

Full Length Research Paper

## Using the $p$ -median location model to set up aerodromes for coverage fertilization of eucalyptus plantations

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In this study an economic analysis was conducted to evaluate problem associated with a geographical information system (GIS) using facility location models for the  $p$ -median in order to optimize the location of aerodromes for the aerial fertilization (coverage fertilization) of eucalyptus plantations. The location model was tested on a 9,095.65 ha farm located in the Três Lagoas municipality in the Mato Grosso do Sul State, in Brazil. The non-capacitated  $p$ -median location model, available in ArcGIS, was evaluated in the location-allocation module. Simulations were performed based on one to five aerodromes. The fertilization and setup costs were calculated for each scenario. The results showed that the  $p$ -median location model was efficient in determining the optimal location of aerodromes. The economic analysis of the location model found that the lowest costs are incurred when using three aerodromes.

**Key words:** Combinatorial optimization, operational research, aerial fertilization.

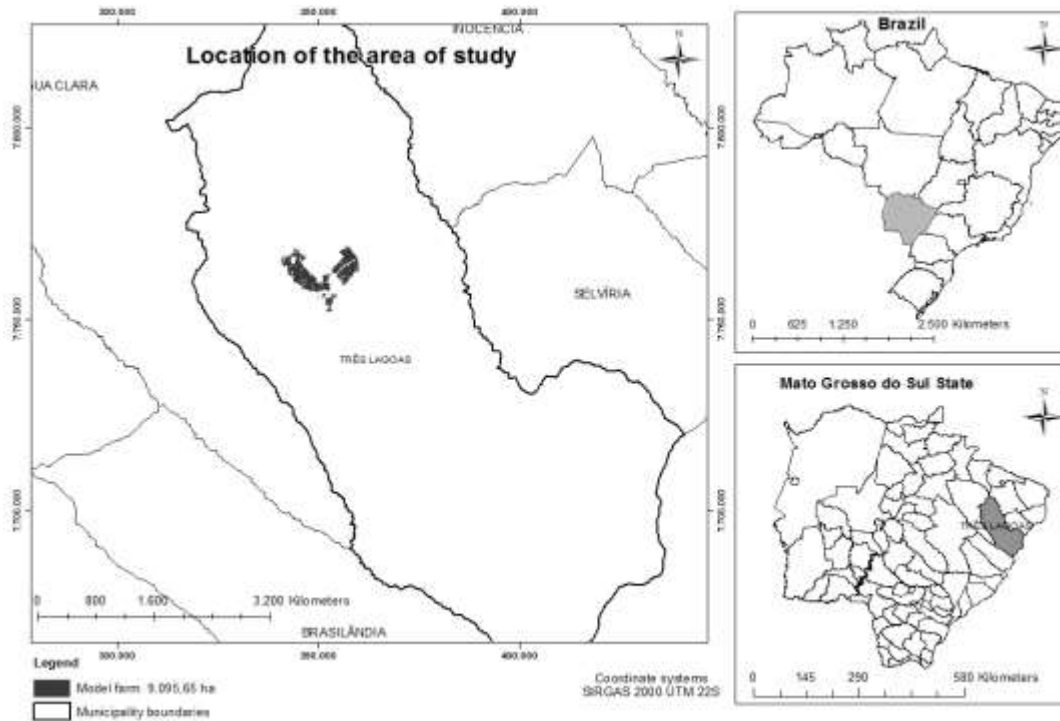
### INTRODUCTION

Forest management can be regarded as management on multiple analysis levels, each involving many decisions based on several resources, most of which are scarce

(Church et al., 1998). The productivity of a particular plantation is limited by one or more nutrient sources and forest nutrition is a key part of managing commercial

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**Figure 1.** Location of the clonal *Eucalyptus* plantations.

forests (Stape et al., 2006). Fertilization is required because the soil is not always able to provide all the nutrients required by the plants to grow properly (Gonçalves, 1995).

