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Modelling crown volume in *Acacia mearnsii* stands**Carlos Roberto Sanquetta^{1*}, Alexandre Behling¹, Ana Paula Dalla Corte¹, Augusto Arlindo Simon², Aurélio Lourenço Rodrigues¹, Guilherme Camacho Cadore¹ and Sérgio Costa Junior¹**

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The aim of the present study was to model the crown volume in black wattle stands (*Acacia mearnsii* De Wild.) in Rio Grande do Sul, Brazil. The study was carried out in plots installed in black wattle stands in area where the plantations of the species are common. Two trees in each plot of average diameter were felled to determination total height, crown length (crown height) and crown width taken twice, each measurement oriented perpendicular to the other across the bole axis and parallel to the base of the crown. To calculate crown volume (cv) widths were taken every meter starting at the base of the tree bole. Stepwise, backward, and forward variable selection methods were used to formulate the volume equations. The combined variable squared diameter and crown height (d^2ch) was the most highly correlated with crown volume ($r=0.84$). This in turn was the variable that was integrated into the selected model, both in the stepwise and forward selection methods. The equation $cv = b_1 d^2ch + \epsilon_i$ was the model that provided the best fit for predicting crown volume in black wattle stands, both in age rotation and in young stands.

Key words: *Acacia mearnsii*; morphometry; stepwise; models.

INTRODUCTION

Black wattle (*Acacia mearnsii* De Wild.) is a tree species which is prominent today in the state of Rio Grande do Sul, and stands of the species rank among the most widely planted in region behind the genera *Eucalyptus* and *Pinus*. According to Simon (2005) black wattle is the primary source of bark for the global plant-based tannin industry, used mostly in leather tanning. The high-quality wood from this species is ideal for pulp and paper production, and most of the wood is consumed in these industries (Stein and Tonietto, 1997). The species is

cultivated by more than 10,000 small farms and therefore plays an important socioeconomic role in the region (Oliveira et al., 2006).

One of the characteristics of black wattle stands is the use of intercropping, especially in the early years including annual crops, such as watermelon, corn and cassava, and in later years along with cattle (Fleig et al., 1993; Mora, 2002; Muller, 2006). In intercropping systems where the soil nutrient conditions, temperature and water availability are not limiting factors and pests

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and disease are not significant factors, growth and yield of intercropped species depends primarily on the total solar radiation intercepted by the plants during the growing period (Monteith 1978). Therefore, knowing the amount of light that reaches the plant system is critical for managing intercropped species (Sinoquet and Bonhomme, 1992).

The amount of radiation intercepted in a stand is determined by the space that the crown occupies, which can be determined by its volume. Once determined, the crown volume can be used for various purposes, such as predicting the amount of light reaching the interior of the stand (important in the case of intercropping) or the amount of light intercepted by the crown (a factor that regulates growth and production of the population), and also as an input variable in simulating growth and crop yield (Meng et al., 2007).

The crown profile and volume are generally obtained by making approximations based on geometric figures such as cones, whose volumes are easily calculated. However, models using simple geometric shapes do not accurately represent the actual volume of the crown, since in most cases the crown departs from common geometric shapes (Durló et al., 2004; Crecente-Field, 2008). An alternative to this procedure is to estimate the volume of the tree crown and fit models with independent variables whose data are easily obtained, and so making the process less costly and more reliable.

However, the great difficulty in fitting a model well lies in meeting the assumptions required of a regression, such as parameter linearity, normality, homoscedasticity and the absence of error autocorrelation. Furthermore, mean errors of the regression model should be zero and the regressed variables should not exhibit collinearity (required exclusively of multiple regressions), according to Werkema and Aguiar (2006) and Hair Jr. et al. (2009). Such requirements in the regression analysis have not been well researched, though it is an essential procedure for assessing the quality of fit and the reliability of the model.

In light of the importance of estimating crown volume and finding models that meet the statistical requirements, this study sought to model crown volume in black wattle stands.

MATERIALS AND METHODS

To conduct this study we drew data from temporary plots installed in commercial plantations of black wattle in regions of high concentrations of the species in the state of Rio Grande do Sul, Brazil, in the municipalities of Cristal and Piratini. In each region, stands were studied at the end of the rotation, or at seven years old.

