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Multi-objective cropping pattern in the Vaalharts irrigation scheme

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This study presents the multi-objective cropping pattern modeling of a farm in the Vaalharts irrigation scheme (VIS) in South Africa. The cropping pattern model presents three objectives and three constraints. The objectives are to maximize the total net benefit (NB) in monetary terms (South African Rand, ZAR) generated by planting four different crops, maximize total agricultural output (tons) and minimize the irrigation water use (m$^3$). The total farm size is 77.1 ha while the total available water for irrigation is 9,140 m$^3$ per ha/annum. Multi-objective differential evolution algorithm (MDEA) which is a stochastic multi-objective evolutionary algorithm recently developed was used to solve the multi-objective model in this study. The model produced non-dominated solutions that converge to Pareto optimal front. The averages of total net benefit, total agricultural output, total irrigation water and total area planted are ZAR 882 890.63, 3 439 518.75 tons, 702 522.50 m$^3$ and 661 444.06 m$^2$ respectively with corresponding average planting areas of 416 680, 53 030, 87 620 and 212 410 m$^2$ for maize, groundnut, Lucerne and Pecan nuts respectively. It is concluded that MDEA is a good optimizer for multi-objective cropping pattern model for generating maximum agricultural output for the farmers in the area with the constraints of land and water availabilities.

Key words: Cropping pattern, irrigation, agricultural output, differential evolution, non-dominated solutions.

INTRODUCTION

Vaalharts irrigation scheme (VIS) is important to agricultural production in South Africa as one of the largest irrigation schemes in the world. It covers an area of 36 950 ha. Water is provided to some 680 farmers. The scheme is supplied with water abstracted from Vaal River. Agricultural productivity is of importance in the scheme to provide sufficient food to South Africans. Water use in agriculture in South Africa is always emphasized because of shortage of irrigation water in the country. South Africa is a dry country with less than 500 mm rain on average annually over about two-third of her area. Farmers should therefore adopt a cropping pattern, in a multi-crop environment, to maximize the agricultural output, minimize the irrigation water use and maximize the total profit from farming.

Usually, irrigation demand is higher than available water in South Africa. Therefore, cropping pattern determination is a serious challenge to prevent loss to farmers when irrigation water requirements are not met. Cropping pattern determination presents itself with many conflicting objectives and constraints. Among the constraints are irrigation canal capacities, available water for irrigation, land availabilities, labour requirements, farming equipment, machinery for farming operations etc. Irrigation performances are influenced by deficit allocation among the competing crops in a water scarce environment (Janga and Nagesh, 2008).

Many researchers have worked on irrigation planning...
and crop planning using single and multi-objective techniques (Adeyemo and Otieno, 2009; Bergez et al., 2019; Chang and Chang, 2009; Croley and Rao, 1979; Doppler et al., 2002; Grove, 2006; Janga and Nagesh, 2007a; Madsen et al., 2006). Several studies have reported the use of evolutionary algorithms in water management in agriculture. Ines et al. (2006) present an innovative approach to explore water management options in irrigated agriculture considering the constraints of water availability and the heterogeneity of irrigation system properties. They set up a soil-water-atmosphere-plant model (SWAP) in a deterministic-stochastic mode for regional modeling. Genetic algorithm (GA) was used in data assimilation and water management optimizations. Their results showed that under limited water condition, regional wheat yield could improve further if water and crop management practices are considered simultaneously and not independently.

In another study by Chen et al. (2008), parallel GEGA was constructed by incorporating grammatical evolution (GE) into the parallel GA to improve reservoir water quality monitoring based on remote sensing images. The GE automatically discovers complex nonlinear mathematical relationships among observed Chl- a concentrations and remote-sensed imagery. A GA was used afterward with GE to optimize the appropriate function type. The performance of parallel GEGA was found to be better than that of the traditional linear multiple regression with lower estimating errors. Kerachian and Karamouz (2007) also studied water quality management in reservoir-river systems. Optimal operating rules for water quality management in reservoir-river system are developed using a methodology combining a water quality simulation model and a stochastic GA-based conflict resolution technique.

