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Full Length Research Paper

Generalized height-diameter models for *Populus tremula* L. stands

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Using permanent sample plot data, selected tree height and diameter functions were evaluated for their predictive abilities for *Populus tremula* stands in Turkey. Two sets of models were evaluated. The first set included five models for estimating height as a function of individual tree diameter; the second set also included six models for estimating height as a function of individual tree diameter and some stand-level-attributes. The inclusion of the stand-level-attributes (basal area, dominant height, dominant diameter, number of trees) and *BAL* index (which simultaneously indicates the relative position of a tree and stand density) into the base height-diameter models increased the accuracy of prediction for *P. tremula*. As a result, the second set models gave high performance than the first models. On average, by including stand level attributes, root mean square values were reduced by 21 cm. In the second set, the best results were obtained by the Schnute's function. In this function, dominant diameter and dominant height independent variables in addition to tree diameter were found significant at 0.01 significant level (R^2= 0.949, $S_{y.x} = 1.226$, $P < 0.01$). Root mean square was reduced 35 cm Schnute's function alone. Thus, a generalized height-diameter model based on Schnute's function was developed for *P. tremula* L. stands in Turkey. Based on the residual plots and fit statistics, the model can be recommended for estimating tree heights for *P. tremula* L. in Turkey. The model coefficients are documented for future use.

Key words: Schnute's function, *h-d* models, *Populus tremula* L.

INTRODUCTION

Individual tree heights and diameters are essential measurement in forest inventories and are used for estimating timber volume, site index and other important variables related to forest growth and yield, succession and carbon budget models (Peng, 2001). Considering that diameter of breast height is relatively cheap and can be more accurately obtained than total tree height, usually only a sub sample of heights is measured. Height-diameter equations are then used for accurate prediction of the heights of the remaining trees, reducing the cost of data acquisition. For this reason, the quantitative relationship between tree height and diameter is considered to be one of the most important characteristics of a stand or plot, and essential for describing its structure (Dorado et al., 2006).

The height-diameter relationship varies from stand to stand, and even within the same stand the relationship is not constant over time (Curtis, 1967). Therefore, a single curve can not be used to estimate all the possible relationships that can be found within a forest. To minimize this level of variance, *h-d* relationships can be improved by taking into account of stand variables that introduce the dynamics of each stand into the model (Curtis, 1967; Larsen and Hann, 1987; Lopéz Sánchez et al., 2003; Sharma and Zhang, 2004; Temesgen and Gadow, 2004, Dorado et al., 2006). The estimation of stand development over time relies on accurate height-diameter functions. Many growth and yield models require height-diameter as basic input variables, with all or part of the tree height predicted from measured diameters (Burkhart et al., 1972; Wykoff et al., 1982; Huang et al., 1992a). When actual height measurements are not available, height-diameter functions can also be used to indirectly predict height growth (Larsen and Hann, 1987). These equations do not include additional variables that may influence the height-diameter relation in different stands