In each stand a North-facing slope was selected where one plot was demarcated in the each of the upper, middle and bottom thirds of the slope. The plot size was 9 m wide by 14 m long, with four rows of 10 plants per row.

In each plot the circumference at breast height was measured for all trees using a graduated tape measure. Two trees of average

diameter (d) in each plot were selected for morphometric variable evaluation. This procedure was adopted to gather additional data for use in other studies that made use of this data set.

The morphometric assessment consisted of measuring (using a tape measure) the total height (H), length of the crown (plant height) and crown width twice measured, each measurement oriented perpendicular to the other across the bole axis and parallel to the base of the crown.

The calculation of crown volume was performed using the following expression:

$$cv = \left(\frac{a_1 + a_2}{2} \right) l + \left(\frac{a_2 + a_3}{2} \right) l + \dots + \left(\frac{a_{n-1} + a_{sn}}{2} \right) l$$

Where: cv = sum of the volumes through to the last section of the crown (m³); a₁ = crown area section 1 (m²); a₂ = crown area section 2 (m²); a₃ = crown area section 3 (m²); a_{n-1} = crown area of the penultimate section (m²); a_n = crown area of the last section (m²); and l = length of section (m). To calculate the cross-sectional area of the crown in each section (a) we used the following expression:

$$a = \frac{\pi \cdot dc^2}{40000}$$

Where: a = cross-sectional area of the crown (m²) and dc = diameter of the crown of the corresponding section (m).

The last measured section was considered a cone, and the formula for the volume of a cone was used to make the calculation, where:

$$v_{co} = \frac{1}{3} * a * lc$$

Where: v_{co} = the volume of the cone of the crown (m³); a = cross-sectional area of the last section "n" (m²), and lc = length of the cone. Thus, the total volume of the crown of each individual was calculated by summing the volume of all sections plus the volume of the cone.

We used stepwise, backward and forward variable selection methods (Draper and Smith, 1966) to obtain equations to estimate crown volume at 5% error probability. The three methods were tested while bearing in mind that the result of the variable selection may not always be the same. Statistical analyses were conducted in SAS 8.1 (SAS, 2002).

The independent input variables in the models were: diameter at breast height (d, in cm), total height (h, in meters) and crown height (ch, in meters), as well as combinations of these variables and log transformations. The dependent variable in the studied models was the crown volume (vc, in cubic meters).

Initially, a Pearson correlation analysis was performed between crown volume and dendrometric variables, both in their original, combined and transformed form. The results of formed the basis for the use of stepwise, forward, and backward selection methods. The *t* test, at a 5% probability level, was used to verify the existence of a linear relationship between variables.

The equations were evaluated and selected by: adjusted coefficient of determination (R²_{adj}, in %), the standard error of estimate (S_{y_x}, in %), F value for p < 0.05, and the significance of all the coefficients and the residual plot as a function of the estimated values (%).

The models generated were also tested with respect to the regression conditions using the White test (homoscedasticity), Shapiro-Wilk test (normality) and Durbin-Watson test (independence). For the multiple linear models, a tolerance level was determined and consequently a variance inflation factor (VIF),

since the latter is an indicator of the effect that other independent variables have on the standard error of a regression coefficient. High VIF values (greater than 10) indicate a high degree of collinearity or multicollinearity (Hair Jr. et al., 2009), or increasing variance in the coefficients given by the correlation between the independent variables.

The performance of the best model, using the procedures described above, was tested in varying age of black wattle stands. Therefore, further evaluations were conducted in the same manner and used in the construction of the model, in one, three and five year old stands. The evaluation of fits was based on the cited accuracy statistics as well as the chi-square test (χ^2) at 5% probability.

Therefore, models were constructed using 12 trees from 6 plots, and the performance was evaluated based on 36 other trees from another 18 plots.

RESULTS AND DISCUSSION

Using simple linear correlation analysis, squared diameter and crown height (d^2ch) combined variable was found to be the most highly correlated with crown volume ($r = 0.84$). The other variable significantly correlated with crown volume was diameter at breast height (d), both untransformed and transformed as $1/d$, $\ln d$, and $\ln 1/d$ ($r = 0.80$, -0.80 , 0.80 and -0.80 , respectively). Other variables showing significant correlations with crown volume were the combination of the squared diameter with the total height (d^2ht) and crown height (ch).