Guan et al. (2009) proposed a resource assignment and scheduling based on a two-phase metaheuristic for a long-term cropping schedule. Their simulated results indicated that the formulated schedule has high ratio of resource utilization in sugarcane production. Their approach also contributes a referential scheme for applying the metaheuristic approach to other crop production scheduling.

Tittonell et al. (2007) showed that inverse modeling techniques can be used effectively for optimization and trade-offs analysis of farming systems in their study. They analysed the trade-offs between resource productivity, use efficiency and conservation in relation to different patterns of resource allocation for a maize-based simplified case study farm from western Kenya.

A new DE algorithm for multi-objective optimization problem (MOOP) is proposed in this study and called multi-objective differential evolution algorithm (MDEA). The DE algorithm proposed is based on the existing DE algorithm proposed by Price and Storn (1997). The difference is its implementation of multi-objectives. Though the proposed MDEA can be used on any strategy, the strategy used in this study is DE/rand/1/bin which is the most widely used of all the ten strategies of DE.

Mathematical optimization model is used to solve the problem of cropping pattern in a water scarce environment. This will enable a farmer to know the combination of crops to plant on the available area of land using available irrigation water to maximize his agricultural output. The model in this study is adapted to a farmland in the Vaalharts irrigation scheme (VIS) in South Africa.

The objective of the study is to find the corresponding planting areas where each of the four crops namely, maize, groundnut, Lucerne and Pecan nuts should be planted to maximize both the total net benefit (NB) in South African Rand (ZAR) and total agricultural output when a farmer is using the minimum irrigation water. The first objective of the model is to maximize the total net benefit (NB) in monetary terms (South African Rand, ZAR) generated by planting the four crops (with 77.1 ha of land). The second objective is to maximize the total agricultural output while the third objective is to minimize the irrigation water use. The overhead costs per annum, household expenses per annum and fixed liabilities per annum on the average for the selected farm are taken from Grove (2006).

**MATERIALS AND METHODS**

**Multi-objective differential evolution algorithm (MDEA)**

Differential evolution (DE) is a novel heuristic optimization technique which is exceptionally simple evolution strategy, more likely to find a function’s true global optimum, known for fast convergence, robust at numerical optimization and has few control parameters (Price and Storn, 1997b). DE is a simple but yet powerful population based, in direct search algorithm for globally
optimizing functions with real value parameters (Babu and Jehan, 2003). DE has been used for multi-objective optimization especially in water resources with good results (Angira and Babu, 2005; Babu and Jehan, 2003; Reddy and Kumar, 2007; Santana-Quintero and Coello, 2005). Several studies by Deb et al. (2002), Xue et al. (2003), Angira and Babu (2005), Babu et al. (2005), Madavan (2002), Parsopoulos et al. (2004) and Robic and Filipic (2005) have proposed ways of extending DE to handle multi-objectives. MDEA combines the advantages of multi-objective differential evolution (MODE) proposed by Babu and Jehan (2003) and the algorithm proposed by Fan et al. (2006) while overcoming the shortcomings of the algorithms. In this way, MDEA runs faster with better and more Pareto optimal solutions. The description of the MDEA is as follows: The vectors are randomly generated to create initial solutions to the problem; the generated solutions are allowed to undergo mutation, crossover and selection for the number of generations; the solutions that evolve are checked for domination and the dominated solutions are removed. The selection procedure of Fan et al. (2006) is modified. The trial solution survives to the next generation if its objective function is better in at least one objective that is non-dominated to the target solution. MDEA will produce many non-dominated solution on the Pareto front than the algorithm by Fan et al. (2006). Moreover, the algorithm by Fan et al. (2006) has not been modified to handle constraints except bound constraints. MDEA can handle multiple constraints. If any of the constraints is violated, a high value (108) is added to the objective function to make the solution infeasible (Deb, 2001). In this way, the solution will not be selected when compared with other solutions because of high value.