Several strategies can be used for the distribution of fertilizers, but the importance of aerial fertilization has been growing owing to the rapid expansion of cultivated areas and the subsequent need for fertilization. Aerial fertilization is highly efficient, particularly in comparison with using terrestrial machinery.

One of the main factors that affects the efficiency of aerial fertilization is the distance between the aerodrome and the application area. The ideal aerodrome location is a combinatorial problem involving several possible choices, with limitations on flight distance, refueling, and operational stations, among other restrictions.

The network  $p$ -median problem, introduced by Hakimi (1964), identifies facility locations in order to minimize the total weighted distance for trips between the demand areas and the facilities. This problem may be formulated using integer linear programming (Owen and Daskin, 1998) and is useful for predicting the location of industrial plants, warehouses, and public installations (Mladenovic et al., 2007). The tool identifies the locations of facilities in order to minimize the average distance between clients and the nearest service point (Daskin, 1995).

The  $p$ -median problem has great practical importance. In Brazil, studies have used the technique to examine, for instance, locations for public schools (Pizzolato et al.,

2004; Pizzolato and Menezes, 2010), telecommunication antennas (Lorena and Pereira, 2002), public healthcare stations (De Rosário et al., 2001), and urban leisure areas (Brondani et al., 2013; Lorena et al., 2001), as well as for evaluating the integration of location models and Geographical Information Systems (GIS). With regard to forestry areas, the method has been used to identify optimal wood storage locations (Junior et al., 2014; Martinhago, 2012), allocating fire lookout towers in a natural reservoir (Juvanhol, 2015), and for the spatial stratification of compartments in forest harvesting (Gomide, 2013).

This study evaluates using the mathematical model associated with the  $p$ -median problem to identify optimal aerodrome locations in eucalyptus plantations. In addition, an economic analysis is conducted to find the ideal number of aerodromes for the study area.

## MATERIALS AND METHODS

This study focuses on an area of 9,095.65 ha containing clonal eucalyptus plantations in the Três Lagoas municipality in the Mato Grosso do Sul State, Brazil (Figure 1).

In these plantations, 200 kg of nitrogen and potassium per hectare are applied between the third and fifth months after planting, using coverage fertilization. The aircraft used is an AirTractor 502B, which has a load capacity of 1,800 kg. The specifications of the aircraft are shown in Table 1. The runways have standard dimensions (that is, 1,200 m long and 40 m wide), and always run in a north/south direction.

**Table 1.** AirTractor 502B specifications.

Engine	P&W PT6A-34AG	Empty weight, including pulverization equipment	4,860 lbs (2,204 kg)
Engine SHP	750 at 2,200 RPM	Useful load	5,403 lbs (2,450 kg)
Propeller	Hartzell HC-B3TN-3D/T10282NS+4	Hopper capacity	500 U.S. gal(1,893 L)
Takeoff weight	9,400 lbs (4,263 kg)	Fuel capacity	170 US gal (644L)
Landing weight	8,000 lbs (3,628 kg)	Wing area	312 sq. ft (29.01 m <sup>2</sup> )
Wingspan	52 ft (15.84 m)	Dimensions of main wheels	29.00 × 11
Dimensions of rear wheels	5.00 × 5		

The optimal aerodrome location problem is modeled as a location-allocation problem, solved using the p-median model of the Location-Allocation module in the extension Network Analyst of ArcGIS version 10.2.2. The available algorithm combines several techniques, adopting vertex substitution heuristics and fine-tuning metaheuristics in order to achieve a satisfactory, optimal, or near-optimal solution (Sultana and Kumar, 2012; Costa, 2014).

The areas containing clonal eucalyptus plantations are represented by polygons, but in a network context, it is necessary to define demand points in order to draw the routes connecting them to the facility locations. In this case, the demand points are defined as centroids in the intersections between the features of the subject plantation and a grid of 300 m × 300 m polygons created using the Fishnet tool available in ArcToolbox. This made it possible to represent the plot areas on locations corresponding to 9.0 ha plantations, generating a total of 1,013 demand points for the 32 facilities (Figure 2).

