Prediction of somatic growth of tropical fish using a simple mathematical formula

Anderson Mon

School of Applied Sciences and Statistics, Koforidua Polytechnic, Ghana.

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Age determination or growth rate of fishes is a critical component of a proper fishery management. Current methods of fish age determination or growth rate have several drawbacks, including, large numbers of fish needed over time for analyses, subjective interpretations during age determination or the use of expensive instrument for analysis. In this report, a simple theoretically derived mathematical formula is assessed for prediction of fish growth in length. Secondary data of length measures of 16 different species of tropical marine fish was used to assess the accuracy of the formula for predicting fish growth. The data comprised of relative length-at-age of 14 different fishes and absolute length (in centimeters) of 2 other different fishes. For each species, two of the length measures together with the corresponding ages were used to estimate two constants in the formula and the formula used to predict the remaining lengths. The accuracy of the formula for prediction was assessed by evaluating the discrepancies between observed data and corresponding predicted data. The biasness as well as the accuracy of the formula was also assessed. In all the species studied, discrepancies between observed data and the corresponding formula predicted values were minimal and fluctuated between negative and positive values. The mean signed value of the discrepancies (a measure of biasness) for all the 16 species was -0.2 ± 1.7, while the mean of the absolute discrepancies (a measure of accuracy) was 1.2 ± 1.4. The fluctuations of the discrepancies between negative and positive values demonstrate that the discrepancies are not systematic errors of prediction. The signed mean discrepancy of -0.2 is close to 0, thus indicating minimal biasness of prediction. Also the absolute mean discrepancy of 1.2 suggests prediction accuracy within 1 unit of actual measurement, indicating high accuracy.

Key words: Tropical fish, demersal marine fish, fish length prediction, growth rate, age determination, discrepancies.

INTRODUCTION

Age determination of fishes is a critical component of a proper fishery management; and some conventional techniques have been used for direct age determination of fishes. These include the growth zones analysis of skeletal-structures (Bagenal, 1974; Brennan and Cailliet, 1989; McForlane and Beamish, 1995), the tag-recapture technique (Cailliet, 2001) and the length-frequency analysis technique (Ricker, 1975). All these methods...
require collecting large numbers of fish over time (in some cases) for analyses. In addition to this time consuming process of studying large numbers of fish over a period of time in order to determine the age structure, the tag-recapture and the length-frequency analyses are prone to subjective interpretations and/or tagging artifacts, thus leaving the skeletal-structure analysis technique the preferred method (Fowler, 1990), though that is not to say that the skeletal-structure analysis technique is without drawbacks. As noted in a previous report, several factors, such as maturation and spawning, crowding conditions, population density, photoperiod, productivity of the ecosystem, chemical components in the water, temperature fluctuations, amount and quality of food, as well as almost any environmental biotic or abiotic factors, all affect the way fish hard structures can be interpreted during age determination studies (Buesa, 1987). Another method, which utilizes radioactive decay activities of some elements (notably lead-210 (210\text{Pb}) and radium-226 (226Ra) in the ooliths of bony fish to determine the age, has also been used (Bennett et al., 1982; Campana et al., 1990; Calliet et al., 2001). Though valuable for determining the ages of long-lived fishes and deep water fishes (which are difficult to sample and keep alive), the technique is expensive to use, as it requires the use of spectrometers as well as scintillation counters to measure the radioactivity of 210\text{Pb} and 226Ra. In addition to this drawback, three assumptions must be met in order to successfully apply the technique in fish age determination: These assumptions are that: (1) The hard structure of the fish (e.g., ooliths) is a closed system for radium and its daughter products; (2) The initial activity ratio of 210\text{Pb} to 226Ra in the structure should be close to zero, and (3) uptake rate of 226Ra is proportional to mass growth of the structure during the lifetime of the fish (Calliet et al., 2001). Violations of some of these assumptions resulted in failures of the technique in determining the ages of sharks (Welden et al., 1987) and the sturgeon fish when using it pectoral fin rays (Burton et al., 1999).