**Study area**

The study area is a farmland in the Vaal harts irrigation scheme (VIS). The size of the farm is 77.1 ha. Vaal harts irrigation scheme is in the east of Fhaap Plateau on the Northern Cape and North West province border in South Africa. VIS covers about 36 950 ha and is one of the largest areas in the world. Water is provided to some 680 farmers (Grove, 2006). The scheme is supplied with water abstracted from Vaal River at the Vaal harts’ weir about 8 km upstream of Warrenton. A canal is used to convey the water to the scheme. VIS is located in a summer rainfall area. This area battles with low, seasonal and irregular rainfall. The average rainfall is 442 mm per year (Jager, 1994). The average precipitation in summer months, October to February differs between 9.1 and 9.6 mm/day while in July precipitation is only 3.6 mm/day. The capacity of the canals in Vaal harts is 4 mm/day which is less than the desired. The common crops grown in the area are wheat/barley, maize, groundnuts, cotton and other permanent crops like Lucerne, Pecan nuts, grapes, olives and some other fruits.

**Model formulation**

The farm used as a case study has an area of 77.1 ha of land. It is supplied with 9,140 m³ per ha/annum of water. A farmer plants 4 different crops namely maize, groundnuts, Lucerne and Pecan nuts. Each crop is planted in at least 5 ha of land and at most in known maximum irrigated areas. These constitute the boundary constraints of the problem. The minimum planting areas ensure the availability of all the crops in the market while the maximum planting areas ensure that a farmer will not have storage or selling problem if the yields exceed the storage facilities available or if the demand is less than the supply which could cause the selling price to fall. The monthly estimated gross irrigation water requirement (mm/ha) for the selected crops under flood irrigation in Vaal harts in Groove (2006) are used in this study.

**Solution method**

The problem is solved using the proposed MDEA. The pseudo code of MDEA is presented below. The proposed MDEA methodology is summarized in the following steps:

1. Input the required DE parameters like population size, crossover constant, scaling factor maximum generation, number of objectives, bound constraints and so on.
2. Initialize all the vector populations randomly in the limit of bound constraints.
3. Set the generation counter, G = 0.
4. Perform mutation and crossover operations on all the population members:
   a. For each parent, select 3 distinct vectors from the current population. The selected vectors must not be selected from the parent vector.
   b. Calculate new mutation vector using the expression,
      \[ V_i(g) = X_{i0}(g) + F(X_{i1}(g) - X_{i2}(g)) \]
   c. Perform crossover using binary crossover method because of the strategy DE/rand/1/bin used in MDEA.
5. Evaluate each member of the population and check the offspring for domination. Replace the parents with offspring if the offspring dominate the parents in the next generation otherwise, the parents proceed to the next generation.
6. Increase the generation counter, G, by 1. That is G = G+1 and check if G = GMAX. If G < GMAX, then go to step 4 above and repeat mutation, crossover and selection. If G = GMAX, then go to step 7.
7. Use naïve and slow method (Deb, 2001) of removing the dominated solutions in the last generation. A solution is dominated if there is another solution which is better in it in all the objectives.
8. Output the non-dominated solutions.

**The pseudo code for multi-objective differential evolution algorithm (MDEA)**

*Initialize the values of D, NP, Cr, F, k (number of objective functions) and maximum generation (MAXGEN)*

*Input the boundary constraints of the problem*

*Initialize all the vectors of the population randomly*  
For i = 1 to NP  
For j = 1 to D  
\[ X_{ij} = \text{Dlower}_{ij} \text{ + random number (0,1)} \times (\text{Dupper}_{ij} - \text{Dlower}_{ij}) \]  
Next j  
Next i

*Initialize gen = 1*  
For gen = 1 to MAXGEN  
For i = 1 to NP  
For m = 1 to k  
Evaluate the objective function values  
Next k  
Check the constraints violation  
If violated objective function value = objective function value + 8*10⁸  
(Deb K, 2001)  
For j = 1 to D  
(Mutation stage)
Select 3 different vectors for perturbation different from i such that \(i \neq 1 \neq 2 \neq 3\)

\[ V_{i, gen} = X_{i, gen} + F(X_{1, gen} - X_{2, gen}) \]

