variables, crown variables, combination of stem and crown variables, and stem volume. We used a dataset comprised of 170 trees from 12 temporary plots located in forest stands in southern Brazil. Models composed of stem variables presented weaker predictive ability. The best model contained crown variables, which explained 78.95% of observed variability. However, implementation of such model is bounded by its independent variables, which are not often measured in forest inventories. The model composed by diameter at breast height and crown length proved to be an adequate modeling approach. The predictive capability was kept by model $v_c = \beta_0 dbh^{\beta_1} c l^{\beta_2} + \varepsilon_i$, which is composed by most easily measured variable in a forest - diameter at breast height, also by the most easily acquirable crown variable - crown length. In our suggested model, estimates of β_1 and β_2 are coefficients that convert volume of a regular geometric solid - RGS is dbh^2 times crown length) - into crown volume, whilst estimate of β_0 is an allometric constant.

Key words: Crown modeling, diameter at breast height, crown length, crown volume.

INTRODUCTION

Tree crowns are responsible for light interception, thus contribute to the regulation of individual growth, and stand yield (Burkhart and Tomé, 2012; Oliver and Larson, 1996; Cluzeau et al., 1995). Tree crown is an important variable to elucidate what occurs in forest stands and its

dynamics and, thus, a great effort has been devoted towards quantification of tree crowns (Burkhart and Tomé, 2012), as well as to its modeling (Godin, 2000). Therefore, crown volume turns to be an adequate variable to assess forest dynamics and to improve

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reliability of growth and yield models (Bragg, 2001). Several authors have been exploring three main methods of estimating crown related variables, particularly: 1) approximations to geometric shapes; 2) modern remote sensing techniques aided by computer procedures; and 3) regression methods.

With respect to geometric shapes approximations, Osborn (1962) estimated crown volume using volume of cones while Mawson et al. (1976) compared 15 different Euclidean geometric shapes and obtained crown volumes based on measurements of crown width, length and crown radii. Tucker et al. (1993) estimated crown volume by means of irregular pyramids based on eight measured crown radii. Despite their pioneering approximations of geometric shapes to crown shape and volume, nowadays these techniques are believed to be too restrictive for estimating crown variables because crowns do not necessarily assume geometrical shapes (Crecente-Campo et al., 2009). Due constraints and assumptions of using these shapes, such approximations started to be substituted by more flexible and sophisticated models. Smith (1994) used a distance-dependent individual-tree model to calculate crown shape of Loblolly pine (*Pinus taeda*) in relation to a tree's competitor's crowns. Dubrasich et al. (1997) fitted crown area, taper, width and length models of mixed conifer and mixed conifer-hardwood stands. Hann (1999) proposed adjustable models to predict crown profile for Douglas-fir stand-grown trees by using measurements or predictions of largest crown width. Bragg (2001) developed nonlinear models for crown width estimation of 24 species based on tree diameter at breast height and on local basal area. Bechtold et al. (2002) pointed that regression models were a better alternative for estimating mean crown diameter of hardwoods relatively to field measurements and ocular estimates. Gill et al. (2000) developed individual tree crown radius models for several species to predict canopy cover. Crecente-Campo et al. (2009) used both geometrical shapes and equations to model crown profile. Rupšys (2015) developed stochastic models of crown widths. Power et al. (2012) proposed equations of crown length, profile, shape and surface area of black spruce (*Picea mariana*) and white spruce (*Picea glauca*) to characterize crown characteristics of these species. Sattler and LeMay (2011) proposed a simultaneous system of nonlinear equations to predict crown length and crown radius in structurally complex stands.

Recently, remote sensing methods provide competitive approaches for acquiring not only faster measurements than previous methods, but also accurate estimates of crown (Strîmbu and Strîmbu, 2015; Hu et al., 2014; Eerikäinen, 2009; Næsset, 2002; Hyyppä et al., 2000; Næsset, 1997). However, modern estimation methods based on remotely sensed acquired data may not be affordable in many circumstances due to challenges presented by high computational power, big data,

logistics, data acquisition costs (Strîmbu and Strîmbu, 2015; Wulder et al. 2012) and software licenses (Sönmez, 2009; Zhang et al., 2007; Song et al., 2003). Therefore, traditional methods of estimation still show potential. The study of crowns is of considerable importance to assess a forest system. However, major part of studies have been focusing on modeling crown variables other than crown volume. Even though several crown volume studies were carried out, most kept estimating crown volume based on cones volume, cylinders, paraboloids, ellipsoids, hemispheres, or spheres (Villacorta et al., 2015; Fernández-Sarría et al., 2013; Leites et al., 2013; Velázquez-Martí et al., 2012; Pérez-Cruzado and Rodríguez-Soalleiro, 2011; Roberts et al., 2003; Montgomery and Chazdon, 2000; Van Pelt and North, 1996; Jack and Long, 1992; McPherson and Rowntree, 1988; Kuuluvainen, 1988).