In addition to the above mentioned techniques, several mathematical models have been advanced for modeling fish growth (von Bertalanfy, 1938; Kozlowski, 1996; Jørgensen and Fiksen, 2006; Lester et al., 2004; Day and Taylor, 1997; Economo et al., 2005; Dumas et al., 2010). However, all these models contain several parameters, requiring statistical parameter optimization for good fit to growth data to be achieved. Also, in most of these models, prior knowledge of some of the parameters is required in order to do curve fitting.

Therefore, any method of determining fish age and growth rate that could overcome some of the drawbacks of the techniques mentioned above will be very important in fishery management and in research. Buesa proposed the use of regression analyses based on exponential functions for the estimation of growth rate and age of fishes (Buesa, 1987). However, with this method, knowledge of the feeding habit of the fish is needed so that a feeding correction factor can be imposed on the regression estimates to achieve good prediction of age and/or growth rate of the fish. Recently, a simple mathematical formula was derived for prediction of human brain tissue volume re-growth/recovery in sustained abstinent alcohol dependent individuals (Mon et al., 2011) and for prediction of individual child growth (human growth) in both boys and girls across ethnically diverse children (Mon et al., 2013). For individual growth data, the formula relies on an intrinsic factor, known as growth rate factor or growth coefficient \((k)\) to predict growth of the individual. The value of \(k\) of an individual is governed by the combined effect of genetics, environmental and any other factors that modulate growth. However, the formula can be used for group data analysis, in which case, \(k\) is the growth rate factor assumed for all the members of the group. In this study, the applicability of the formula for prediction of fish growth in length is assessed, using group data of 16 tropical marine fish species.

**METHODS**

In terms of fish growth in length, the formula proposes a square root dependence of the length of the fish on time (that is, age); as follows:

\[
\frac{1}{2}L^2 = kt + C
\]

where \(L\) is the length of the fish at time \(t\). \(k\) is the growth rate factor/growth coefficient, and \(C\) is the intercept on the \(L\) axis. If \(L\) is measured at two well separated time points \(t_1\) and \(t_2\), the values of \(k\) and \(C\) can be estimated from:

\[
k = \frac{1}{2(t_2 - t_1)}(L_{t_2}^2 - L_{t_1}^2)
\]

and

\[
C = \frac{1}{2}L_{t_1}^2 - kt_1
\]

or

\[
C = \frac{1}{2}L_{t_2}^2 - kt_2
\]

When the estimated values of \(k\) and \(C\) are substituted back into the formula, all other loci of points on the growth curve of the fish can be predicted. The units of \(k\) and \(C\) are cm\(^2\) per unit time and cm\(^2\) respectively if \(L\) is measured in centimeters (cm).

The formula was assessed for fish growth (in length) using secondary data of linear growth of 16 different species of tropical marine fishes from one published work (Buesa, 1987) and one unpublished work (http://fishing.about.com/od/bassfishing/a/How-Fast-Do-Alabama-Bass-Growth.htm)). The growth measurements for the 16 tropical marine fishes in the two reports were established values obtained from repositories, therefore no sample sizes are provided in the reports. The relative length-at-age data of 14...
different species of tropical demersal fishes (Table 1) was extracted from Buesa’s article (Buesa, 1987). Relative
length-at-age \( L_{t} \) (a dimensionless quantity) is defined as
\[
L_{t} = \left( \frac{L_{t}}{L_{\text{max}}} \right) \times 100,
\]
where \( L_{t} \) is the length at age \( t \) and \( L_{\text{max}} \) is the maximum total length of the species for the area where
the fish came from (Buesa, 1987). The fishes were from several geographical locations including the United States.
of America, Japan, Brazil, Cuba, the Atlantic Ocean, the Caribbean, Panama and Venezuela. The growth measures used in this report covered a minimum of 5 years and a maximum of 8 years from hatch. The absolute lengths of 2 species were measured using the marginal increment method, 1 using the length-frequency method and the rest using the growth-zone method (otoliths, scales and urohyal bone).

