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Review

Mathematical modeling on tomato plants: A review

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Mathematical models allow for predictions of behavior under specific handling and environmental conditions, and are particularly useful in expensive studies or in studies where long term effects may be difficult to monitor. In mathematical modeling there are two main types of models: descriptive models and mechanistic models; the first are relationships between response and predictor which are not ruled by biological processes; the latter takes into account the basic processes in plants by means of differential equations to account for the development of plants. This requires a deeper knowledge of the physiological development of plants. This work reviews mathematical modeling on tomato plant. The TOMGRO model is modular and has been widely studied and calibrated under several climatic conditions which demonstrates that it is a robust model. As a future research the TOMGRO model is proposed to be adapted to other crops.

Key words: Differential equations, descriptive models, mechanistic models.

INTRODUCTION

An important trend in greenhouse production involves mathematical modeling, which can provide a description of changes in plant growth caused by environmental conditions, such as temperature and light intensity (Caliskan et al., 2009), epidemic dynamics (Contreras-Medina et al., 2009) and other factors. Mathematical modeling was initiated at the end of the 1960s as a result of the integration of plant physiological knowledge of internal processes with the development of computer systems (Bouman et al., 1996). A mathematical model for a particular crop consists of a set of mathematical expressions describing the changes in a state variable in response to physiological processes active on the plant (e.g. biomass changes resulting from photosynthesis and respiration). Crop mathematical modeling therefore represents a quantitative way to assess concurrently active processes in the plants. In this sense, crop mathematical modeling, as part of biotechnology, can help to eliminate malnutrition and hunger through evaluation of improved crops without much effort or economical resources. Biotechnology can be applied to improve agriculture and food production and to support the human population in an environmentally sustainable manner (Tonukari and Omotor, 2010). The objective of this work was to review mathematical modeling for tomato plants.

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STRUCTURE OF A GROWTH MODEL

A growth and development crop model can be defined through a mathematical equation:

$$\frac{dx}{dt} = f(x, u, p), x(t_o) = \beta \qquad (1)$$

The state vector (x) consists of *n* of crop variables, such as dry weight, number of fruits and foliar area. The input vector (u) deals mainly with the climatic conditions (air temperature, CO_2 concentration, humidity and photosynthetically active radiation (PAR)). The vector (p) represents biological parameters or constants for the model, and (β) represents the vector of initial conditions of the state variables. As (f) is a vector of *n* functions the model does not have a general analytical solution and must be solved by numerical methods or through computer simulations (López et al., 2005).

MATHEMATICAL MODELING

A mathematical model is an abstraction of reality that describes processes, the aim of which is the study and analysis of a system under varying conditions (Mason and Dzierzon, 2006). In order to create a successful mathematical model, the modeler must choose which mathematical principles and techniques to use. The solution also needs to be tested against experimental data (Crouch and Haines, 2004). Models sometimes simplify systems to reduce the datasets required to estimate parameters (Lentz, 1998; Lisson et al., 2005). Mathematical models allow for predictions of behavior under specific handling and environmental conditions, and are particularly useful in expensive studies or in studies where long term effects may be difficult to monitor (Fraisse et al., 2006). In this way, a mutual dependency between basic crop physiology research and model development can be demonstrated (Lisson et al., 2005).

Plant development involves processes working within efficiencies of scale. The representation of these processes and related interactions in a model is a particular challenge. Ecophysiological orientation is needed to predict plant composition and function on the basis of physiological traits. The modeling of physiological traits can help to improve yield and to facilitate decisions that optimize the use of available resources (Soltani et al., 2001). Crop growth models do exist for many horticultural crops, which are often distinguished between descriptive and mechanistic models (Pronk et al., 2003; Vázquez-Cruz et al., 2010). Descriptive models reflect little or none of the mechanisms causal to system behavior, whereas mechanistic models incorporate quantitative description of these mechanisms.