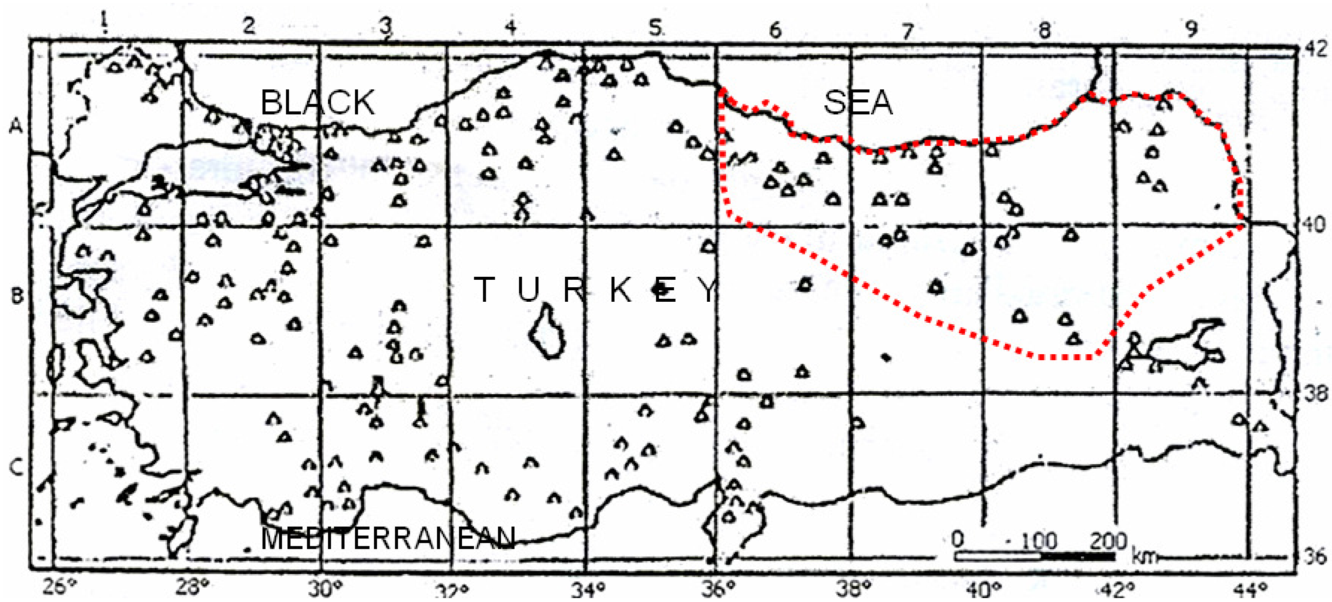

Figure 1. Natural distribution of *P. tremula* L. (Davis 1965).

(Temesgen and Gadow, 2004).

A generalized height-diameter function estimates the specific relationship between individual tree heights and diameters using stand variables such as basal area per hectare, quadratic mean diameter, dominant height and dominant diameter, mean height and number of trees. The reason for using them is to avoid having to establish individual diameter-height relationships for every stand (Curtis, 1967). Thus, height can be estimated by means of only a function. Although in Europe, generalized height-diameter functions have been used since 1930's (Lang, 1938; Kramer, 1964; von Laer, 1964; Kennel, 1972; Nagel, 1991; Hui and Gadow, 1993; Temesgen and Gadow, 2004), in Turkey, until now no generalized height-diameter functions were developed. The objective of this paper is to assess the predictive abilities of selected generalized height-diameter models in natural trembling aspen stands. The study specifically evaluates the contribution of stand attributes in estimating height-diameter relationships.

Data

This study was conducted in trembling aspen natural stands in east and northeastern Anatolia region in Turkey (Figure 1). Altitude of this area ranges from 500 - 2000 m above the sea level with an average slope of 40%. Data from 46 sample plots were collected from even-aged *Populus tremula* L. stands, covering the existing range of stand densities and sites. Permanent sample plots were developed to document tree and stand growth and mortality over time in east and northeastern Anatolia trembling aspen forests. They represent a variety of stand structures, densities and site qualities.

Plots were stratified into 10-year age classes and sampled with an effort to equal allocation of at least three sample plots to each age group. For each age group, effort was also made to include the full range in site index (at an index age of 30 years) from 4.1 - 14.8 m. The site index for each site was determined using Fabbio method (Yavuz et al., 2006). The mean site index for sample plots was 9.4 m. The data used to develop the generalized height-diameter models were obtained in two different sources. In 2000, 46 sample plots were established in even aged trembling aspen natural stands in east and northeastern Anatolia in Turkey. The plot size ranged from 200 - 800 m^2, depending on stand density, in order to achieve a minimum of 30 trees per plot. Diameter at breast height (*dbh*) was measured in all trees using calipers. Total tree height (*h*) was measured using a height meter and in additional sample including the dominant trees (the proportion of the 100 thickest trees per hectare, depending on plot size). Diameter at breast height was measured to the nearest 0.1 cm.

The plots were re-measured in 2005. The data were the two inventories carried out in 2000 and 2005, resulted in a total of 5540 diameter and height observations from 1385 trees. All of the plots were measured in different stands, thus the nested correlation between plots and stands was not considered. The sample plots were randomly split into two groups using the RANUNI function of the SAS statistical package (SAS Institute, 2004). 80% of these (37 sample plots) were used for model fitting and the remaining 20% (9 sample plots) were reserved for model evaluation. Since data set is large enough, the 80 - 20% split used is unlikely to reduce the precision of the parameter estimates compared with those obtained with

Table 1. Statistical evaluation of stands in the study area (Characteristics of the fitting and evaluation data sets).

	Fitting data set (No. of plots: 37, No. of trees: 1115)				Evaluation data set (No. of plots: 9, No. of trees: 270)			
Variable	**Min**	**Max**	**Mean**	**S.D.**	**Min**	**Max**	**Mean**	**S.D.**
Dbh (cm)	1.6	51.4	9. 7	7.11	1.9	45.4	8.9	7.18
D_{dom}	8.0	48.4	22.2	9.48	8.0	33.5	20.6	8.17
H_{dom}	5.5	18.9	11.5	2.87	9.1	15.3	11.0	2.62
h (m)	1.8	38.5	7.7	3.44	2.6	19.8	8.8	4.32
$\bar{d}_q$ (cm)	4.5	21.0	10.3	3.80	4.6	19.5	9.0	2.59
BA (m^2/*ha*)	3.7	52.4	26.5	10.88	5.5	85.6	21.8	10.24
N (*trees/ha*)	875	5150	2679	1381	1025	4850	3558	1322
BAL (m^2/*ha*)	0.0	52.4	26.5	18.57	0.0	85.6	34.8	21.21
SI (*m*)	4.1	14.8	8.3	2.96	7.2	11.0	7.9	3.01

the model built from the entire data set (Uzoh and Oliver, 2006).