Absolute length data (in centimeters) for the first 5 years from hatch of 2 other species were extracted from the unpublished report by the Alabama Division of Wildlife and Freshwater Fisheries (http://fishing.about.com/od/bassfishing/a/How-Fast-Do-Alabama-Bass-Growth.htm). These species were the Alabama Largemouth Bass and the Alabama Spotted Bass. The method used to measure the lengths of these two species is not stated in the report.

The proposed formula suggests that \( L^2 \propto t \); that is, the square of the length is proportional to age. Thus in the analyses, the linear dependence of the square of length on age was first verified, by plotting the square of the observed length measures against age for each species. Then for each species, two of the length measures together with the corresponding ages at which the measurements were taken, were used to estimate \( k \) and \( C \). The formula was then used with the estimated values of \( k \) and \( C \) to predict the lengths for the remaining ages of each species. The discrepancies \((\Delta L_i)\) between the measured data and their corresponding predicted data were then estimated from \( \Delta L_i = L_{\text{pred}} - L_{\text{meas}} \), where \( L_{\text{pred}} \) is the predicted length and \( L_{\text{meas}} \) is the corresponding measured value. The biasness of the formula for prediction of length for all the different fishes across all ages was estimated by summing all \( \Delta L_i \) (signed values). The accuracy of the formula for predicting growth of all the species was also estimated by taking the sum of the absolute values of all the \( \Delta L_i \). The biasness and accuracy of prediction for each species are not included in the results as these can easily be appreciated by inspection of Table 1.

RESULTS

Figure 1 shows plots of the square of length measures against age for all the 16 fish species analyzed. As can be seen, all the plots are fairly linear, indicating that indeed the square of fish length is linearly dependent on age \((L^2 \propto age)\). Table 1 shows the observed relative length data and the formula predicted values, as well as the discrepancies between corresponding observed and predicted values for the 14 fish species taken from Buesa (1987). The discrepancies marked ‘**’ (0) correspond to the measures that were used to calculate \( k \) and \( C \). These discrepancies are zero since the fit of the formula passes through the centers of the two measurements used to calculate \( k \) and \( C \). As can be seen in the table, most of the \( \Delta L_i \) corresponding to all other measurements are minimal, except in a few cases where discrepancies are several centimeters large. For each species, \( \Delta L_i \) varied between negative and positive numbers, indicating that
the discrepancies are not systematic prediction errors. The mean signed value of $\Delta L_r$ (excluding all $\Delta L_r$ marked ***) for all the 14 species was $-0.2 \pm 1.7$, indicating minimal bias. The mean of the absolute $\Delta L_r$ was $1.2 \pm 1.4$, suggesting prediction accuracy within 1 unit of actual measurement. Figure 2 shows a plot of both the observed data and the formula prediction against age for the Alabama largemouth bass (AB) and Alabama spotted bass (ASB) fishes. The curves represent the formula’s description of the growth trajectories of the fishes (solid curve for AB and dotted curve for ASB), while the marks represent the observed data (circular marks for AB and star marks for ASB). Similar to the results in Table 1, the figure demonstrates the closeness of the formula’s fits to the observed data of the Alabama bass fishes too. The discrepancies between the measured data and corresponding predicted data for these two fishes range from $-0.1$ to $+0.4$ cm, with a signed mean value of $0.1 \pm 0.2$ cm and a mean absolute value of $0.2 \pm 0.2$. These means also demonstrate minimal bias and high accuracy of the formula’s prediction of growth for the two Alabama bass fishes.

DISCUSSION

In this report the applicability of a simple mathematical formula for determining fish growth rate was assessed using longitudinal length measures of 16 different marine fish species as proof of concept. As can be seen in Table 1 and Figure 2, the formula predicts growth of all the 16 fish species with high accuracy; and Figure 1, which demonstrates a linear dependence of the square of the observed length data on age, is consistent with the theory that growth rate of a fish is inversely proportional to current length of the fish. It is important to note that prediction accuracy was similar and minimal in most cases across all the species (except in a few cases where discrepancies were several units large) and fluctuated between negative and positive values, demonstrating that the discrepancies were not systematic errors of prediction. The signed mean of the discrepancies was $0.2 \pm 1.7$ cm, while the mean absolute value was $1.2 \pm 1.4$ cm. The signed mean, which is close to 0, suggests very little bias, while the absolute mean suggests accuracy within 1.0 cm.