The existence of a high correlation between the tree crown diameter with bole diameter is already widely understood (Hemery et al., 2005) and based on the results obtained it is clear that the diameter of the bole also has a significant correlation with the crown volume. Meng *et al.* (2007) modelled the crown volume of *Pinus contorta* Engelm. and found that the diameter at breast height had the strongest independent correlation with crown volume, accounting alone for 62% of the total variation. But the fact that the variable d^2ch presented the highest correlation with the volume is justified since it generated a virtual parallelogram with respect to the squared diameter, referring to the area occupied by the crown. The variable is combined with the height of the crown, referring to its length and therefore, strongly correlated with volumetric form.

The transformation of all the independent variables did not improve the correlation with crown volume. Therefore, with the aim of compiling a single matrix mainly for the backward method, these variables were excluded from the process. Thus, total height (h), crown height (ch), diameter at breast height (d) and the combination between the diameter (d) squared with the total height (d^2ht) and diameter (d) squared with crown height (d^2ch) were all part of the modelling process.

With the inclusion of the independent variable most highly correlated with crown volume (d^2ch), the stepwise variable selection procedure generated a single step, resulting in the equation $cv = -0.56767 + 0.01467 d^2ch$ (Table 1).

It was observed that the coefficient b_0 of the obtained equation was not significant ($p > 0.05$), resulting in an increase in the standard error of the coefficients. Thus the exclusion of this coefficient was tested, resulting in the equation $cv = 0.01431 d^2ch$. The change yielded an improvement of approximately 11% in the adjusted coefficient of determination (R^2_{adj}), lower standard error of the estimate (S_{yx}), and reduced standard error (and consequently an improved confidence interval) of the coefficient.

Comparing the two equations, both showed adequate distribution of residuals (Figure 1a), and met the conditions of the regression tested, given the non-significance ($p > 0.05$) of the White, Shapiro-Wilk and Durbin-Watson tests. Since the improvement of statistical fit was obtained in the model without intercept, the following model was obtained by the stepwise method: $cv = b_1 d^2ch + \epsilon_i$

The backward variable selection method resulted in two steps: the first with the full model (including all variables) and the second where the crown height variable was excluded. The statistics obtained for the two steps in the fit are shown in Table 1. The fits resulted in a difference of less than 1% with respect to R^2_{adj} and S_{yx} . Both equations also showed adequate distribution of residuals along the estimated line and a similar trend (Figure 1).

In the first step, the coefficients b_3 and b_5 were not significant and were excluded from the model. However, when the ch variable was excluded in the second step, all coefficients of the model become significant ($p < 0.05$) resulting in the equation:

$$cv = -572.58561 + 14.71806 h + 43.69779 d - 0.08388 d^2h + 0.01103 d^2ch.$$

The results of the White, Shapiro-Wilk, and the Durbin-Watson tests for the model obtained by the backward method ($cv = b_0 + b_1 h + b_2 d + b_3 d^2h + b_4 d^2ch + \epsilon_i$) were not significant, indicating that the model met the requirements of normality, homogeneity of variances and independence.

The values of the variance inflation factor (VIF) and the high linear correlations found between the independent variables present in two steps suggest a multicollinearity effect in the multiple regression proposed by this method, because the VIF values are greater than 10 (Table 1). Multicollinearity occurs when any independent variable is highly correlated with a number of other independent variables present on the same model as in the present case is between DBH, ch , and the combination d^2ch .

The forward variable selection method resulted in three steps: The first began with the independent variable most highly correlated with volume and therefore the same equation obtained by the stepwise method. In the next two steps the variables diameter at breast height and crown height were added (Table 1).

The three steps resulted in similar fits, with R^2_{adj}

Table 1. Statistical accuracy of models obtained by stepwise, forward, and backward methods to estimate crown volume in black wattle.

