African Journal of Agricultural Research Vol. 2 (7), pp. 300-304, July 2007
Available online at http://www.academicjournals.org/AJAR
ISSN 1991- 637X © 2007 Academic Journals

Full Length Research Paper

A weighted relative density model applied to loblolly pine (*Pinus taeda* L.) stands

John M. Lhotka[1] and Edward F. Loewenstein[2]

[1]University of Kentucky, Department of Forestry, 105 T. P. Cooper Building, Lexington, KY 40546-0073.
[2]Auburn University, School of Forestry and Wildlife Sciences Building, Auburn University, Alabama 36849-5418.

Accepted 21 June, 2007

We present a weighted, individual-tree relative density approach whose reference conditions have the same distribution of crown areas and species as a subject stand. An initial evaluation of the method's efficacy was performed within an even-aged loblolly pine (*Pinus taeda* L.) plantation. Species-specific reference density equations were developed using plot data and existing crown width equations. Diameter growth prediction was evaluated using data from a thinned 20-year-old plantation. Regression analysis was used to related annual diameter increment and the approach's density indices. The predictive ability of the presented method was compared to existing approaches such as Stand Density Index (SDI) and Crown Competition Factor (CCF). Results suggest the weighted, individual-tree approach predicts diameter increment as well as traditional indices. These findings suggest an individual tree approach may not be necessary within an even-aged loblolly pine plantation with a narrow diameter range.

Key words: relative density, size-density, *Pinus taeda* L., loblolly pine

INTRODUCTION

The interaction of tree size and stand density dictates the amount of growing space available for each tree and affects tree growth and mortality. By understanding size-density relationships and concepts of self-thinning, silviculturists have the ability to manage stand density and to influence available growing space, tree size, growth, and mortality. Researchers have developed quantitative approaches to aid in the understanding of size-density relationships and to evaluate alternative management regimes (Jack and Long, 1996; Newton, 1997). Such methodologies generally incorporate a relative density component that compares observed stand density to some reference level of competition (Curtis, 1970).

Because tree species can be dissimilar in terms of crown architecture and physiological response to the forest environment, size-density relationships can differ among species (Zeide, 1987; Puettmann et al., 1993; Hynynen, 1993; Osawa, 1995). Due to variability among species, application of density management aids, which do not include species characteristics, may result in inappropriate thinning regimes and suboptimal yield (Pretzsch and Biber, 2005; Pretzsch, 2006). Sterba and Monserud (1993) found that the skewness of a diameter distribution could influence size-density relationships. Therefore, relative density approaches may need to incorporate species composition and diameter structure to appropriately characterize size-density relationships in mixed species stands and/or those with skewed diameter distributions. Other authors highlight that relative density approaches must also have utility in stand management and have the ability to predict growth (Dean and Baldwin, 1996; Ducey and Larson, 2003).

Authors have presented relative density approaches that account for irregular diameter distributions (Long and Daniel, 1990; Shaw, 2000) and incorporate species compositions (Stout and Nyland, 1986; Torres-Rojo and Velázquez-Martínez, 2000). However, the successful development of an approach that accounts for variable diameter structure and species mixture has been limited. In response, a weighted, individual-tree relative density (Wit-RD) framework has been under development. Our goal has been to create a relative density approach whose density reference conditions have the same distribution of

*Corresponding author. E-mail: john.lhotka@uky.edu

canopy areas and species as a subject stand. The first step to examining the approach's efficacy is to evaluate its behavior and utility under the most basic stand conditions, an even-aged mono-specific plantation. Loblolly pine (*Pinus taeda* L.) was chosen because it is commercially important and is commonly used in single-species plantations within the southeastern United States.

We will outline the structure of the WitRD framework and apply it to loblolly pine by developing the appropriate relative density reference equations. Because the ability to predict growth is an important trait of a relative density index (Dean and Baldwin, 1996; Ducey and Larson, 2003), we evaluate the relationship between the approach and diameter growth of plantation-grown loblolly pine. The predictive ability of the proposed approach is compared to the ability of existing metrics such as Reineke's (1933) Stand Density Index (SDI) and Crown Competition Factor (CCF) (Krajicek et al., 1961).