Different from approximating volumes, Rautiainen and Stenberg (2005) and Rautiainen et al. (2008) fitted a curve of Lamé curve family to represent crown profile above maximum crown radius, and assumed a cylindrical form for crown below its maximum radius. Dubravac et al (2009) obtained crown volume assuming it to be a cylinder multiplied by a form factor. It is important to note that only one previous study regarding crown modeling of black wattle (*Acacia mearnsii*) was found (Sanquetta et al., 2015). In addition, this species is the third major cultivated species in Brazil, whose bark is the main source of tannin in the world and whose crown biomass has recently gained interest for energy purposes as raw material for production of pellets (Dunlop, 2005).

This paper proposed four approaches for modeling black wattle crown volume by means of acquired stem and crown related variables that could potentially present a biological and mathematical sense to explain crown volume. Our foremost objective was to suggest an operational and reliable crown volume model.

MATERIAL AND METHODS

Field sampling and data

Data was gathered in 2014, during June and July in black wattle stands in the state of Rio Grande do Sul, Brazil. More specifically, this dataset was collected in Cristal, Encruzilhada do Sul and Piratini counties, these are the regions where major part of black wattle commercial stands are found in the country (Figure 1).

In each stand, four 10 m diameter circular plots (covering 78.54 m²) were randomly located, totaling 12 plots and 170 trees. Sampled stand ages in Cristal, Encruzilhada do Sul and Piratini averaged between 9 and 11 years old - near the end of black wattle's rotation age. In each plot, all trees were felled and the following variables were measured: diameter at breast height - *dbh* (cm and m) measured at 1.30 m above the ground, total height - *ht* (m), crown diameter - *cd* (m), and crown length - *cl* (m), distance attributed to the distance from first branch at the base of the crown to tree tip (Figure 2). All crowns and stems were measured relative to their length so that we could obtain their volume (m³).

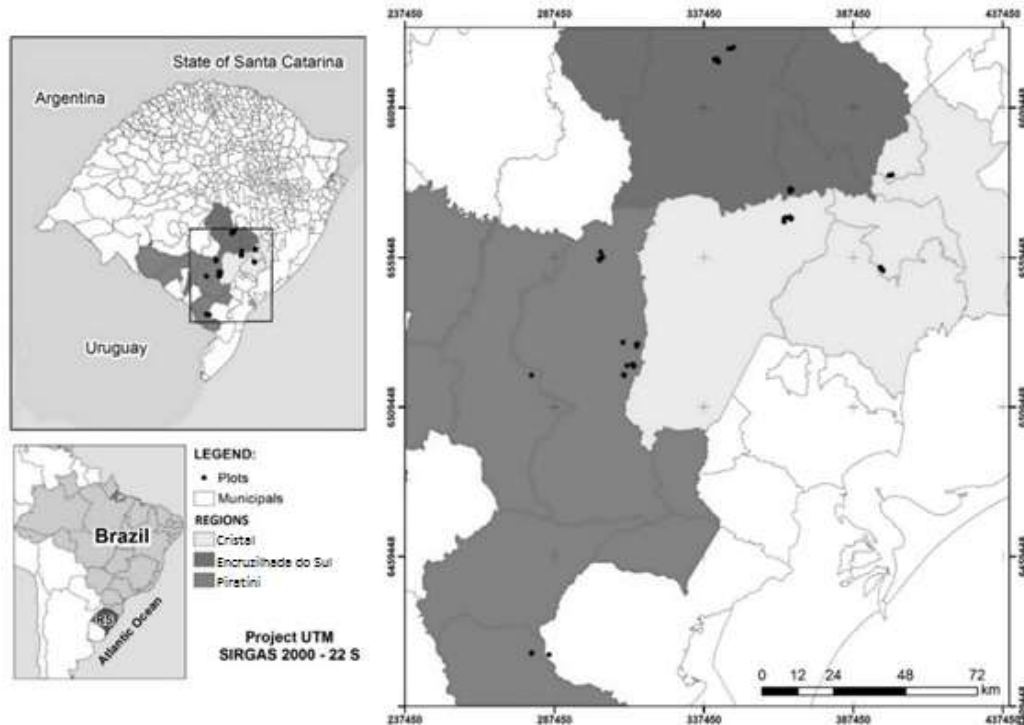


Figure 1. Map of sampled black wattle stands in Cristal, Encruzilhada do Sul and Piratini counties in the state Rio Grande do Sul, Brazil counties.

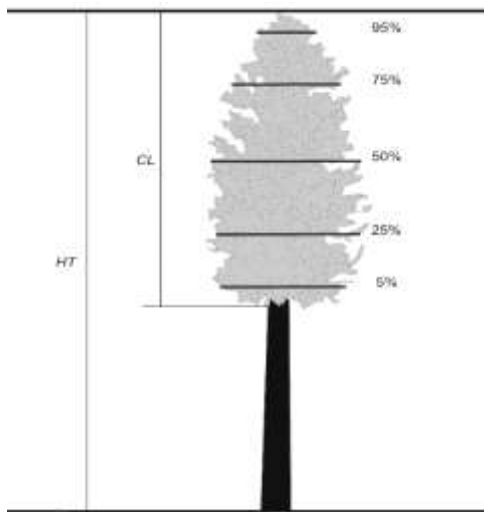


Figure 2. Scheme of how crowns were dissected to obtain volumes of intermediate sections, as well as variables directly measured on the tree, except *dbh*.