The study uses the geographical information system ArcGIS® version 10.2.2. The data were created and stored in a geobasis, a native structure of ArcGIS, and stored in a Feature Dataset of the SIRGAS 2000 UTM 22S coordinate system. The extensions Spider Tools, Data Management Tools, Editing Tools, and Network Analyst were used for database processing and structuring, as well as for the required feature editions.

According to Reville and Swain (1970) and Church and Reville (1976), in a network context, the p-median problem can be defined as follows: minimize the total weighted distance of the trip associated with a network of demand nodes to locate p-facilities in the network (on vertices or on nodes), where each demand vertex is served by its nearest facility.

In the following mathematical model,  $i$  denotes the group of demand points (nodes);  $i \in I$  is a defined client or vertex;  $j \in J$  is a potential facility or median;  $p$  is the number of service facilities or medians to be located;  $w_i$  is the weight or importance of client  $i \in I$ ;  $[d_{ij}]_{i,j}$  is a symmetrical matrix of the distances between each client  $i$  and facility  $j \in J$ ;  $[x_{ij}]_{i,j}$  is an allocation matrix for client  $i$ , where  $x_{ij} = 1$  if a facility is located at candidate node  $j \in J$  and 0 otherwise;  $y_{ij} = 1$  if demand node  $i \in I$  is assigned to facility at candidate node  $j \in J$  and 0 otherwise. According to Reville and Swain (1970), the problem may be formulated with objective of minimizing the demand-weighted total distance as follows:

$$\begin{aligned} & \text{Minimize } \sum_{j \in J} \sum_{i \in I} w_i d_{ij} y_{ij} \\ & \text{s.t.} \\ & \sum_{j \in J} y_{ij} = 1 \forall i \in I, \\ & y_{ij} - x_j \leq 0 \forall i \in I, \forall j \in J, \\ & \sum_{j \in J} x_j = p, \\ & x_{ij} \in \{0,1\} \forall j \in J, \end{aligned}$$

