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Full Length Research Paper

Characteristics and mechanics of tall vegetation for resistance to flow

M. Fathi-Moghadam

School of Water Sciences Engineering, Chamran University of Ahwaz, Ahwaz, Iran. E-mail: fathi49@gmail.com.

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Characteristics and mechanical properties of vegetation have a considerable effect on the resistance to flow for both water in vegetation zone of rivers and air in forest canopies. For tall vegetation, these properties include leaf density, shape and the general flexibility of tree species. To estimate the resistance coefficients (i.e., coefficient of drag, the Darcy-Weisbach friction factor, *f*, or Manning's *n* value) of flow inside tall and flexible vegetation, an index is required to account for the effects of vegetation type and properties. In this study, an index is proposed to characterize physical properties of tree species. The index is derived based on the resonance frequency of the first mode of vibration of the trees and a fundamental relationship for the homogeneous beams. The estimated indexes for four species of coniferous trees were used in a mathematical model for estimation of the friction factor (*f*), and were able to account for the differences due to the leaf density, shape and rigidity of the tree species.

Key words: Mechanics of vegetation, index, natural frequency, flow resistance.

INTRODUCTION

Vegetation resistance to flow of water and air is an important subject dealing with land-water conservation, crop protection, wind load and fences. Estimation of water level in flood events and examination of the ability of trees to withstand high winds are examples which show the applicability range of the resistance to flow study. Resistance to flow for vegetated surface is very sensitive to the flow velocity and physical condition of vegetation. The resistance to flow decreases rapidly as the flow velocity increases, due to the streamlining and the resulting reduction of the frontal area of vegetation. The fundamental vegetation properties to be considered in establishing a resistance equation are leaf density, shape, flexibility and manner of defection of the tree species (Kouwen and Fathi-Moghadam, 2000; Jarvela, 2004). A mathematical model has been developed by Fathi-Moghadam (2006) to estimate the flow resistance coefficients (including the Darcy-Weisbach friction factor, *f*, and Manning's *n* value) for flow inside tall and flexible vegetation in rivers. Using dynamic similarity between the flow of water and air through vegetation, the model is capable of estimating the drag coefficient of vegetation in forest canopies. Details of the modeling can be found in Fathi-Moghadam and Kouwen (1997). The model is based on a dimensional analysis supported by a series of experiments in water and air conducted on four species of coniferous trees including cedar (*Thuja occidentalis*), spruce (*Picea glauca*), white pine (*Pinus strobus*) and Austrian pine (*Pinus palustris*). Using the model, effects of flow depth and land slope on the friction factor was investigated in Fathi-Moghadam, (2006) for non-submerged vegetated lands.

In order to develop a single mathematical model to estimate flow resistance coefficients for all species of coniferous trees, a vegetation index (symbolized as ξE) is required in the model to account for the effects of leaf density, shape and rigidity of individual trees. The purpose of this study is to present a practical procedure for estimation of the vegetation index (ξE), and identifying the vegetation for the above model calculations.

MATERIALS AND METHODS

Theoretically, any beam, tree or plant stem with mass and elasticity may exhibit one or more resonance frequencies of vibration depending on damping (Timoshenko and Gere, 1961; McMahon and Kronauer, 1976; Niklas and Moon, 1988). For small damping, these resonance frequencies are close to the natural frequencies of the beam. Natural frequencies result from the cyclic exchange of kinetic and potential energy when a structure such as a beam or plant stem is vibrated. The kinetic energy is proportional to the square of the velocity of the structural mass, while the potential energy is proportional to the square of the elastic strains. The rate of exchange between kinetic and potential energy is the natural frequency of vibration.