In this study, crown volume was considered as a group of components represented by a set of truncated cones and was represented globally, as defined in a review of Godin (2000). Therefore, crown volume was measured based on Huber’s method and it was dissected applying Hohenadl’s method. The measurements of crown diameter were taken orthogonally with a tape to

obtain an average of measured diameters in a single section. These measurements were taken at positions 5, 25, 50, 75 and 95% of crown length (Figure 2). The measurements were taken after tree felling (Hann, 1999; Biging and Wensel, 1990), since there were no major crown deformations.

The volume of top (95 to 100%) and base (0 to 5%) crown sections were obtained separately and will be further explained. The volume of *i* intermediate sections (for example, sections 5 to 25%, 25 to 50%, 50 to 75% and 75 to 95% of crown length) were calculated by the following expression:

$$v_{sc} = \sum_{i=1}^n \frac{\pi}{4} d_i^2 l_i \tag{1}$$

where v_{sc} is the volume of intermediate sections of the crown (m^3), d is the average of orthogonal crown diameters (m), and l is the length of section i (m). Base and top sections volumes of the crown were obtained using the disc method and the volume of a solid generated by rotating the area bounded by the crown axis of symmetry (y-axis), crown length and the function of crown section profile. Volumes of top and base sections were calculated as a concave paraboloid, and were generalized as:

$$v_p = \int_0^l \pi x^2 dy \tag{2}$$

where v_p is the volume of the top or base section of the crown (m^3) calculated separately, l is the section length (m), and x is the maximum radii of the section (m). These sections were centered in a Cartesian plane with center on O (0, 0), and symmetry of

parabola is given by the y-axis. The radius measurements were considered half of average diameter of each section and were used as coordinates over the plane. Thus, we could express profile from of top and base sections as a quadratic equation. These coordinates were substituted in the general parabola form function. Crown top and base profile sections were expressed by equation 3:

$$y = a x^2 \quad (3)$$

where y is the crown profile equation (m), a is the coefficient associated with de changes of radius along the section length, and x is crown radius along the length of the section (m). Since discs were integrated along the sections' length, radius had to be expressed in terms of section length (y):

$$x = \sqrt{\frac{y}{a}} \quad (4)$$

After integrating the found expression for each tree top and base sections profiles, volumes were obtained by the following expression:

$$v_p = \pi \left[\frac{y^2}{2a} \right]_0^h \quad (5)$$

Finally, crown total volume was calculated by:

$$v_c = v_p + v_{sc} + v_b \quad (6)$$

where v_c is tree crown total volume (m³), v_p is volume of crown base or top sections (m³) calculated separately and v_{sc} is volume of a crown intermediate sections (m³). Stem volume (v_s) was obtained using Hohenadl's method and it was measured according to Huber's method. The measurements were taken with a diameter tape along the stem at positions 5%, 15%, 25%, 35%, 45%, 55%, 65%, 75%, 85% and 95% of the total height. The volume of n stem sections was calculated using the following expression:

$$v_s = \sum_{i=1}^n \frac{\pi}{4} d_i^2 l_i \quad (7)$$

where v_s is the volume of sections of stem (m³), d is the mean diameter of section i (m), and l is the length of section i (m). The data set utilized in this study is summarized in Table 1.

Strategic crown volume models

In this paper, we evaluated four different strategies to accurately estimate crown volumes from easily measured variables. These strategies were based on different approaches of crown volume model using stem and crown as independent variables. A Pearson correlation matrix for all independent variables against each other and against crown volume was built to assess relationships. Independent variables were also plotted against tree crown volume aiming to graphically assess explanatory relationships. Our models were adapted to each modeling approach based upon observed relationships.

Strategy 1: stem variables

Initially, we estimated crown volume as a function of diameter at

breast height (dbh), and total height (ht). Four models were proposed and are presented in (8), (9), (10) and (11).

$$v_c = \beta_0 (dbh^2)^{\beta_1} + \varepsilon_i \quad (8)$$

$$v_c = \beta_0 (dbh)^{\beta_1} (ht)^{\beta_2} + \varepsilon_i \quad (9)$$

$$v_c = \beta_0 (dbh^2 ht)^{\beta_1} + \varepsilon_i \quad (10)$$

$$v_c = \beta_0 (1/dbh)^{\beta_1} ht^{\beta_2} + \varepsilon_i \quad (11)$$

Where v_c is crown volume (m³), dbh is diameter at breast height (cm), ht is tree total height (m), β_i are the coefficients of the models and ε_i is the associated error (m³).