$$y_{ij} \in \{0,1\} \forall i \in I, \forall j \in J.$$

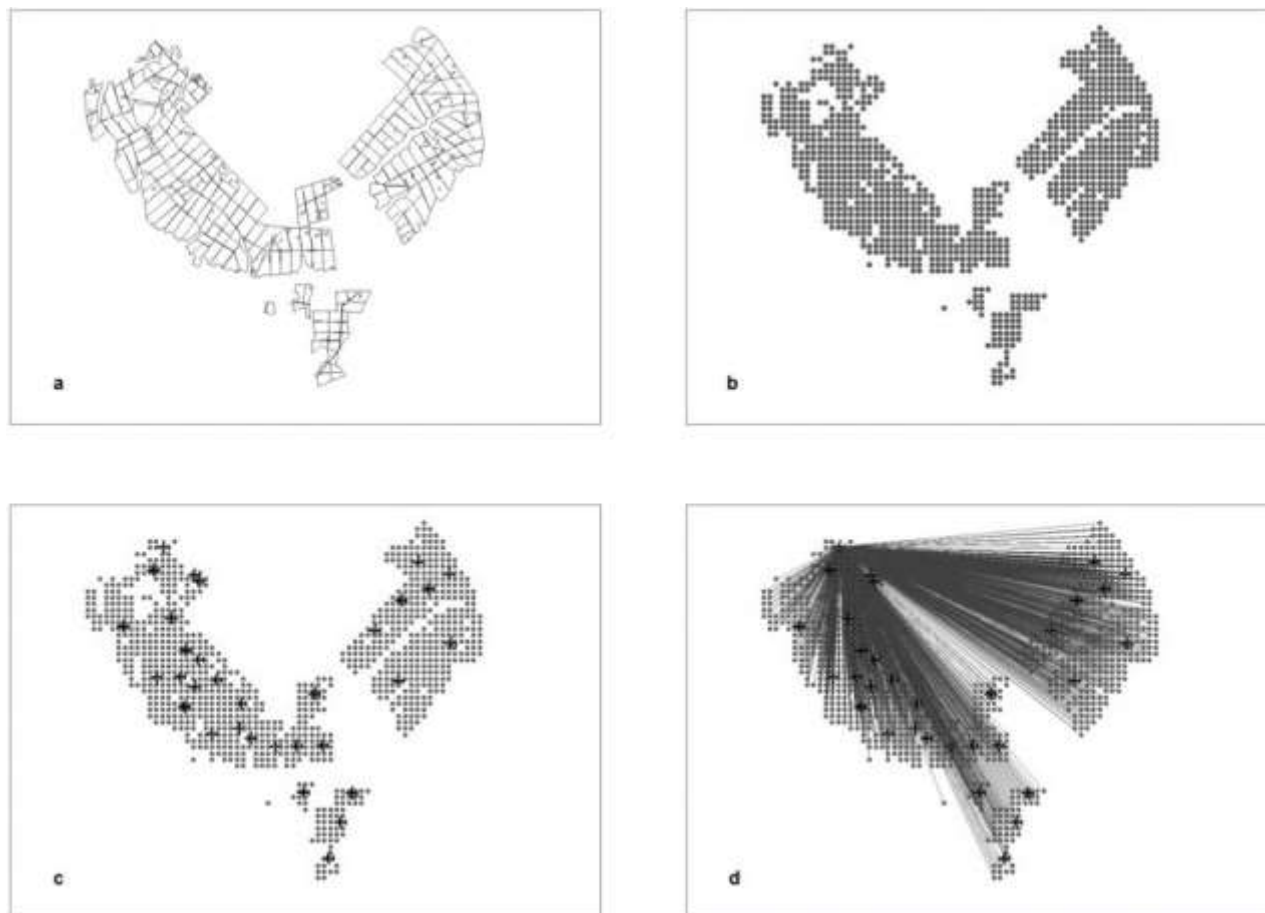
The solution to a facility location problem can be found using a geographical information system integrated with optimization or simulation algorithms. ArcGIS uses heuristic methods to solve the p-median problem, because most location problems are classified as NP-hard. These heuristics were developed by Densham and Rushton (1992), Teitz and Bart (1968), and Arakaki and Lorena (2006).

In order to perform the economic analysis, the costs associated with setting up an aerodrome are calculated. Fertilization costs were provided by the company. The cost of coverage fertilization per hectare varies with the coverage radius of an aerodrome. Here, the distances from the plots are distributed into four classes: 2,500, 5,000, 7,500, and 10,000 m. The values provided by the company for the fertilization activity costs, as a function of the distance between the aerodromes and the plots, for radii of 2,500, 5,000, 7,500, and 10,000 m are R\$97.5, R\$110.0, R\$126.9, and R\$143.8 per ha, respectively.

The total cost of the activity is the sum of the aerodrome setup cost (the same for all points in the study area) and the fertilization cost per hectare.

## RESULTS

In order to identify the lowest-cost configuration and number of aerodromes, the evaluated location models are used to calculate the aerodrome setup cost and the coverage



**Figure 2.** Polygons denoting the plantations (a), 9.0 ha demand points (b), candidate aerodromes (c), and routes between the candidate aerodromes and the demand points (d).

fertilization cost, which are added together to give the total cost of the activity. Table 2 presents the items making up the aerodrome setup cost, as well as their respective values.

A summary of the activity costs is as shown in Table 3. The results show that the scenario with the lowest costs, according to the  $p$ -median model, comprises three aerodromes, with a total cost of R\$1,031,980.5.

When conceptualizing the setup of aerodromes for aerial fertilization, the use of a GIS is important because it enables the results to be presented as illustrative maps. These maps help with the interpretation and comprehension of the results, which are as shown in Figure 3. These results refer to the simulations of setting up one to five aerodromes and to the spatial distribution given by the  $p$ -median model.

## DISCUSSION

The location of facilities is a critical problem in strategic planning for several public and private companies. For

example, this may involve a producer choosing where to build a warehouse serving a new market in a retail chain, or an urban planner selecting locations for fire stations. In each case, decisions are constrained by the allocation of spatial resources (Owen and Daskin, 1998). Mathematical modeling and optimization have proven to be effective in helping decision-makers with location and transport matters (Kelley et al., 2013).