Model			R ² _{adj}	S _{yx}	F	α (F)	W	SW	DW	
cv = b ₀ + b ₁ d ² ch + εi			67.86%	17.37%	24.23	6.03E-04	3.08 ^{ns}	0.97 ^{ns}	1.91 ^{ns}	
cv = b ₁ d ² ch + εi			88.58%	16.58%	463.00	1.05E-09	2.8 ^{ns}	0.97 ^{ns}	1.95 ^{ns}	
Coefficients			S _{yx}	t	α (t)	Confidence Interval				
Stepwise method	cv = b ₀ + b ₁ d ² ch + εi									
	b ₀	-0.56767	4.63	-0.122	0.9	-10.90	≤ Y ≤		9.76	
	b ₁	0.01467	0.00	4.922	6.4E-04	0.00	≤ Y ≤		0.02	
	b ₁	0.01431	6.65E-04	21.51	2.4E-10	0.01	≤ Y ≤		0.01	
Model			R ² _{adj}	S _{yx}	F	α (F)	W	SW	DW	
cv = b ₀ + b ₁ h + b ₂ d + b ₃ ch + b ₄ d ² h + b ₅ d ² ch + εi			88.00%	10.62%	17.140	1.70E-03	12.00 ^{ns}	0.96 ^{ns}	1.98 ^{ns}	
cv = b ₀ + b ₁ h + b ₂ d + b ₃ + d ² h + b ₄ d ² ch + εi			87.16%	10.99%	19.660	6.58 E-04	12.00 ^{ns}	0.88 ^{ns}	1.74 ^{ns}	
Coefficients			S _{yx}	t	α (t)	Confidence Interval				
cv = b ₀ + b ₁ h + b ₂ d + b ₃ ch + b ₄ d ² h + b ₅ d ² ch + εi										
Backward method	b ₀	-530.8521	151.29	-3.50	1.2E-02	-901.04	≤ Y ≤	-160.65	-	0
	b ₁	19.5751	5.66	3.45	1.3E-02	5.72	≤ Y ≤	33.42	0.004	233
	b ₂	40.5999	11.32	3.58	1.2E-02	12.90	≤ Y ≤	68.29	0.008	125
	b ₃	-12.5966	10.30	-1.22	2.7E-01	-37.80	≤ Y ≤	12.60	0.001	643
	b ₄	-0.1166	0.03	-3.34	1.6E-02	-0.20	≤ Y ≤	-0.03	0.001	600
	b ₅	0.0922	0.06	1.38	2.2E-01	-0.07	≤ Y ≤	0.25	0.000	1341
	cv = b ₀ + b ₁ h + b ₂ d + b ₃ d ² h + b ₄ d ² ch + εi									
	b ₀	-572.5856	152.51	-3.75	7.1E-03	-933.23	≤ Y ≤	-211.94	-	0
	b ₁	14.7180	4.17	3.52	9.6E-03	4.85	≤ Y ≤	24.58	0.008	118
	b ₂	43.6977	11.41	3.82	6.5E-03	16.70	≤ Y ≤	70.69	0.008	118
b ₃	-0.0838	0.02	-3.63	8.4E-03	-0.13	≤ Y ≤	-0.02	0.004	244	
b ₄	0.0110	0.00	2.92	2.2E-02	0.00	≤ Y ≤	0.01	0.249	4	
Model			R ² _{adj}	S _{yx}	F	α (F)	W	SW	DW	
cv = b ₀ + b ₁ d ² ch + εi			67.86%	17.38%	24.230	6.03E-04	3.08 ^{ns}	0.97 ^{ns}	1.91 ^{ns}	
cv = b ₀ + b ₁ d + b ₂ d ² ch + εi			70.44%	16.67%	14.107	1.68E-03	4.73 ^{ns}	0.95 ^{ns}	2.14 ^{ns}	
cv = b ₀ + b ₁ d + b ₂ ch + b ₃ d ² ch + εi			72.38%	16.11%	10.607	3.67E-03	5.55 ^{ns}	0.90 ^{ns}	1.52*	
Coefficients			S _{yx}	t	α (t)	Confidence Interval				
Forward method	cv = b ₀ + b ₁ d ² ch + εi									
	b ₀	-0.56767	4.63	-0.12	9.1E-01	-10.90	≤ Y ≤	9.76	-	-
	b ₁	0.01467	0.00	4.92	6.1E-04	0.00	≤ Y ≤	0.02	-	-
cv = b ₀ + b ₁ d + b ₂ d ² ch + εi										
b ₀	-38.17384	27.84	-1.37	2.0E-01	-101.06	≤ Y ≤	24.81	-	0	

Table 1. Contd.