Strategy 2: crown variables

As an alternative strategy, we proposed the same aforementioned models, however, using the largest measured crown diameter (cd , m) and crown length (cl , m) at this time. Rautiainen et al. (2008) recommended using crown length and maximum crown radius, arguing that these variables were the only ones required to model crown shape. However, in this study, as our objective was to estimate volume, we focused on using crown maximum diameter instead, once it represents a full unidimensional measure of the crown. The models are presented in (12), (13), (14) and (15).

$$v_c = \beta_0 (cd^2)^{\beta_1} + \varepsilon_i \quad (12)$$

$$v_c = \beta_0 (cd)^{\beta_1} (cl)^{\beta_2} + \varepsilon_i \quad (13)$$

$$v_c = \beta_0 (cd^2 cl)^{\beta_1} + \varepsilon_i \quad (14)$$

$$v_c = \beta_0 (1/cd)^{\beta_1} (cl)^{\beta_2} + \varepsilon_i \quad (15)$$

Where v_c is crown volume (m³), cd is crown diameter (m), cl is crown length (m), β_i are the coefficients of the models and ε_i is the associated error (m³).

Strategy 3: stem and crown variables

Our third approach uses stem and crown variables (i.g. dbh and cl) to model crown volume. The fitted models were the same ones as in previous strategies, except for the model which has only diameter as independent variable - model (8) and (12). Models are listed in (16), (17) and (18).

$$v_c = \beta_0 (dbh)^{\beta_1} (cl)^{\beta_2} + \varepsilon_i \quad (16)$$

$$v_c = \beta_0 (dbh^2 cl)^{\beta_1} + \varepsilon_i \quad (17)$$

$$v_c = \beta_0 (1/dbh)^{\beta_1} (cl)^{\beta_2} + \varepsilon_i \quad (18)$$

Where v_c is crown volume (m³), dbh is diameter at breast height (m), cl is crown length (m), β_i are coefficients of the models and ε_i is the associated error (m³).

Strategy 4: stem volume expansion to crown volume

The last proposed alternative uses of estimates of stem volume (\hat{v}_s) for modeling crown volume. For this purpose, Schumacher-Hall model of stem volume (model 9) was fitted and its outputs were

Table 1. Summary of descriptive statistic of the data set used to model crown volume of black wattle stands in Southern Brazil (n=170; 12 sample plots).

Variable	Minimum	Maximum	Mean	Standard deviation	Mean error
<i>dbh</i> (cm)	3.8	23.6	12.9	3.88	0.30
<i>ht</i> (m)	7.7	21.9	16.5	2.92	0.22
<i>cl</i> (m)	0.8	15.6	7.4	2.86	0.22
v_s (m ³)	0.0055	0.4773	0.1250	0.08	0.01
<i>cd</i> (m)	0.7	5.6	2.3	0.87	0.07
v_c (m ³)	0.2097	150.3095	23.2775	22.22	1.70

Note: *dbh* is diameter at breast height (cm), *ht* is total height (m); *cl* is crown length (m), v_s is stem volume (m³), v_c is crown volume (m³).

used as independent variable in a simple entry crown volume model. The models are presented in (19) and (20).

$$v_s = \beta_0(dbh)^{\beta_1}(ht)^{\beta_2} + \varepsilon_i \quad (19)$$

$$v_c = \beta_0 + \beta_1 v_s + \varepsilon_i \quad (20)$$

Where v_s is stem volume (m³), *dbh* is diameter at breast height (cm), *ht* is tree total height (m), v_c is crown volume (m³), β_i are the coefficients of the models and ε_i is the associated error (m³).

Model fitting

All models were initially fitted using MODEL procedure (SAS Institute Inc., 2002). We have modeled the structural variance of residuals to obtain weights that afterwards were applied to fitted coefficients obtained by *Estimated Nonlinear Generalized Least Squares*, since model outputs resulted in heteroscedastic error terms (Parresol, 2001; Harvey, 1976).

Model comparison and selection

The resulting equations were assessed based on indicators of goodness of fit: adjusted coefficient of determination (R^2_{adj}) and root mean squared error (RSME). Graphical analyses of residuals accounting for model lack of fit was also performed by means of plots of observed versus predicted values, and dispersion and distribution of residuals, following recommendations of Steel et al. (1997). In addition, the White's test accounting for homogeneity of residual variance (White, 1980) and Durbin-Watson's test for independence of residuals (Durbin and Watson, 1950; 1951) were evaluated, as suggested by Greene (2011).

Statistical measures of fit are adequate for original data used, and effectiveness and validation of fitted equations can only be assessed with an independent dataset (Rawlings et al., 1998). Kozak and Kozak (2003) demonstrated that, with seven data sets, cross validation and double cross validation by splitting data rarely granted any additional information on regression models. Thus, due the size of this data set and lack of an additional one, no split data validation analysis was conducted.

In total, thirteen models were fitted: eight models for strategy 1 and 2, three models for strategy 3, and two models for strategy 4 (i.e. one model for estimating stem volume and another for crown volume). To reduce the extensiveness of this study, only the best model for each strategy was discussed. Best overall model was chosen after comparing the best models across proposed

strategies.

RESULTS

Strategic crown volume models

The correlation matrix created for all independent variables against each other showed that majority of independent variables were clearly correlated (Table 2). The relationships observed between these variables demonstrated that combinations of inputs should be carefully chosen, once several combinations were highly correlated and that was likely to produce biased regression models due multicollinearity effect. Regarding crown volume, most independent variables presented high capability of explaining the variability of dependent variable, except for *ht*, $1/dbh$, *cl* and $1/cd$, which presented lowest correlations. The observed relationships indicated that most of the chosen variables for this study presented potential to be effectively utilized for modeling crown volume.