The resonance frequencies, f_j (with $j = 1,2,3,...,n$, where f_1 is the fundamental or base natural frequency and $f_{2,..,n}$ are higher modes of natural frequencies) of a linear and homogeneous beam depend upon its length (l), mass per unit length (m), second moment of inertia (I), modulus of elasticity (E), as well as a dimensionless parameter (λ_j) which is a function of beam geometry and the boundary conditions under which the beam is tested. The relationship between the resonance frequencies and the above variables is given by the following equation (Timoshenko and Gere, 1961; Clough and Penzien, 1975):

$$f_j = \frac{\lambda_j^{\ 2}}{2\pi}\left(\frac{EI}{ml^4}\right)^{½} \qquad (1)$$

Where EI is flexural stiffness.

The values of (λ_j) have been theoretically calculated for a variety of beam geometry (i.e., prismatic, non-prismatic and tapered beams) and methods of attachment (i.e., boundary conditions) which can be found in advanced dynamic and vibration standard texts. Wang and Worley (1966) presented numerically based tables of natural frequencies and nodes for transverse vibration of tapered beams. The tables cover the whole range of elliptical tapering.

The published values for λ_j, E, and I can be used in equation 1 with sufficient accuracy for linear and homogeneous beams. If the geometry of beam does not resemble the geometry of beam presented in the literature, then a model can be constructed with specified dimensions and known material properties, i.e., known EI; after measuring f , the value of (λ_j) can be computed and then used to determine flexural stiffness EI for a given beam.

Niklas and Moon (1988) measured flexural stiffness and modulus of elasticity of flower stalks using multiple resonance frequency analysis (MRFA) of spectra. The flower stems were attached to a shaker and their vibrations were tracked by an Optron camera and analyzed by a spectrum analyzer. For small and symmetric shape vegetation elements such as flower stems, the use of published E, I, and λ_j-values in equation 1 to estimate a material property may be appropriate. Obviously, such a method or methods for simple structures (like beams) are less applicable for large scale and complex structures like trees.

Development of a semi-empirical method to characterize tall vegetation

In classical mechanics of materials, a vegetative biomass is classified as a non-homogeneous visco-elastic material (Niklas, 1992). For large trees, this non-homogeneity will be much greater than short grass or analysis of small plant segments. Trees have different classes of branches and significant difference in ratios of hardwood and softwood in their segments. It should be noted that the vegetal drag coefficient for the leafy trees was found to be three to seven times that of the leafless trees (Jarvela, 2004).

The complexity and large non-homogeneity of the visco-elastic materials of large trees disqualify the use of theoretical values of E, I, and λ_j to characterize a tree species. This defines a need to derive a semi-empirical relationship based on extensive tests on various tree species for the estimation of a vegetation index and quantification of the physical properties of species.

In order to avoid errors resulting from the use of theoretically-based values for λ_j, I, and E in equation 1 and to minimize the number of unknowns, several simplifications have been made in this study. The dimensionless ratio of ($I\ l^{-4}$) in equation 1 together with the parameter [$\lambda_j^2\ (2\pi)^{-1}$] can be assumed to be a single parameter, symbolized as ξ for the base mode of vibration. This parameter characterizes height, mass or leaf density and the moment of inertia of a tree. Substituting the tree's height (h) for the beam length (l) in equation 1 and transferring the measurable parameters to the right side, equation 1 for the first mode of the natural frequency (f_1) will be:

$$\xi E = f_1^2\left(\frac{m_s}{h}\right) \qquad (2)$$

where $m_s = m\ .\ h$, the total mass of the tree and ξE is called the "vegetation index" of a tree species in this study. Measuring a tree's height, mass, and recording its natural frequency of the first mode of vibration, the vegetation index can be estimated by equation 2. The developed vegetation index includes all physical properties of a tree species for their leaf density, shape, stiffness, and manner of deflection.

Experiment procedure and apparatus

The heights, weights, and natural frequencies of the first mode for four species of coniferous trees were measured for 30 samples in three categories of size including small, mid-size and full size trees with average heights of 0.3, 1.2 and 3.0 m respectively. The natural frequency of small and medium sized categories was recorded using an accelerometer, dynamic analyzer and a frequency spectra plotter. A small silicon accelerometer with a manufacturer reported bias of $\pm$ 0.1% was used for small and medium sized model testing. The natural frequency of the first mode of vibration was clearly defined in the spectrum by shaking models from side to side. The recordings were started when the models were released from loading and were allowed to freely vibrate.