Several recent studies have attempted to define and characterize localization problems and have proposed models to approach and solve these problems (Daskin, 1995; Eiselt and Marianov, 2011). A number of studies use geographical information systems, developing localization models for the establishment of biomass industries (Zhang et al., 2011; Sultana and Kumar, 2012; Costa, 2014; Teixeira et al., 2018).

Serra and Marianov (1998) formulated the  $p$ -median problem for the location of new facilities in the case of uncertain demand, in terms of time or the distance of trips, where it is possible to define scenarios that represent differences in travel times or demand in the region of interest. For example, the model has been

**Table 2.** Composition and values of the items in the aerodrome setup cost.

Items	Unit	Value
Width	m	40.00
Length	m	1,200.00
Total area	ha	4.80
Terrain price	R\$/ha	5,000.00
Construction costs	R\$/ha	20,000.00
Annual interest rate	%	12
Road duration (years)	year	20.00
Depreciation	ha/year	1,000.00
Interest on the construction value	R\$/ha/year	1,200.00
Interest on the terrain value	R\$/ha/year	600.00
Maintenance	R\$/ha/year	2,000.00
Cost per ha	R\$/ha/year	4,800.00
Total Cost	Per aerodrome	23,040.00

**Table 3.** Fertilization, implantation, and total costs results as a function of the number of aerodromes.

Name	Number of aerodromes	Fertilization costs (R\$)	Aerodrome setup costs (R\$)	Total costs (R\$)
<i>p</i> -medians	1	1,153,514	23,040	1,176,554
<i>p</i> -medians	2	1,012,449	46,080	1,058,529
<i>p</i> -medians	3	962,861	69,120	1,031,981
<i>p</i> -medians	4	941,633	92,160	1,033,793
<i>p</i> -medians	5	923,847	115,200	1,039,047

applied to identify the optimal locations of fire brigades in Barcelona. Note that for this study on the location of aerodromes, uncertainty was not taken into account in the evaluated models. Even though there is considerable research on applying the *p*-median problem in different scenarios, no studies have examined the setting up of aerodromes in the context of the aerial fertilization of eucalyptus plantations. This technique has proven to be very efficient in finding optimal locations for aerodromes, helping to reduce the associated costs. The results showed that the lowest cost is achieved with three aerodromes, with a reduction of 12.3% in the allocation costs of using one aerodrome (Table 3).

The limitation of using the *p*-median mathematical model is the need to specify the number of facilities being established. In other location models, such as the coverage model, this information is not required; that is, the number of aerodromes is not an input variable in the model. However, this limitation can be overcome by performing simulations that consider different scenarios.

The optimal number of aerodromes depends on the plot distribution in the area. In the present study, the distribution favored the location of three aerodromes. Therefore, the decision on runway allocation is directly correlated with the hierarchical planning. Restricting the number of harvest blocks may favor a smaller number of

aerodromes, while excessive specialization may result in the allocation of more aerodromes.

Even though they are not analyzed in the present work, the spatial distribution and the number of aerodromes also depend on the physiographical characteristics of the region where the plantations are located, as well as on the size of the forest. However, the methodology employed in this study remains valid and enables the determination of the number and optimal distribution of runways.

Only one aircraft type was taken into account in the present study. Given the possibility of acquiring more than one model, the choice of which aircraft to buy can be made based on the total cost (Ellram, 1993), while also considering the useful life of each aircraft type.

## Conclusions

The mathematical location model associated with the *p*-median problem proposed in the present study is efficient in identifying the optimal location of aerodromes for the coverage fertilization, by aerial fertilization, of eucalyptus plantations.