When plotted against crown volume, all independent variables proposed in this paper have presented interesting graphical relationships (Figure 3). Crown variables were the most strongly related variables to crown volume. However, even though stem variables did not present relationships as strong as crown related variables, observed behaviors between stem variables, transformed stem variables and combination of stem and crown variables indicated that reasonable equations were likely to result from models based on such variables.

Strategy 1: stem variables

Models (9) and (11) revealed no statistical significance for both estimates of β_0 and β_2 (Table 3). In model (10) the β_0 estimate was not significant as well. These models presented highest adjusted coefficients of determination and lower RMSE's relatively to model (8). However, when these were refit, no improvements were produced and

Table 2. Pearson correlation matrix for all independent variables and crown volume of black wattle commercial stands in Southern Brazil.

Variable	<i>dbh</i>	<i>ht</i>	<i>dbh</i> ²	<i>dbh</i> ² <i>ht</i>	1/ <i>dbh</i>	<i>cd</i>	<i>cl</i>	<i>cd</i> ²	1/ <i>cd</i>	<i>cd</i> ² <i>cl</i>	<i>dbh</i> ² <i>cl</i>	<i>v</i> _s	<i>v</i> _c
<i>dbh</i>	1												
<i>ht</i>	0.87	1											
<i>dbh</i> ²	0.98	0.80	1										
<i>dbh</i> ² <i>ht</i>	0.97	0.81	0.99	1									
1/ <i>dbh</i>	-0.91	-0.90	-0.82	-0.80	1								
<i>cd</i>	0.77	0.61	0.79	0.79	-0.64	1							
<i>cl</i>	0.29	0.32	0.26	0.26	-0.31	0.28	1						
<i>cd</i> ²	0.72	0.53	0.76	0.77	-0.55	0.97	0.25	1					
1/ <i>cd</i>	-0.73	-0.68	-0.69	-0.67	0.70	-0.87	-0.33	-0.73	1				
<i>cd</i> ² <i>cl</i>	0.66	0.50	0.71	0.72	-0.51	0.88	0.51	0.92	-0.64	1			
<i>dbh</i> ² <i>cl</i>	0.83	0.67	0.86	0.86	-0.68	0.72	0.66	0.70	-0.60	0.84	1		
<i>v</i> _s	0.97	0.83	0.99	1.00	-0.81	0.79	0.27	0.76	-0.68	0.72	0.85	1	
<i>v</i> _c	0.71	0.53	0.76	0.77	-0.54	0.87	0.52	0.89	-0.66	0.97	0.88	0.76	1

Note: *dbh* is diameter at breast height (cm), *ht* is total height (m); *cl* is crown length (m), *v*_s is stem volume (m³), *v*_c is crown volume (m³).

worse fit statistics were observed. Differently from previous fitted models, model (8) generated an equation in which all of its estimators were significant. Additionally, this model did not violate the assumptions of regression adjustments, as indicated by White's and Durbin-Watson's statistics. Model (8) was able to explain 58% of crown volume variability and presented a mean error of the estimate of 14.37 m³, or 61.73% relatively to mean crown volume. Thus, model (8) was the best one of this strategy.

Strategy 2: crown variables

Models (13), (14) and (15) presented a great capability of explaining crown volume variability, once their adjusted coefficients of determination were, respectively 93, 94 and 93%. Additionally, mean errors for these models were 5.76, 5.27 and 5.76 m³. However, despite models (13), (14) and (15) presented satisfactory fit statistics, these were dropped from this strategy since modeling of residual variance did not correct problems of heteroscedasticity of error terms. On the other hand, model (12) presented lower adjusted coefficient of determination for this strategy (79%) and the higher mean error (10.19 m³, or 43.78%), was the best model for crown volume based on crown related variables. This model was chosen because it presented no problems with any of evaluated assumptions, as previously indicated in Table 3, even though its statistics of fit were not best ones of this strategy.

Strategy 3: stem and crown variables

Model (17) was the first to be dropped because of model

singularities, as its large variance calculations and its parameter estimates did not converge. The results obtained for this strategy pointed that remaining models were able to produce satisfactory and reliable estimates, and only slight differences were observed when compared relatively to each other. All estimators were significant, and any models violated assumptions of regression. Model (18) presented somewhat worse statistics of fit when compared with those obtained for model (16). Therefore, model (16) was selected to represent the strategy for modeling crown volume inputting both stem and crown variables. The resulting equation explained 78% of crown volume variability, producing estimates with lower mean error of 10.50 m³ (45.09 %) for this modeling approach.

Strategy 4: stem volume expansion to crown volume

The model based on stem volume estimates showed no heteroscedasticity problems, neither with correlated error terms as indicated by White's and Durbin-Watson's statistics. All coefficients were significant. This model explained 56% of crown volume variability, having resulted in a mean error of 14.77 m³, or 63.43% relatively. In this strategy, no other models were fitted and comparisons across strategies will be subsequently discussed.