Descriptive models

Until 1960, agricultural research was almost completely reliant on experimental and empirical work combined with statistical analyses (Van Ittersum et al., 2003). Initially, function oriented models were designed to describe and analyze plant growth without an underlying model for physiological processes (Renton et al., 2005). Descriptive models use a relatively short computing time and normally contain few stated variables (Mirschel et al., 2004). Although the predictive value of descriptive models can be high, important limitations exist. For example, they are unable to simulate plant adaptability and response to different conditions. Also, adding new input factors necessitates building a new model based on an extended data set (Renton et al., 2005). Unlike descriptive models, mechanistic models are ruled by biological principles and involve breaking the system down into components that are modeled separately. Generally, descriptive models only try to describe relationships between response and predictor variables as economically as possible for a particular data set. Essentially, descriptive models are accounts of observational data, most often associated with curve fitting and regression. They are not ruled by biological processes and often do not require detailed knowledge of respiration, photosynthesis or assimilation mechanisms in plants (Domijan et al., 2006).

Mechanistic models

Mechanistic models are used for testing hypotheses and synthesizing knowledge of complex systems based on physiological processes that respond to climatic conditions, such as photosynthesis, assimilation and respiration (Brainard and Bellinder, 2004; Thornley and Cannell, 2000). These models are increasingly being used to investigate the impacts of weather and climate variation on crop growth and production (Tao et al., 2009).

Photosynthetically active radiation (PAR) is the driving force for evapotranspiration and photosynthesis (Dutilleul et al., 2007; Sentelhas and Gillespie, 2008). In photosynthesis- based models, the interception of light by leaf area is used to simulate the production of photosynthates Carbon is allocated according to organ demands through carbon leaf assimilation and mobilization of carbon from reserves. Subsequently, photosynthates are used to ascertain respiration, conversion into structural dry matter (DM) and fresh weight, as estimated from the dry weight (Jordan-Meille and Pellerin, 2004). Partitioning of carbon into various compounds (that is, sugars, other fruit compounds and respired CO_2) can also be taken into account. These processes are nearly always described in terms of a set of differential equations.

Prediction of leaf area index is required to estimate interception of solar radiation and biomass production (Soltani et al., 2006). In field crops, there is often a linear relationship between cumulative intercepted PAR and accumulated biomass (Zhang et al., 2008). Insufficient nitrogen (N) levels reduce leaf area development, decrease mass accumulation and lead to early maturation (Sinclair et al., 2003). Temperature can affect plant leaf area through its effects on the rate of leaf growth (Singels et al., 2005). Although respiration is one of the main energy sources in growing plants, it has been studied much less than photosynthesis (Kuretz et al., 2003). Respiration has been modeled according to the concepts of growth and maintenance. Short-term observations generally show that respiration is highly sensitive to temperature variations and that CO_2 may affect the growth coefficient (Challinor and Wheeler, 2008). In simulation models, the growth coefficient is usually independent of environmental factors (Urban, 2003; Bannayan et al., 2005).

Mathematical models that could successfully predict product composition as a function of climatic variables would be useful tools in achieving more desirable sensorial characteristics in the final crop product (Heredia and Andres, 2008).

Tomato growth models

Tomato growth (TOMGRO) is a complex mechanistic model, initially, of 69 state variables that compute the development and production of tomato plants (Jones et al., 1991). The model has an input vector of climatic variables: air temperature, solar radiation (PAR) and CO_2 concentration. The output vector consists of seven main groups of state variables: number of leaves, number of new branches, number of fruits, dry weight of leaves, dry weight of new branches, dry weight of fruits and foliar area. The model uses a source-demand function to identify carbohydrates for the growth of different parts of the plant.

The latest version of TOMGRO (v3.0) consists of 574 state variables, simulating in great detail the development of fruit, with each fruit having specific positions on the clusters. The development of the fruit is modeled separately from the growth in biomass, which enables size to be handled as a variable (Kenig and Jones, 1997). Moreover, Jones et al. (1999) proposed a relatively simple model for tomato crops which behaves similarly to the more complex TOMGRO model, but with the advantage of having only five state variables: number of nodes on main stem, index leaf area, total plant weight, fruit weight and weight of ripe fruit (Ramírez-Arias, 2005). This simplified version was evaluated with data from several experiments, including data collected in a commercial greenhouse and the results showed that the model can accurately describe the growth and yield of tomatoes in different locations and timeframes. Some researchers have used the TOMGRO model or variants to implement a control strategy to optimize greenhouse climates.