Figure 1 shows a plot of the natural frequency spectrum for the medium sized Austrian pine tree No. 1 listed in the Table 1. The average of five excitations was used to mark the first and second modes of vibration. Similar plots were recorded for all the small and medium sized samples and species which are reported in Table 1. The frequency spectrum for the model support was recorded before testing any of the models to avoid any confusion in later analysis of the spectra. In general, the second mode of the natural frequencies was damped much faster than the first mode due to its interaction with the vibration of the tree's laterals. Although the second mode of vibration was not taken into account for the analysis in this study, future work is needed to ascertain its lack of importance. Within the tested species, cedar had a better response and clearer distribution between the first and second modes of vibrations compared to the others. Spruce had the least distinct response and the highest damping.

Full size trees with an average height of approximately 3.0 m were only tested for their first mode of vibration. A silicon micro-machined accelerometer Model 3145 with a standard range of $\pm$ 2 g and $\pm$ 2 volt output (i.e., precisely 988 mv/g), stable up to frequency of 250 Hz, and a manufacturer reported bias of $\pm$ 0.2% was used for full size tree measurements. The accelerometer was attac-

Table 1. Physical properties and vegetation indices of coniferous trees.

General Size	Tree Species	Sample Name	Height (h), m	Weight (W), N	Mass (m), Kg	ms Kg m^{-1}	1st mode Nf, s^{-1}	2nd mode Nf, s^{-1}	Index ξE N m^{-2}	Average ξE N m^{-2}
small size models	Cedar	C93	0.30	1.03	0.10	0.35	3.75	15.50	4.90	4.90
	Aus. Pine	AP93	0.30	1.27	0.13	0.43	4.00	19.75	6.92	6.92
mid-size models	Cedar	C1	1.40	17.50	1.78	1.27	1.38	3.75	2.41	
		C2	1.15	8.62	0.88	0.76	1.63	5.00	2.02	
		C3	0.85	4.19	0.43	0.50	1.73	5.25	1.49	1.97
	Spruce	S1	1.55	22.02	2.25	1.45	1.65	3.38	3.94	
		S2	1.25	16.82	1.72	1.37	1.60	4.00	3.51	
		S3	0.65	4.91	0.50	0.77	1.73	12.50	2.30	3.25
	Aus. Pine	AP1	1.25	25.60	2.61	2.09	1.13	4.25	2.64	
		AP2	1.15	18.79	1.92	1.67	1.38	4.25	3.15	
		AP3	0.75	8.65	0.88	1.17	2.25	7.75	5.95	3.91
Full size trees	Cedar	CW1	2.95	62.07	6.33	2.14	1.05	-	2.34	
		CW2	3.30	83.40	8.50	2.58	1.02	-	2.68	
		CW3	3.10	83.87	8.55	2.76	0.92	-	2.35	
		CW4	2.85	51.93	5.29	1.86	1.08	-	2.17	
		CW5	2.50	33.78	3.44	1.38	1.12	-	1.73	
		CW6	2.20	31.78	3.24	1.47	1.07	-	1.67	
		CW7	1.90	28.59	2.91	1.53	1.09	-	1.81	2.11
	Spruce	SW1	3.45	103.59	10.56	3.06	1.10	-	3.67	
		SW2	2.35	46.30	4.72	2.01	1.43	-	4.11	
		SW3	3.15	81.36	8.29	2.63	1.13	-	3.33	
		SW4	2.65	40.22	4.10	1.55	1.24	-	2.39	
		SW5	3.85	267.81	27.30	7.09	0.71	-	3.57	3.41
	White Pine	WPW1	2.80	102.42	10.44	3.73	0.86	-	2.73	
		WPW2	2.15	86.29	8.80	4.09	0.92	-	3.43	
		WPW3	1.90	54.58	5.56	2.93	0.98	-	2.81	2.99
	Aus.Pine	APW1	2.95	116.44	11.87	4.02	1.09	-	4.78	
		APW2	3.20	133.30	13.59	4.25	0.94	-	3.75	
		APW3	3.10	149.83	15.27	4.93	1.05	-	5.43	
		APW4	3.25	186.80	19.04	5.86	1.02	-	6.10	5.02