DISCUSSION

Strategic crown volume models

When comparing the best selected models of each

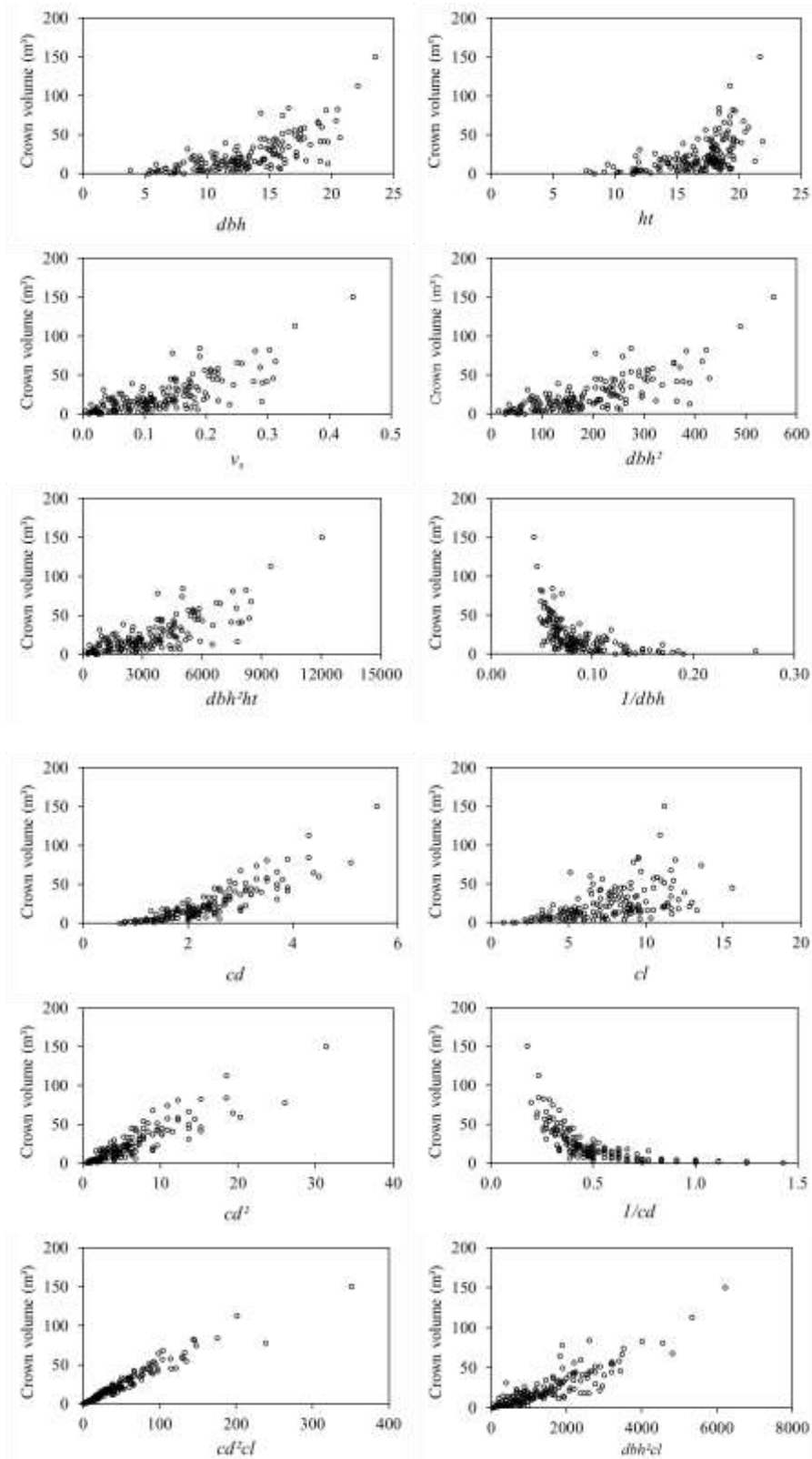


Figure 3. Graphical relationships between independent variables and crown volume in black wattle stands, in estate of Rio Grande do Sul. Where: dbh is diameter at breast height (cm), ht is total height (m); cl is crown length (m), v_s is stem volume (m^3), v_c is crown volume (m^3).

Table 3. Summary table of crown volume models of black wattle stands in Southern Brazil.