De Koning (1994) developed a model of 300 state variables to predict the distribution of dry matter with tomatoes grown in greenhouses. The number of growing tomatoes was estimated by predicting the initiation, abortion and harvesting of individuals. The demand for photoassimilates was based on potential growth rate. The dry matter distribution in the model was found to be proportional to potential growth rate. The model can reasonably predict the formation of fruit clusters, the rate of fruit growth, and the distribution of dry matter. The prediction of the number of fruits per cluster; however, was not found to be acceptably accurate. Even so, this model has been used as a basis for the development of simplified models that can be used in research on optimization and control of greenhouse climates (Tap, 2000).

Tomsim is another model developed for tomato with a modular structure, which predicts the growth and development of tomato. The production of dry matter is predicted by a sub-module based on the estimation of photosynthesis (Ramírez-Arias, 2005). The functions for the development of fruits were adapted from Koning (1994).

Development, calibration and validation of greenhouse tomato models

Dayan et al. (1993a, b) conducted a detailed analysis of the TOMGRO model using calibration and validation for climatic conditions in Israel, but did so without conducting a sensitivity analysis. They found that the model takes into account the most important phenomenas that occur inside a greenhouse and therefore can be used to study the effects of environmental conditions and management practices for fruit production. This model is both schematic and modular, meaning it can be easily adapted and that subroutines can be replaced or combined with a more comprehensive model. It can also be used in economic optimization studies. The TOMGRO model was modified to allow the modeling of growth and development of individual plant organs, allowing fairly accurate simulations of the number and weight of fruits per cluster. Adjustments were also made to more accurately describe leaf area expansion and to improve user interface, allowing specification of parameter values and initial conditions prior to simulation (Gary et al., 1995).

On the other hand, iterative procedures were applied to derive the parameters for the functional responses of various processes to temperature, radiation intensity and CO_2 concentration. The model was subsequently validated on the basis of the results of completely independent experiments. It was shown that the model accounts for the major phenomena observed under greenhouse conditions and may be used therefore with confidence to examine the effects of environmental conditions and management practices on tomato fruit yield.

Dimokas et al. (2009) performed calibration and validation studies for the adaptation of the TOMGRO model to a short term cropping technique and to conditions in Greece. They were able to model the current practice of topping tomato plants for short term cultivation with the TOMGRO model, modifying it accordingly and calibrating it with winter 2005 data. As compared to data from winter 2007, results showed that dry matter partitioning in the plant was not altered by topping, but the duration of fruit life from setting to maturity was shortened. Good agreement was observed between the measured and simulated plant development indicators, biomass and fruit production. Satisfactory agreement was also obtained for plant leaf area, normally a weak point of the TOMGRO model. Based on these results, they concluded that this adaptation of the model accurately simulated the development of short-term tomato crops grown in greenhouses and could therefore be used for decision support to help growers optimize greenhouse operations.

Cooman et al. (2007) analyzed the variation of the TOMGRO model in response to solar radiation intensity, greenhouse air temperature and CO_2 concentration. The differences in model output were apportioned to the sources of variation to obtain a sensitivity analysis. A similar degree of variability on the prediction of fruit dry weight was evidenced for the three climate variables, with solar radiation and air temperature being the most sensitive. Fruit dry weight increased with solar radiation and CO_2 concentration, while an optimal range was detected for air temperature. Variance decomposition showed that within the climate variables, dry fruit weight was most sensitive to solar radiation, followed by air temperature and CO_2 concentration. The development of vegetative plant parts was more sensitive to air temperature than to solar radiation and CO_2 concentration.

FUTURE RESEARCH

TOMGRO model which is a simplified model for potential growth of tomatoes takes into account environmental variables such as photosyntetically active solar radiation (PAR), air temperature and CO_2 air concentration and also takes into consideration physiological process such as crop photosynthesis and respiration, which are inherent to any plant. For these reasons it seems possible to adapt the TOMGRO model to other crops. Hernández-Hernández (2009) adapted the five state variables TOMGRO model to the sweet pepper crop having good agreement with the measured variables. In general, the procedure was to find new parameters for the functions involved.

CONCLUSIONS

The TOMGRO model is both schematic and modular in form. This means it can be adapted easily, and most of its subroutines can be replaced easily by others as improved descriptions become available. It can also be combined with a more comprehensive model describing greenhouse climate and appears robust for use in economic optimization of greenhouse climate conditions and management. It may also form the basis of decision support systems, aiming at recommendations for crop management.

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