hed to the top half of the main stem. Trees were fixed at their base and were shaken from side to side at the top of the tree. The sinusoidal vibration of the shaken trees was converted to an output signal by the accelerometer, amplified ten times and recorded. The high frequency of data acquisition (one hundred readings per second) provided a smooth sinusoidal graph and the number of swings per second could be easily determined. Thus an estimate of the first mode of the resonant frequency was found. Figure 2 shows a plot of the natural frequency of the first mode for a full size spruce tree No. 5. The average of three excitations (as shown in Figure 2) was used to calculate the frequency of the first mode of vibration. Similar plots were recorded for other full size trees as reported in Table 1.

RESULTS

The measured height, mass, natural frequency and the calculated vegetation index (ξE) from equation 2 is recorded in Table 1 for each tested sample. A simple averaging technique was applied first to the calculated indexes (ξE) of each species in each category of sizes and then over the size categories. The resulting representative indexes are 2.07, 3.36, 2.99, and 4.54 $N\,m^{-2}$ for cedar, spruce, white pine, and Austrian pine, respectively. The representative indexes (ξE) are used to normalize the velocity. In a relative sense, the representative indexes are in agreement with the reported value of modulus of elasticity (*E*) in Niklas (1992). The Austrian pine and cedar have the maximum and minimum rigidity respectively, while the white pine and spruce are in the medium range of rigidity within all species of coniferous trees. It should be noted that because the small sized models of cedar and Austrian pine were low in number (Table 1), they were not used in the averaging for calcul-

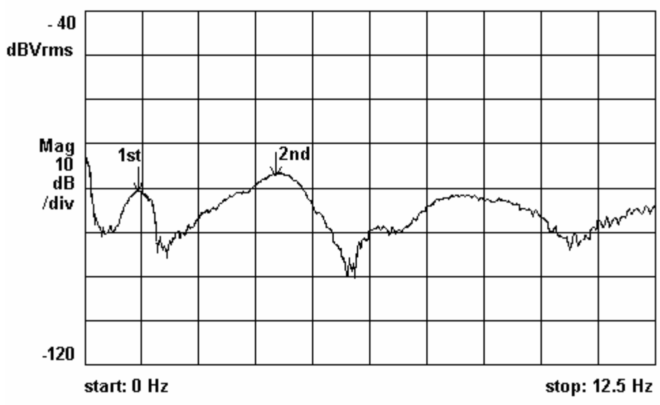

Figure 1. Frequency spectrum of medium sized Austrian pine tree No. 1 recorded by a dynamic analyzer.

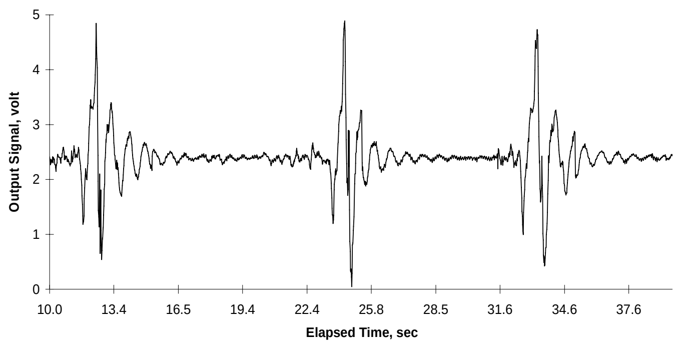

Figure 2. Output signal of natural frequency of first mode for full sized spruce tree No. 5.

ations of the representative indexes.