Strategy	Model	Parameter estimates			White's statistic	D-W statistic	R ² _{adj}	RMSE (m ³)	RMSE (%)
		β_0	β_1	β_2					
1	8	0.059233*	1.141983*	-	3.82*	1.7704	0.5816	14.37	61.73
	9	0.2534	2.774462*	-0.96481	4.06*	1.7363	0.5834	14.34	61.60
	10	0.016939	0.895621*	-	6.42*	1.8093	0.5566	14.79	63.55
	11	0.255409	-2.77661*	-0.96957	3.26*	1.7360	0.5836	14.34	61.59
2	12	3.05095*	1.126167*	-	1.27*	2.0353	0.7895	10.19	43.78
	13	0.484012*	1.98941*	1.019193*	53.97	1.8512	0.9328	5.76	24.74
	14	0.67959*	0.926881*	-	94.16	1.9465	0.9437	5.27	22.65
	15	0.483975*	-1.98944*	1.019219*	53.43	1.8511	0.9328	5.76	24.74
3	16	154.2672*	2.070689*	1.089476*	5.42*	1.8704	0.7768	10.50	45.09
	18	163.3499*	-2.09431*	1.085761*	3.18*	1.9347	0.7745	10.55	45.32
4	20	1.657697*	173.0236*	-	6.38*	1.8106	0.5582	14.77	63.43

Note: Asterisk (*) indicates significance at 95% level of confidence ($p < 0.05$).

modeling approach, it was possible to rank them according to their statistics of fit. The best overall strategy for modeling crown volume in terms of goodness of fit was strategy 2: crown related variables. Model (12) was the one with greater capability of explaining tree crown volume, accounting for 79% of observed variability, and produced estimates with lower mean error across strategies, with an error of 10.19 m³ (43.78%). The subsequent model regarding quality of fit was model (16). This model has produced similar statistics of fit to model (12). It was able to explain 77% of crown volume variability, with a mean error of 10.55 m³ (45.32%). Differently from these, models (8) and (20) have produced worse fit statistics, substantially smaller than in previously discussed models, as presented in Table 3, thus these were not compared. Residual plots of models (12) and (16) are presented in Figure 4.

Regarding operationality of each fitted model, the most operational ones are model (8) and model (20), once these strategies are based only on the most easily acquirable variables (that is, *dbh* and *ht*), even though model (20) requires an intermediate equation to estimate stem volume. However, as already discussed, these models' statistics were not satisfactory, and were ones with poorest predictive capability, thus they were not recommended despite their operational advantages.

Although model (12) fit statistics were the best ones in this study, this model does not present operational potential. This model's inputs are *cl* and *cd*, and, even though *cl* can be easily obtained when compared to *cd*, measuring this *cd* in all sample plots would greatly enlarge time spent on field. In addition, *cd* was the most difficult variable to be measured in this study, once it was obtained only after felling trees. The use of this model

would only be appropriate if a consistent procedure for predicting maximum diameter or crown radius was developed and used with the *cl* variable as explanatory variables in a crown volume model. It is also important to note that a model with multiple estimates as inputs makes it difficult to obtain errors of final estimates, and, for this reason, this approach should be carefully used. Alternatively, model (16) was indicated as the most reliable and operational model for predicting black-wattle crown volume. The resulting equation produced marginal estimates relatively to model (12), even though its statistics of fit were somewhat worse. Model (16) requires *dbh* and *cl* as inputs and these variables make this model operationally advantageous over model (12), since these are easier to measure. Diameter at breast height is the easiest measurable variable on a tree and is the most used variable estimate tree's variables, such as crown volume. On the other hand, even though *cl* is not as commonly measured as *dbh* is, it can be easily calculated as the difference of tree *ht* and height to live crown base - commonly named *hblc* (Crecente-Campo, 2008; Rautiainen and Stenberg, 2005). Also, *hblc* can be modeled and estimated for other trees, aiming to reduce the time spent measuring it, in same the fashion as commonly done with hypsometric equations of even-aged single species stands.

Regarding number of variables needed as inputs, model (16) requires only two independent variables - a reasonable number of inputs, what grants model simplicity according to Rawlings et al. (1998), differing from models built by automated procedures of variable selection such as stepwise regression method. Such selection method could yield a "better" model regarding quality of fit, however hardly comprehensible most of

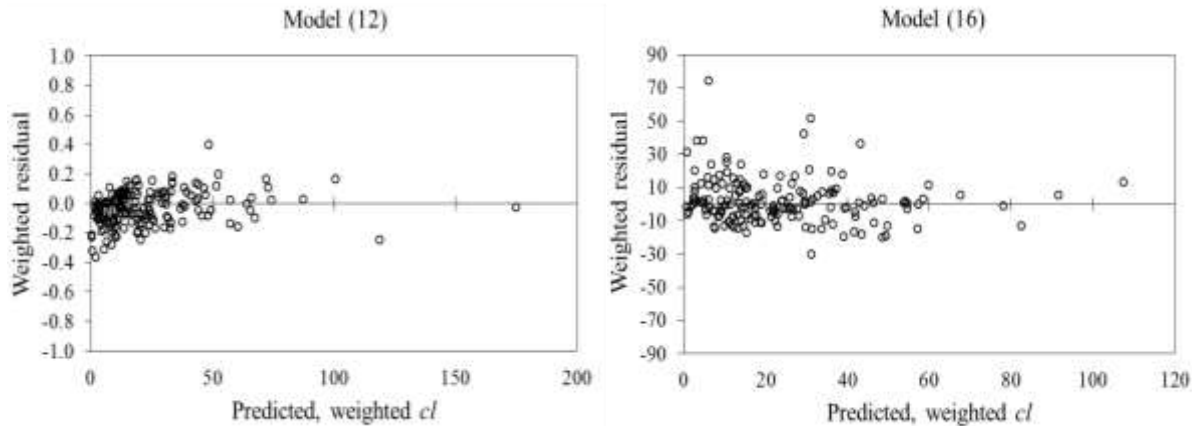


Figure 4. Residual plots of each selected model of different strategies for modeling crown volume of black wattle stands in Southern Brazil.

times due the quantity and different natures of selected input variables (Draper and Smith, 1966). Models that require a greater number of inputs can easily become costly and time demanding due to the number of variables that would have to be measured on field.