METHODS

A weighted, individual-tree relative density approach was developed to assess size-density relationships in stands possessing varied diameter and species distributions. The approach's primary relative density measure, Relative N_{MCA}, was defined as the proportion of observed stand density (N) to the minimum density of full site occupancy (N_{MCA}).

$$\text{(1) Relative } N_{MCA} = \frac{N}{N_{MCA}}$$

Where,
N = observed stand density
N_{MCA} = minimum density of full site occupancy (trees ha^{-1})

Krajicek et al. (1961) defined N_{MCA} as the maximum density at which each tree still possesses its maximum open-grown crown area and is the density where CCF equals 100. N_{MCA} can also be defined as the density where crown competition begins. Following Krajieck et al. (1961), the reference density assumes uniform tree spacing and round crowns with sufficient plasticity to fill all canopy voids without diminishing the crown area of individual trees. Unlike CCF (Krajieck et al., 1961), our approach includes a secondary density reference, percent N_{AMax}, to describe how stand density compares to average maximum density (N_{AMax}) of normally stocked stands (Gingrich, 1967).

$$\text{(2) Percent } N_{AMAX} = \frac{N}{N_{AMax}}$$

Where,
N = observed stand density
N_{AMax} = average maximum density (trees ha^{-1})

Normal stands are defined as those whose density fluctuates around an equilibrium level (Zeide, 2004).

Tree-by-tree calculations are used to help account for the influence of stand specific diameter structure on the computed relative density indices (Long and Daniel, 1990; Shaw, 2000). The approach incorporated species composition by utilizing species-specific N_{MCA} and N_{AMax} reference density equations and was developed so any species or species group could be included. Species-specific, stand-grown crown width models are used to estimate the growing space proportion occupied by each subject tree and these estimates are used to adjust the overall N_{MCA} and N_{AMax} calculations. A weighted summation approach was used to help incorporate the influence of individuals trees on a stand's size-density characteristics and to assure the relative density reference conditions have the same distribution of crown areas and species as the subject stand.

Our goal was to evaluate the utility of the outlined relative density approach in pure, even-aged stands. Loblolly pine was chosen because it commercially important and is commonly used in single-species plantations. To apply this approach to loblolly pine, three model components were needed:

1) Minimum density of full site occupancy (N_{MCA}) equation.
2) Average maximum density (N_{AMax}) equation.
3) stand-grown crown width equation. N_{MCA} was calculated by dividing the square meters in a hectare (10 000 m^2) by the maximum crown area for a given tree diameter (Krajieck et al., 1961; Assmann, 1970).

For loblolly pine, crown area was calculated using the maximum open-grown crown width (MCW) model (R^2 = 0.948) presented in Smith et al. (1992).

$$\text{(3) MCW} = 0.738 + 0.254(\text{dbh}) - 0.000809(\text{dbh}^2)$$

Where,
CW_{open} = maximum open-grown crown width (m) for a given tree diameter (cm)
dbh = tree diameter at breast height (cm)

Using equation 4, minimum density of full site occupancy is determined for diameters between 5 and 50 cm.

$$\text{(4) } N_{MCA} = \frac{10\,000}{MCW^2 \left(\frac{\pi}{4}\right)}$$

Where,
N_{MCA} = minimum density of full site occupancy
MCW = maximum open-grown crown width (m) for a given tree diameter (cm)

A loblolly pine average maximum density (N_{AMax}) equation was developed using plot data from the United States Forest Service - Forest Inventory and Analysis Database (FIADB) (U.S. Department of Agriculture Forest Service, 2005). In an attempt to more appropriately represent the size-density characteristics of loblolly pine in the southeastern United States, data were geographically restricted to the following states: Alabama, Georgia, South Carolina, and North Carolina. Quadratic mean diameter, trees ha^{-1}, and basal area (m^2 ha^{-1}) were calculated for all trees greater than 5 cm. All plots with a density and basal area less than 90 % loblolly pine were removed to limit the influence of associated species on data analysis. This resulted in the selection of 1,549 FIA plots.