(Crossover Stage)

\[ u_{i,j,gen} = \begin{cases} v_{i,j,gen} & \text{if } (rand) \leq Cr \text{ or } j = n \\ x_{i,j,gen} & \text{otherwise} \end{cases} \]

and \(n \in \{1, ..., D\}\)

Next j
(Selection stage)
Start comparing vectors \(U_{i, gen}\) and \(X_{i, gen}\)
For \(m = 1\) to \(k\)
Evaluate \(k\)th objective function, \(f_k(U_{i, gen})\)
If \(f_k(U_{i, gen}) \leq f_k(X_{i, gen})\)
Select vector \(U_{i, gen}\), the trial vector
\((X_{i, gen+1}) = (U_{i, gen})\)
Go to next step
Else

Next m
End if
Select \(X_{i, gen}\) the current population member for the next generation
\((X_{i, gen+1}) = (E_{i, gen})\)
Next m
Next i
Next gen
*remove the dominated solutions from the last generation using naïve and slow method proposed by Deb (2001).

Print the results.

The objective functions are formulated as thus explained.

Objective function 1: Maximization of total net benefit

\[ \text{Max NB} = \sum_{i=1}^{N} (T_i \times A_i) - (A_i \times CWR_i \times C_{HE} + C_{FL} \times C_{HE}) \]

(1)

Where, NB is the overall net benefit on the whole farm; \(N\) is the number of crops; \(T_i\) is the total income of ith crop in rand per annum; \(A_i\) is the area where ith crop is grown in m²; \(CWR_i\) is the crop water requirements for crop i; \(C_{HE}\) is the cost for water per m³ = 8.77 cents; \(C_{FL}\) is the overhead costs per annum; \(C_{HE}\) is the household expenses per annum; \(C_{FL}\) is the fixed liabilities per annum.

To compute the total income (ZAR/m²) from each crop, the selling price (ZAR/ton) (Agriculture, 2008) of crop (i) is multiplied by yield (ton/ha) (Agriculture, 2008) and divided by 10 000.

\[ T_i(ZAR/m^2) = \text{Price(R/ton)} \times \text{Yield(ton/ha)} / 10 000 \]

(2)

Objective function 2: Maximization of agricultural output

Total agricultural production of all the crops are to be maximized for meeting the demands of the consumers:

\[ \text{Max AP} = \sum_{i=1}^{N} \left( \frac{[Y_i \times A_i]}{10000} \right) \]

(3)

Where, AP is the agricultural output (tons/m²) and \(Y_i\) is the yield of ith crop (tons/ha)

Objective function 3: Minimization of irrigation water

The volume of water used for irrigation is minimized:

\[ \text{Min Vol} = \sum_{i=1}^{N} (CWR_i \times A_i) \]

(4)

The three objectives above are subjected to the following constraints as thus explained.

Constraint 1: Total area

The sum of all the areas of land where the crops are grown is less than or equal to the total area, \(A\), available for farming and should be less than 77.1 ha:

\[ \sum_{i=1}^{N} (A_i) \leq 771000 \]

(5)

Constraint 2: Monthly release

The irrigation in any month cannot exceed the canal capacity. The canal capacity can be converted to volumetric units, m³, so that it becomes compatible with releases to the canal. According to Grove (2006), water is supplied to the farm for 5½ days a week. The water available for one hour is 150 m³ because of the canal capacity constraint. Therefore the maximum volume of water that may be available on the farm is:

150 m³/h × 24 h × 5.5 days × 4 weeks = 79 200 m³ monthly

Therefore,

\[ \text{IRD}_t \leq 79200; t = 1, ---12. \]

(6)

Where, \(\text{IRD}_t\) is the irrigation demand in month; \(t\) is the total of crop water requirements for all the crops in month t.

Thus the total amount of water available in a year using the canal capacity as constraints is 79 200 × 12 = 950 400 m³. The model minimizes the total volume of irrigation water. The volume of water is required to be less than 950 400 m³ per annum.