In the literature of crown volume modeling, Velázquez-Martí et al. (2012) fitted crown volume models for mandarin (*Citrus reticulata*) that explained between 61 and 77% of crown variability using crown diameter and crown length as inputs. In the study of Rautiainen et al. (2008), estimates of crown volume presented mean error ranging from 14.6 to 21.0 m³ for a Scots pine (*Pinus sylvestris*) and from 22.4 to 48.1 m³ for Norway spruce (*Picea abies*), for different approaches of modeling crown volume that were previously presented. Crown volume models of Pedunculate oak (*Quercus robur*) and Common hornbeam (*Carpinus betulus*) for stands of different age classes proposed by Dubravac et al. (2009) explained 44 to 79% and 23 to 65% of crown volume variability for these species, respectively. Meng et al. (2007) suggested crown volume models for *Pinus contorta* that were build based on uniform stress theory. When using *dbh* to estimate crown volume, this model accounted for 62% of the variability. In his second proposed model, when inputting *dbh*, distance between center of crown and wind speed as independent variables, it has accounted for 70% of crown volume variability. Our proposed model was likely to be significantly advantageous regarding previous studies due its capability of producing consistent estimates of crown volume, and its simplicity of inputting *dbh* and *cl* to do so. Additionally, in relative terms the proposed model was able to explain variability with a similar performance to the models for other species.

Besides being simultaneously the most operational and reliable model of this study, model (16) was biologically and mathematically sound. Model (16) is biologically

reasonable because crown volume has implications on *dbh* (Sprinz and Burkhart, 1987), which is also intrinsically related to *cl* (Taiz and Zeiger, 1991). A large crown has potential of intercepting greater amounts of radiation because it comprises greater amounts of active foliage, which are responsible for increasing photosynthesis rates, therefore affecting *dbh* growth (Burkhart and Tomé, 2012; Ottorini et al, 1996; Cluzeau et. al, 1995; Ottorini, 1991). Mathematically, since our data are experimental, b_1 and b_2 are compensatory coefficients, whose product generates the volume of a regular geometric solid – RGS ($(dbh)^2$ times *cl*), whilst the estimate of b_0 may represent a constant of shrinkage from the volume of such solid to the volume of the crown.

The final model (21) with its coefficients and plots of observed crown volume versus predicted values and its residuals distribution plot are presented in Figure 5.

$$v_c = 0.010553 (dbh)^{2.095694} (cl)^{1.085087} + \varepsilon_i \quad 21$$

Conclusions

This study was able to suggest a simple, likely unbiased and reasonably accurate crown volume model, relatively to other crown volume models found in literature. Models composed by stem variables did not present a satisfying predictive capability. The model based only on crown related variables was the one with greater predictive ability. However, its independent variables are most onerous to be measured in the field. The predictive capability was kept by model (16), inputting both stem and crown variables. The resulting model strategy using stem and crown variables presented an operational advantage once it only requires *dbh* and *cl* as independent variables. In addition, it is both biologically and mathematically sound, due its inputs and intrinsic

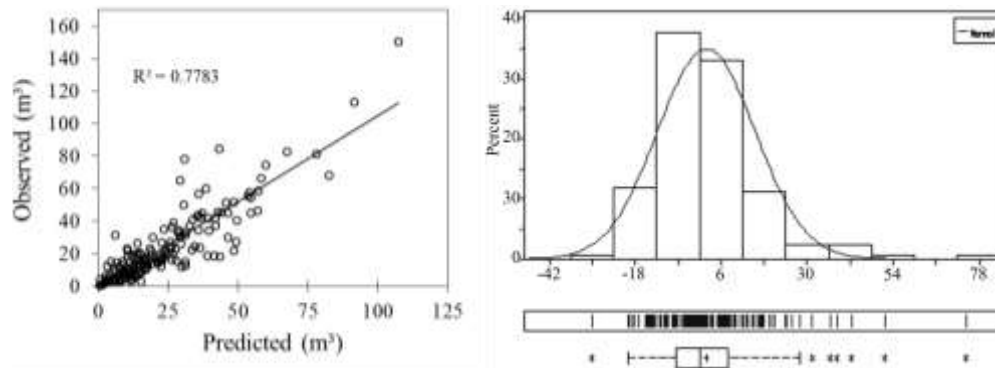


Figure 5. Plot of observed crown volume by predicted values and histogram of residuals of the best overall crown volume model of black wattle stands in Southern Brazil.

mathematical and biological relationships with respect to crown volume.

Conflict of interest

The authors have not declared any conflict of interest.

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