Average maximum density equations were fit using plots with a relative density greater than 70%. This secondary filtering criterion selected the most fully stocked plots (as outlined in Solomon and Zhang 2002). N_{AMax} equations were developed using Reineke's (1933) size-density model and Reduced Major Axis (RMA) regression (Leduc, 1987; Solomon and Zhang, 2002).

$$\log(N) = b_0 + b_1\log(D_q)$$

Where,
N = stand density
D_q = quadratic mean diameter
log = natural logarithmic function
b_0, b_1 = species-specific coefficients

Following initial fit, the resulting RMA regression slope coefficient was used to recalculate the relative density of all plots using the procedure outlined in Solomon and Zhang (2002). Again, plots pos-

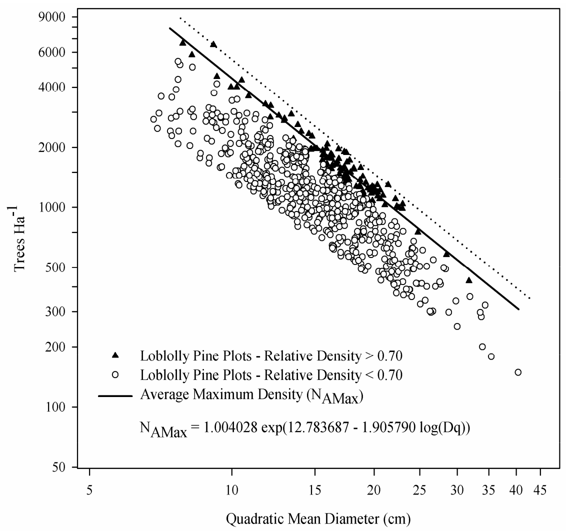

Figure 1. Average maximum density (N_{AMax}) relationship for loblolly pine. Reference equation relating N_{AMax} and plot quadratic mean diameter (Dq) was fit using plots possessing a relative density greater than 0.70. Plots with a relative density greater than 0.70 are represent by the' ▲ 'symbols, while plots with a relative density less than 0.70 and an observed density greater than the minimum density of full site occupancy (N_{MCA}) are denoted by the ' ○ ' symbol. The dotted line serves as a reference for the observed maximum size-density relationship within the data evaluated. Axes are log-log transformed, but labeled in actual units.

sessing relative densities less than 70% were removed and RMA regression was repeated. This procedure was continued until resulting RMA regression coefficients stabilized. The iterative process was completed to limit potential model bias which could result from using a relative density selection criteria derived with coefficients that did not accurately represent the data. Data used to fit the final average maximum density model (Equation 6) are illustrated in Figure 1.

(6) $N_{Amax} = 1.004028 \exp(12.783687 - 1.905790 \log(dbh))$

Where,

N_{AMax} = Average maximum density (trees ha^{-1}) for a given diameter (cm)

1.004028 = logarithmic bias correction factor (Baskerville, 1992)

exp = exponential function

To evaluate the relationship between diameter increment and relative density, data were collected from a 20-year-old loblolly pine plantation (site index = 23 m) located in the Alabama Piedmont. At age 18, the stand had received three treatments: thin from below with 9 m^2ha^{-1} residual basal area, thin from below with 16 m^2ha^{-1} residual basal area, and no thin with 36 m^2ha^{-1} residual basal area. The resulting structural conditions allowed diameter growth to be evaluated over a range of relative densities. Sixteen, 10-m-radius plots were installed across the thinning treatments and were centered on randomly selected co-dominant loblolly pine. Diameter (cm) and distance to plot center (m) were recorded for each tree. Increment cores were taken from each center tree and current year's (2006) diameter growth was measured to the nearest 0.01 mm using digital calipers.

Relative density of the sixteen plots was determined by applying our weighted individual tree approach. First, growing space occupied by each tree was estimated using stand-grown crown width models (Table 1, Column 3). Loblolly pine's stand-grown crown width model (R^2 = 0.66) was drawn from Bechtold (2003).