For the model farm evaluated in this study, the optimal number of aerodromes that minimizes the total cost of the

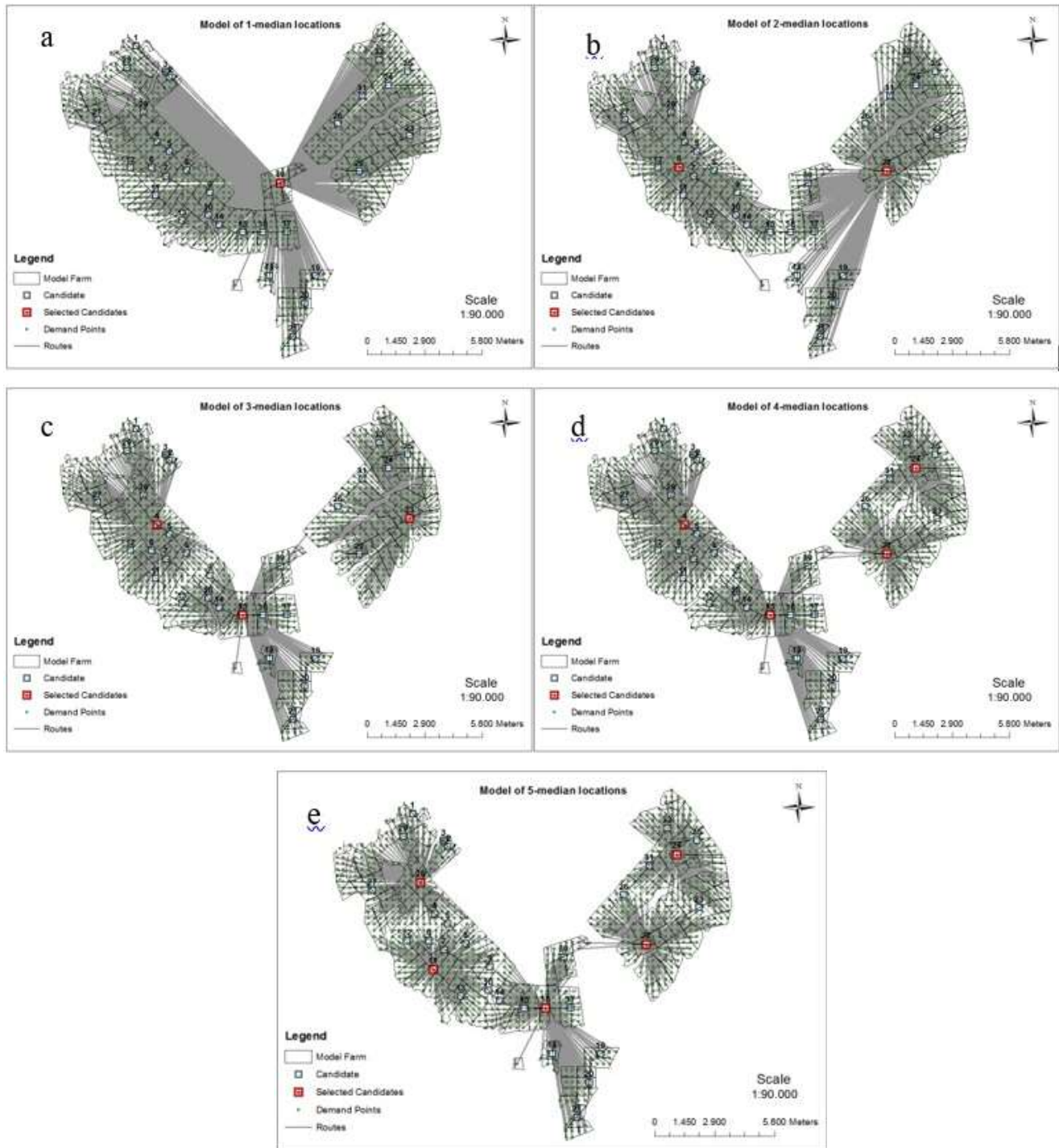


Figure 3. Simulation results, using the p median model, of setting up one to five aerodromes (a to e).

fertilization activity is three.

**CONFLICT OF INTERESTS**

The authors have not declared any conflict of interests.

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