Validation of the vegetation index

Figure 3 shows the relationship between correlated Darcy-Weisbach friction factors (*f*) and average velocity for four species of coniferous trees tested in air and water (Fathi-Moghadam and Kouwen, 1997). The curves in Figure 3 are averages of testing 10 - 12 samples in water flume and 3 - 5 samples in air flow for each species of coniferous trees. The air velocity data were converted to their equivalent water velocity based on a dynamic similarity (same Reynolds number) in Figure 3. A noticeable difference in the Darcy-Weisbach friction factor (*f*) from one species to another in Figure 3 may lead to criticism of the application of just one mathematical model for all species of evergreen trees. The difference in friction factors in Figure 3 is due to variation in shape, leaf density, and material properties which are entirely accounted for by the vegetation index (ξE).

To incorporate the vegetation index and for the sake of a more general mathematical model, the difference betw-

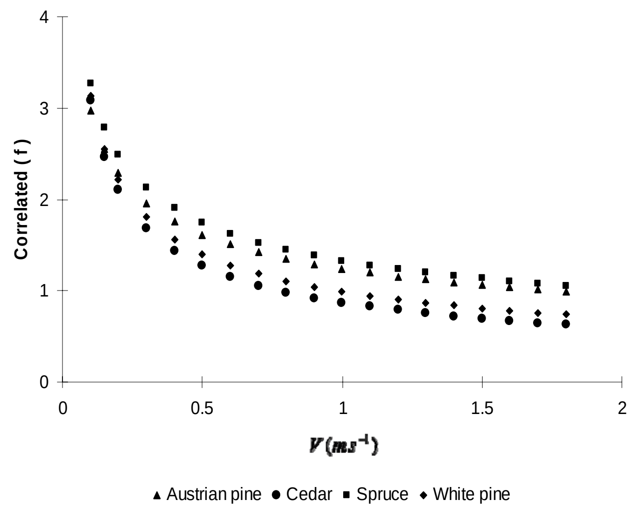

Figure 3. Correlation of Darcy-Weisbach friction factor (f) with velocity (V) for four species of coniferous trees tested in air and water flow.

een the curves is minimized by correlating the friction factor (*f*) and the normalized velocity parameter in a form of $[V\,\rho^2\,(\xi\,E)^{-2}]$ for each species, where *V* is average flow velocity (m s^{-1}), and ρ is mass density of fluid (Kg m^{-3}). Using unit mass density for the same fluid flow, the resulting relationships are plotted in Figure 4 for all species. As was expected, the best fit curves for each species in Figure 4 are approximately 50% closer together than those in Figure 3. This will allow the curves to be combined for an average curve (coniferous index curve) that represents the physical behavior of most species of coniferous trees. The final result of a linear regression of the data for flow of water (ρ_w =10^3 Kg m^{-3}) through coniferous trees is:

$$f = 3.98\left[V\,\rho_w^{\;2}\,(\xi\,E)^{-2}\right]^{-0.45} \qquad (3)$$

Equation 3 can be used to estimate the Darcy-Weisbach friction factor (*f*) for water flow inside a stand of conifer-rous trees on flood plains or in vegetated zones of rivers. Using an average water velocity equivalent to air speed (i.e., equal Reynolds number), the above mathematical model can also be used to estimate the friction factor and the wind momentum absorbed by coniferous trees in a forest stand. Similar power functions between *f* and *V* as in equation 3 were suggested in the literature (i.e., Freeman et al. 2000), but no index or practical method was proposed to account for the effect of vegetation properties on the friction factor.