b_1	3.42212	2.50	1.36	2.0E-01	-2.23	$\leq Y \leq$	9.08	0.47	2
b_2	0.00988	0.00	2.19	5.6E-02	0.00	$\leq Y \leq$	0.02	0.47	2
$cv = b_0 + b_1 d + b_2 ch + b_3 d^2 ch + \epsilon i$									
b_0	-183.76743	117.14	-1.56	1.6E-01	-453.91	$\leq Y \leq$	86.37	-	0
b_1	14.84143	9.26	1.60	1.5E-01	-6.52	$\leq Y \leq$	36.20	0.02	37
b_2	8.39067	6.57	1.27	2.4E-01	-6.76	$\leq Y \leq$	23.54	0.00	115
b_3	-0.04108	0.04	-1.02	3.4E-01	-0.13	$\leq Y \leq$	0.05	0.00	214

Where: W = White, SW = Shapiro-Wilk, DW = Durbin-Watson, T = tolerance and VIF = variance inflation factor.

variation of less than 5% and S_{yx} variation of less than 1.5% and a similar trend in the distribution of the residuals (Figure 1). Despite the significant values of F , indicating that the fitted equation should explain the variation of a dependent variable as a function of the independent variables, the results for the second and third steps (which theoretically should be the best) resulted in non-significant coefficients, and increased standard error for each coefficient. This increase may indicate the presence of a multicollinearity effect, demonstrated by the VIF values and the high linear correlations found between the independent variables in the third step.

Given the advantage based on the statistics of fit of the model obtained in the first step over the others and it successfully met all of the conditions required by the regression, the model obtained by the forward method was the same as that obtained by the stepwise method ($cv = b_1 d^2 ch + \epsilon i$).

Comparatively, the two equations show fits described in Table 1. The equations result in a variation of less than 1.5% in terms of R^2_{adj} , less than 5% with respect to S_{yx} and similar distribution of residuals along the estimate curve. However, the F values for the equation obtained by stepwise and forward methods were more significant and

yielded lower standard error in the coefficient. That is, the stepwise selection statistical procedure resulted in a more advantageous model than the backward method. These results corroborate those of Draper and Smith (1966), who assert that the stepwise method is one of the most recommended for the careful selection of explanatory variables in model construction.

In order to verify the performance of the best model, evaluations were conducted (in the same manner as that required for model generation) in 1, 3 and 5 year old plantations.

Chi-square values (χ^2) were not significant for any of the ages, implying that there are significant differences between estimated and actual values. Furthermore, the statistical fits represented by R^2_{adj} and S_{yx} ranged from 86.01 to 88.63% and from 16.66 to 22.78%, respectively. These values were similar to those obtained from the fits used in model construction from data taken in seven year old stands, a fact that indicates stability in the model under different conditions. The fit also yielded significant F values, significant coefficients, low standard error for the coefficients, and adequate distribution of residuals over the whole estimate curve.

Meng et al. (2007) formulated models for crown volume of *Pinus contorta* Engelm. based on

uniform stress theory and found that the models were able to represent about 70% of the variation in crown volume. In that study the independent variables were diameter at breast height, distance between the centre of the crown and the diameter at breast height, and wind speed. Davies and Pommerening (2008) concluded that models of crown volume of *Picea sitchensis* (Bong.) and *Betula* spp. using independent variables relating only to the bole were inefficient, especially in the case of suppressed trees. According to the authors, the inclusion of variables related to the spatial distribution of trees and stand structure considerably improved the models, representing up to 77% of the variation in the crown volume of the species.

By comparison, the models used in this study have showed statistical precision superior to those of the cited authors and have included fewer variables more appropriately selected. Such conditions characterize quality equations for estimation of the dependent variable, as defined by Draper and Smith (1966).