7) $CW_{stand} = 0.7830 + 0.165804\ (dbh)$

Where,

CW_{stand} = stand-grown crown width (m) for a given tree diameter (cm)

Each tree's growing space proportion was then estimated by dividing its crown area by total crown area of all trees sampled within a plot (Table 1, Column 4). N_{MCA} (equation 4) and N_{AMax} (equation 6) were calculated for all subject trees and the resulting reference densities were weighted by the estimated growing space proportion occupied by a given tree (Table 1, Columns 5 and 6). Total N_{MCA} and N_{AMax} were computed by summing the adjusted reference densities of all trees. Relative N_{MCA} was calculated by dividing observed density by Total N_{MCA} (Table 1, Column 7) and Percent N_{AMax} was then be calculated by dividing observed density by Total N_{AMax} (Table 1, Column 8).

Relative N_{MCA}, Percent N_{AMax}, basal area (m^2 ha^{-1}), Stand Density Index (SDI) (Reineke, 1933), additive Stand Density Index (SDI_{Σ}) (Long and Daniel, 1990), and Crown Competition Factor (Krajicek et al., 1961) were calculated for each plot. The relationship between these metrics and 2006 diameter increment (incre06) of the sampled loblolly pines was quantified using regression analysis. Coefficient of determination (R^2) was used to assess relationship strength. The relationship between density measures and diameter increment appeared to be linear (n = 16).

RESULTS AND DISCUSSION

Plot relative density for the three thinning treatments were evaluated two growing seasons following harvest. The average Relative N_{MCA} and Percent N_{AMax} in the unthinned plots were 2.63 and 0.98, respectively. These values indicate that the unthinned plots have an average density 2.6 times greater than the N_{MCA} for the given diameter structure. Percent N_{AMax} indicates that the plots are at 98 percent of average maximum density. In comparison, the 16 m^2ha^{-1} residual basal area thinning treatment reduced the average Relative N_{MCA} to 1.18 and the 9 m^2ha^{-1} treatment reduced it to 0.74. The 9 m^2ha^{-1} residual basal area treatment reduced stand density 25% below N_{MCA}, thus creating conditions where residual trees will not fully utilize the available growing space and the canopy will remained gapped for an extended period. Intraspecifc competition will likely have little impact on diameter growth under these conditions (Strub et al., 1975).

Assessment of the proposed approach would not be complete without comparing its behavior with traditional relative density measures. Of the six density indices analyzed, basal area (R^2 = 0.6819) and Relative N_{AMax} (R^2 = 0.6875) explained the least amount of variance in diameter increment (Table 2). The remaining four indices explained over 70 % of the variance and their R^2 differed by less than 0.01 (Table 2). These results suggest that the weighted, individual-tree approach was comparable with SDI, SDI_{Σ}, and CCF in its ability to predict diameter growth within the stand conditions present.

Basal area has been used to guide the management of loblolly pine. However, because basal area does not

Table 1. Computational structure for the weighted, individual-tree relative density (WitRD) approach.

Data Inputs		Tree-by-Tree Calculations				Density Indices	
Diameter (dbh)*	Density (tph)	Crown Area (CA)	Growing Space Proportion (GSP)	Adjusted N_{MCA}‡	Adjusted N_{AMax}††	Relative N_{MCA}	Percent N_{AMax}
dbh_1	tph_1	$CA_{dbh1}(tph_1)$	$\frac{CA_{dbh1}(tph_1)}{\sum CA_i}$	N_{MCA1} (GSP_1)	N_{AMax1} (GSP_1)	$\frac{\sum tph_i}{\sum AdjN_{MCA}}$	$\frac{\sum tph_i}{\sum AdjN_{AMax}}$
dbh_2	tph_2	$CA_{dbh2}(tph_2)$	$\frac{CA_{dbh2}(tph_2)}{\sum CA_i}$	N_{MCA2} (GSP_2)	N_{AMax2} (GSP_2)		
dbh_i	tph_i	$CA_{dbhi}(tph_i)$	$\frac{CA_{dbhi}(tph_i)}{\sum CA_i}$	N_{MCAi} (GSP_i)	N_{AMaxi} (GSP_i)		
	$\sum tph_i$	$\sum CA_i$	$\sum GSP_i$	$\sum AdjN_{MCA}$	$\sum AdjN_{AMax}$		