Constraint 3: Minimum and maximum planting areas

To make sure that all the crops are grown in at least 50 000 m² of land, each area, \(A_i\), must be equal or greater than 50 000 m² and less than or equal to the maximum areas for each crop:

\[ 50000 \leq A_i \leq A_{i,\text{max}} \]

(7)

Where, \(A_{i,\text{max}}\) is the maximum area where each crop should be grown.

This constraint is justified because some crops have high...
Table 1. Minimum, maximum and mean of total net benefit, total agricultural output, total irrigation water and planting areas for each crop from non-dominated solutions.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Total net benefit (ZAR)</th>
<th>Total agricultural output (ton)</th>
<th>Total irrigation water (m$^3$)</th>
<th>Maize</th>
<th>Groundnut</th>
<th>Lucerne</th>
<th>Pecan nut</th>
<th>Total area</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>109 210</td>
<td>1 893 300</td>
<td>439 010</td>
<td>58 050</td>
<td>53 030</td>
<td>50 730</td>
<td>50 110</td>
<td>373 720</td>
</tr>
<tr>
<td>Maximum</td>
<td>1 306 100</td>
<td>4 664 800</td>
<td>1 033 000</td>
<td>465 280</td>
<td>469 120</td>
<td>135 160</td>
<td>336 900</td>
<td>770 300</td>
</tr>
<tr>
<td>Mean</td>
<td>882 891</td>
<td>3 439 519</td>
<td>702 523</td>
<td>232 723</td>
<td>257 518</td>
<td>80 473</td>
<td>90 729</td>
<td>661 444</td>
</tr>
</tbody>
</table>

Figure 1. Pareto optimal set for the crop planning model.

RESULTS AND DISCUSSION

The results of this model are presented in Table 1 and Figures 1, 2, 3 and 4. The maximum, minimum and mean of the 64 generated non-dominated solutions are presented in Table 1. Figure 1 presents the Pareto non-dominated solutions. The total net benefit and total agricultural output are maximized while total irrigation water is minimized.
optimal set for the cropping pattern. In the figure, it is found that the solutions converge to the Pareto front. In the Pareto optimal solution set, each solution is not better than the others in all the objectives. In practice, the decision-maker ultimately has to select one solution from this set for system implementation.
Figure 2 presents the combination of planting areas for the 4 crops in the 64 non-dominated solutions generated for the model. It is found that groundnut and maize have the highest planting areas in the majority of the solutions. Groundnut has the highest planting areas in 31 solutions while maize has the highest in 30 solutions. Lucerne has the highest planting areas in only two solutions while Pecan nut has the highest in just one solution. It is found in this study that Pecan nuts have the highest planting areas in just one solution because of water shortages. This confirms the study of Grove (2006) which reports that if it is feasible to include Pecan nuts, the expected net present value will be higher. However, the impact of water shortages will be more severe. From the analysis of all the solutions in Figures 3 and 4, it can be seen from solution 14 that the maximum total net benefit generated from the 4 crops planted is ZAR 1 306 100 while the corresponding total agricultural output, total volume of irrigation water and total area planted are 2 628 300 tons, 1 033 000 and 729 250 m$^2$ respectively. This solution is not within the constraints. Presently, the irrigation water available to the area is 9 140 m$^3$ per ha/annum amounting to 704 694 m$^3$ (Grove, 2006). This is less than 1 033 000 m$^3$ that is needed to plant the 4 crops in this solution. It is reported by Grove (2006) that the water supplied to the farmers is less than desired. Therefore, for this solution to be adopted by a farmer in the Vaal hart’s irrigation scheme, the water allocation needs to be increased to maximize the land use. It is better for a farmer to choose another solution that will maximize the land use within the available irrigation water but with lower total net benefit. This solution can only be chosen in future if the irrigation water allocation to the area is increased or if a farmer can get water from other sources like underground water withdrawal. For example, a farmer who chooses solution 5 with maximum area of land 770 300 m$^2$, will use irrigation water of 801 790 m$^3$ and the total net benefit of ZAR 1 162 400. In this case, the total area and total irrigation water are within the constraints though the present regulation supplies water less than 801 790 m$^3$, the canal capacity can accommodate the irrigation water needed. If a farmer decides to choose solution 26 which corresponds to the minimum irrigation water use (439 010 m$^3$), it may be impossible for him to utilize all the available land. The total net benefit, total agricultural output and total area are ZAR 436 620, 1 899 800 tons and 373 720 m$^3$ respectively. In this solution, only about half of the farm land will be utilized. Solution 1 with total agricultural output, total net benefit, total irrigation water and total area of 1 893 300 tons, ZAR 678 300, 546 940 m$^3$ and 447 260 m$^2$ respectively will be the best for a farmer who has storage and marketing problem because it corresponds to the minimum agricultural output.