Conclusions

The variable with the highest correlation with the

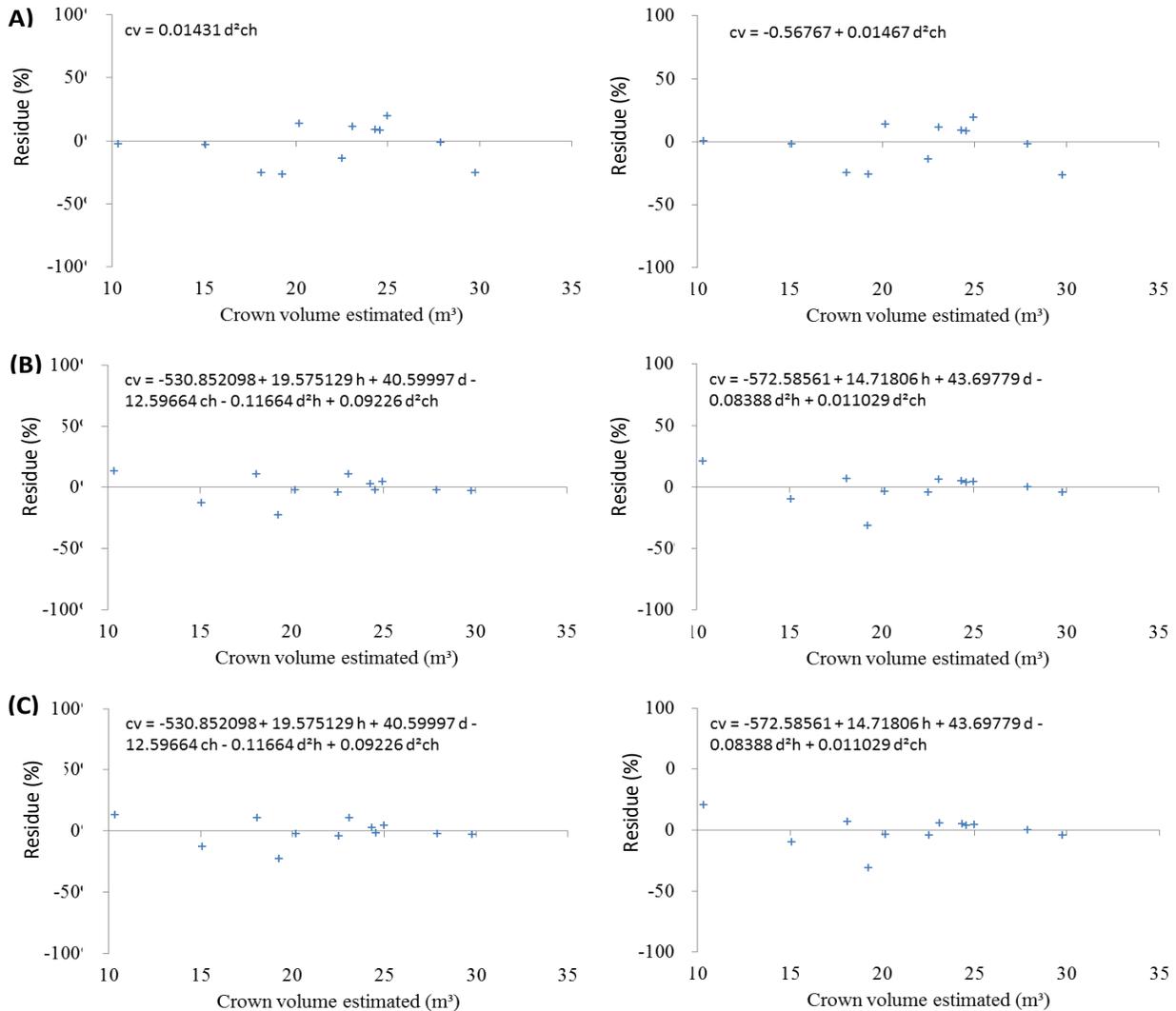


Figure 1. Distribution of residuals of crown volume with respect to estimated volume in stands of black wattle in Rio Grande do Sul, obtained from equations formulated by stepwise (A), backward (B) and forward (C) methods.

volume of the crown of black wattle was the squared diameter at breast height combined with the height of the crown (d^2ch). This variable was therefore selected to compose the model that presented the best fit, resulting both from the stepwise as well as forward methods. The final model resulting from these methodologies ($cv = b_1 d^2ch + \epsilon_j$) made it feasible to obtain crown volume both in aged rotation stands and in young stands, revealing stable statistical precision in both cases. In addition to the characteristics of accuracy, the obtained model includes easily measurable variables, and is therefore suited to activities requiring knowledge of the crown volume in black wattle.

Conflict of interest

The authors declared that they have no conflict of interest.

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