*Tree diameter at breast height in centimeters (dbh). †Crown area (CA) was estimated using the stand grown crown width equation, CW_{stand} = 0.7830 + 0.165804(dbh_i), presented in Bechtold (2003). Equation was converted to metric units. ‡ Minimum density of full site occupancy (N_{MCA}) for a given tree diameter. †† Average maximum density (N_{AMax}) for a Given tree diameter

Table 2. Results of regression analysis (n = 16) relating annual increment (incre06) of co-dominant loblolly pines to the following density measures: basal area m^2ha^{-1} (BA), Stand Density Index (SDI) (Reineke 1933), additive Stand Density Index (SDI_Σ) (Long and Daniel 1990), Crown Competition Factor (CCF) (Krajicek et al. 1961), Relative N_{MCA}, and Percent N_{AMax}*.

Model: incre06 = $b_0 + b_1(x)$	b_0	b_1	R^2	P-value
$b_0 + b_1$ (SDI)	5.37914	-0.004156	0.7095	< 0.0001
$b_0 + b_1$ (CCF)	5.38577	-0.012935	0.7067	< 0.0001
$b_0 + b_1$ (SDI_Σ)	5.38737	-0.004219	0.7061	< 0.0001
$b_0 + b_1$ (Relative N_{MCA})	5.38766	-1.294510	0.7039	< 0.0001
$b_0 + b_1$ (Percent N_{AMax})	5.48820	-3.552910	0.6875	< 0.0001
$b_0 + b_1$ (BA)	5.49660	-0.093130	0.6819	< 0.0001

*Expressed as decimal equivalent of percent.

account for the interacting influence of tree size and density on growing space relationships, its utility in measuring stand stocking is limited to unthinned even-aged stands of a given age, site quality, and developmental history (Clutter et al., 1983). Unlike basal area, analysis approaches integrating Reineke's (1933) stand density index (SDI) account for tree size as well as density. Dean and Baldwin (1993) and Williams (1994) developed loblolly pine density management diagrams using percent of maximum stand density index (SDI) (Reineke, 1933) as a relative density measure. These graphical tools describe the relationship between stand density and quadratic mean diameter. Such tools can also be used to formulate thinning regimes and initial planting densities designed to meet desired tree size or stand yield objectives (Drew and Flewelling, 1979; Long, 1985; Dean and Chang, 2002).

The weighted, individual tree approach uses density reference conditions that have the same distribution of canopy areas as the subject stand, while traditional SDI based diagrams use density reference standards based upon stand averages. The presented approach also incorporates two density indices to highlight how stand density relates to an upper (N_{AMax}) and lower limit (N_{MCA}) of full stocking. Results of this case study suggest the weighted approach predicted diameter increment as well as SDI. While an individual tree approach may not be necessary within an even-aged loblolly pine plantation with a narrow diameter distribution, the potential of this single approach for application in multi-species or irregular stands warrants further study. Its tree-by-tree calculations may allow it to account for non-normal diameter distributions (Long and Daniel, 1990; Shaw, 2000) and assess relative density within uneven-aged loblolly pine stands or even-aged stands possessing highly skewed distributions. Unlike many existing relative density approaches, the framework's design allows species-specific reference equations to be used in combination to account for species mixture. Additional species or species group equations could be developed following the methods out-

outlined for loblolly pine.

Conclusion

Weighted, individual tree adjustment performed comparability to traditional indices in its ability to predict diameter increment. Findings imply the relative density approach may not be needed for even-aged loblolly pine plantations. Because the WitRD model allows additional species-specific reference equations to be incorporated, further research evaluating the approach's utility in mixed-species stands is warranted.

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