Relationship between height and natural frequency

In practice, equation (1) is not an exact relationship bec-

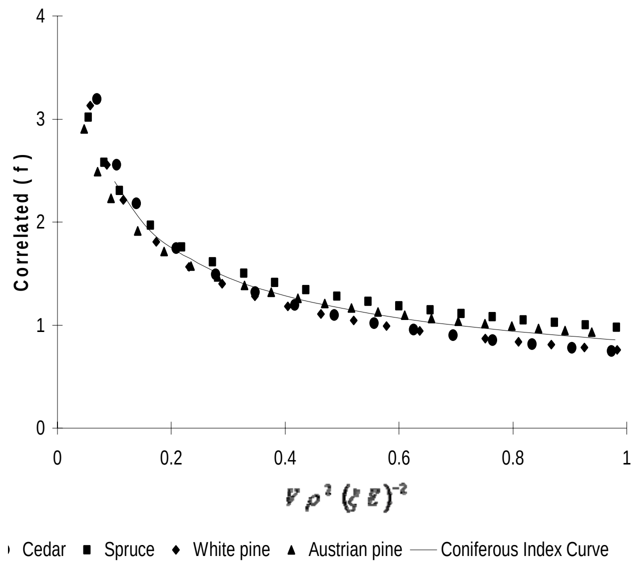

Figure 4. Correlation of Darcy-Weisbach friction factor and normalized velocity for four coniferous species tested in air and water flow.

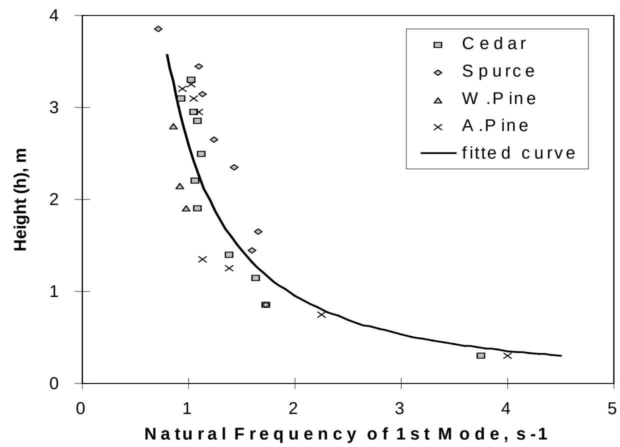

Figure 5. Relationship between natural frequency of first mode and height of trees.

ause damping is not considered. Damping of vibration is not negligible even for homogeneous metal materials. For a highly damped, non-homogeneous and visco-elas-tic tree, a relationship between the height (h) and natural frequency of first mode (f_1) can only be empirically based on a large number of experimental data.

Figure 5 shows a graphical relationship between the height and the natural frequency of the first mode for the whole range of heights (0.3 - 3.85 m) and tree species tested in this study (Table 1). An overall linear logarithmic fitting between the (f_1) and the (h) for all the data results in the following relationship:

$$f_1 = 1.81(h)^{0.58} \quad (4)$$

Since equation (4) is based on the data of four species of coniferous trees, it cannot be used to estimate the (f_1) in equation (2) for a particular species.

DISCUSSION

Characteristics and mechanical behavior of tall vegeta-tion have significant influence on resistance to flow of water in vegetated zones of river as well as on flow of air in forest canopies. In the present study, a vegetation index is developed to account for the effect of vegetation conditions and properties (i.e., leaf density, shape, stiffness, and manner of deflection) on the friction factor (f). The vegetation index (ξE) was used to normalize flow velocity which enables the elimination of the variation of friction factor among different species of coniferous trees. A single correlation between the friction factor and the normalized flow velocity results in a single mathematical model for various species of coniferous trees. The vegetation index showed to be adequately capable of differentiating between tree species and vegetation types in the model. Using the vegetation index, the model estimated friction factors for coniferous trees were consistent with the reported friction factors by Jarvela, (2002) for willows as are compared in Fathi-Moghadam (2006). In the future, estimation of the new vegetation index (ξE) for each species will require extensive sampling and testing of trees having a greater variety in appearance and physical characteristics. Assuming avai-lability of vegetation indices for different tree species, correlating the ground-based measurements and high resolution satellite data will make the analysis of large vegetated areas possible (Rautiainen et al., 2003; Stenberg et al., 2003).

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