Many conventional techniques of age and/or growth rate determination of fish exist, but they are marred by several drawbacks, such as the need to collect large numbers of fishes over a long period of time for analysis (the case of tag-recapture technique (Cailliet, 2001) or the length-frequency technique (Ricker, 1975)), the influence of environmental and other factors on the accuracy of age determination (in the case of growth-zone analysis of skeletal structures (Cailliet, 1997; Brennan and Cailliet, 1989; McForlane and Beamish, 1995)) or the use of expensive instrument for analysis of activities of radio-isotopes ((the case of the radiometric
The mathematical formula presented in this report will help to overcome most, if not all the difficulties encountered in the conventional methods. This is because, for a given geographical location and fish type, the formula requires only two length measurements at two different known ages to determine the growth rate and to predict future growth of the fish. Thus, only two data sets using a conventional method are needed for the calculation of $k$ and $C$, after which the formula can be applied to predict future lengths of the fish.

The formula also improves upon the von Bertalanffy's (1938) and its hybrid models (Kozlowski, 1996; Jørgensen and Fiksen, 2006; Lester et al., 2004; Day and Taylor, 1997; Economo et al., 2005; Dumas et al., 2010) used to describe fish growth. This is because, unlike the current formula where only two constants are required and can easily be calculated, all the existing models contain several parameters, requiring time-involving parameter optimization to achieve good fits to growth data. Most importantly, these models are exponential functions based on the assumption that growth rate is proportional to size. This assumption is inconsistent with practical observations of animal growth (including fish), where growth rate generally decreases with age or size. Schmalhausen studied growth of animals based on the assumption that growth rate is inversely proportional to time (Birnholz, 1980). However, since the size of a growing animal is directly proportional to age (that is, time), it is more appropriate to relate growth rate with the size (length) of the animal rather than time, because size of an animal is uniquely influenced by genetics and environmental factors, which all influence growth rate. Indeed, in fish studies, the well known Gulland and Holt plot shows an inverse dependence of growth rate on length. This together with the demonstration of the linear dependence of the square of length on age and the accuracy of the fits of the current formula to growth trajectories of the 16 fish species studied in this research attest that growth of fish obeys the theory of the formula; that is, growth rate is inversely proportional to current length.

Though the formula is assessed here for prediction of fish growth using data of short-lived tropical fishes, it should be able to also predict growth of long-lived or temperate zone fishes, since somatic growth patterns of all fishes are qualitatively similar. Also, weight of some fishes has a dependence on length (Lester et al., 2004; Jørgensen and Fiksen, 2006); thus the formula may be useful in weight trajectory studies of such species of fish. Furthermore, somatic growth patterns of some aquatic animals, such as crocodiles, sharks, whales and seals are qualitatively similar to somatic growth patterns of fishes; and the formula could also be useful in studies involving somatic growth of these animals. However, direct studies using growth data of these animals are needed to support the above supposition.

It must be noted that for accurate prediction of growth using the formula, the interval between the two measurements used to calculate $k$ and $C$ in the formula must be well separated in time, such that growth between the two time points is greater than measurement error. In this report, the one year time interval between measurements was long enough for two consecutive data points to be used to calculate the constants; however measurements separated by two or more years' interval were also used. Another way to improve upon prediction accuracy is to use the average value of $k$ from two or three estimates of the constant from three measurements. This is because, the possibility of imperfect uniform growth coupled with measurement errors may result in slightly different values of $k$; thus an average value of $k$ from two or three estimates will describe the growth trajectory better than any of the individual values.

In conclusion, a simple mathematical formula has been successfully assessed for prediction of fish growth. The simplicity and accuracy of the formula makes it an attractive method of predicting fish age and growth in length. The accuracy of the formula for predicting brain tissue volume growth, human linear growth and fish linear growth suggests that these three processes share the same trajectory. Given that generally growth trajectories of most animals are qualitatively similar, growth of other animals could also possibly be studied using the formula.

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**Conflict of Interest**

The author has no conflict of interest related to this study.

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**REFERENCES**