In the model, all the three objective functions cannot be satisfied. In a multi-objective optimization, there cannot be a solution that will satisfy all the objectives but instead, there are sets of solutions in one simulation run which corresponds to non-dominated solutions (Deb, 2001). It depends on a farmer to choose the best solution that
suits him from the set of non-dominated solutions. The solutions are optimal in the sense that no other solution in the search space is superior to them when all the objectives are considered. The goal of multi-objective problems is to find as many Pareto-optimal solutions as possible to reveal trade-off information among different objectives (Deb, 2001). Once such solutions are obtained, higher level decision maker will be able to choose a final solution with further consideration like water availability, land area, market situation, and available storage facilities and so on in this study. From Table 1, the averages of total net benefit, total agricultural output, total irrigation water and total area are ZAR 882 890.63, 3 439 518.75 tons, 702 522.50 m$^3$ and 66.144406 ha respectively. From these averages, it is found that the results are within the limits of the constraints. The mean irrigation water of 702 522.50 m$^3$ is less than 704 694 m$^3$ allocated annually to the farm. Also, the mean total area of land irrigated is 66.144406 ha which is also less than 77.1 ha available for farming. The averages of different planting areas are 416680, 53030, 87620 and 212410 m$^2$ for maize, groundnut, Lucerne and Pecan nuts respectively.

**Conclusions**

The application of multi-objective differential evolution algorithm (MDEA) to cropping pattern model is demonstrated in this study. The model results demonstrate the ability of MDEA as applied to multi-objective constrained problem of cropping pattern. Figure 1 shows the convergence of the solutions to Pareto optimal front. Different solutions on the Pareto optimal fronts will generate trade-offs for the problem. The algorithm is suitable for cropping pattern in a multi-crop environment with limited water for irrigation as demonstrated. From the analysis of the solutions, the averages of total net benefit, total agricultural output, total irrigation water and total area planted are ZAR 882 890.63, 3 439 518.75 tons, 702 522.50 m$^3$ and 661 444.06 m$^2$ respectively with corresponding average planting areas of 416 680, 53 030, 87 620 and 212 410 m$^2$ for maize, groundnut, Lucerne and Pecan nuts respectively. Therefore, the model is a good alternative for farmers to obtain the optimal crop planning in a water scarce environment like South Africa. The objective of this study was achieved. The model in this study can be adapted to any irrigation scheme with minor modifications.

**RECOMMENDATIONS**

Agricultural production in the Vaal Hart irrigation scheme can be improved if water allocation to the area can be increased. From the results in Figure 4, the maximum total net benefit of ZAR 1 306 100 in solution 14 can only be achieved, if the water allocation is increased. Moreover, double cropping, which can increase a farmer’s profit, is possible with combination of some of the crops being planted in the area but this is limited by lack of enough irrigation water. In this way, land is being wasted thereby reducing a farmer’s profit. The interim solution to the problem is for farmers to plant crops that are water efficient. A farmer may also seek alternative water supply to supplement irrigation water allocated. Also, the irrigation technology can be improved to reduce water loss. Flood irrigation technique can be water consuming. Farmers in the area can make use of other irrigation techniques to make more water available for other crops to cultivate more.

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