At each measurement time, stand characteristics were computed from individual tree measurements in the stands. In addition, for each individual tree was calculated *BAL* index which summarized basal area for all greater than the subject. Summary statistics, including mean, minimum, maximum, and standard deviation of each of the individual tree and stand variables (basal area (*BA*), quadratic mean diameter ($\bar{d}_q$), mean height weighted by basal area ($\bar{h}_q$), number of trees (*N*), *BAL* index, dominant height (H_{dom}) and dominant diameter (D_{dom})) for both fitting and evaluation data sets are shown in Table 1.

METHODS

Based on literature review, two sets of height-diameter functions were assessed for their predictive abilities. The first set included five models for estimating height as a function of diameter only. The second set also included stand-level attributes. After assessing several height-diameter functions, the following eleven functions were evaluated (Table 2). Model parameters were estimated using the Marquardt-Levenberg Method available in the NLIN procedure with SAS Software (PROC NLIN, SAS Institute, Inc. 2004). In order to find a global minimum, the starting value of each parameter was varied, thus obtaining several runs. The assumption of homoscedasticity was tested using Goldfield-Quandt test (Goldfield and Quandt, 1965).

Model comparison and selection

Selected model was evaluated quantitatively by examining the magnitude and distribution of residuals to detect any obvious patterns and systematic discrepancies and by testing for bias and precision to determine the accuracy of model predictions (Vanclay, 1994; Soares et al., 1995; Mabvurira and Miina, 2002). On the other hands, after parameter estimates were obtained, the predictive abilities of the selected height-diameter functions were evaluated using coefficient of determination for non-linear regression (R^2), the bias and root mean square error (*RMSE*) criteria, the asymptotic *t*-statistics of the parameters and the asymptotic 95% confidence intervals. Although, there are several shortcoming associated with the use of the R^2 in non-linear regression, the general usefulness of some global measure of model adequacy would be seem to override some of those limitations (Ryan, 1997). The expressions for these statistics are summarized as follows:

$$R^2 = 1 - \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(H_{ij} - \hat{H}_{ij}\right)^2}{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(H_{ij} - \bar{H}_{ij}\right)^2}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(H_{ij} - \hat{H}_{ij}\right)^2}{n-p}}$$

$$Bias = \hat{E} = \frac{\sum_{i=1}^{m}\sum_{j=1}^{n_i}\left(H_{ij} - \hat{H}_{ij}\right)}{n}$$

where R^2 is the coefficient of determination, H_{ij} is the measured and $\hat{H}_{ij}$ is the estimated values of tree heights, n_i the number of trees in the *i-th* plot, *m* and *n* the number of plots and the number of observations used to fit the model, respectively, and *p* is the number of model parameters.

The selected model was further validated by an independent control data set. As mentioned on data section, for the present work data were partitioned in two groups, one for model evaluation and one for validation. Both the number of observations determined for model evaluation was made relatively large in order to provide sufficient data for the model evaluation phase and the number of observations in the test data was large enough for validation and appropriate statistical test in this study. The deviations between

Table 2. Height-diameter functions evaluated based on fitting data.

S/No	Model	Expression
Base height-diameter functions		
1	Wykoff et al. (1982)	$H = 1.3 + e^{a + \frac{b}{dbh+1}}$
2	Yang et al. (1978)	$H = 1.3 + a(1 - e^{bdbh^c})$
3	Ratkowsky (1990)	$H = 1.3 + e^{(a + \frac{b}{dbh+c})}$
4	Modified from Hui & Gadow (1993)	$H = 1.3 + ae^{(\frac{b}{dbh+1})}$
5	Modified from Hui & Gadow (1993)	$H = 1.3 + adbh^b$
Generalized height-diameter functions		
6	Temesgen & Gadow (2004)	$H = 1.3 + e^{a + \frac{b}{dbh+1}}$ $a = a_1 + a_2 \times BAL$ $b = a_3 + a_4 \times BAL + a_5 \times N + a_6 \times BA$
7	Temesgen & Gadow (2004)	$H = 1.3 + a(1 - e^{bdbh^c})$ $a = a_1 + a_2 \times BAL + a_3 \times N + a_4 \times BA$ $b = a_5 + a_6 \times BAL$
8	Ratkowsky (1990)	$H = 1.3 + e^{(a + \frac{b}{dbh+c})}$ $a = a_1 + a_2 \times BAL$ $b = a_3 + a_4 \times BAL + a_5 \times N + a_6 \times BA$
9	Modified from Hui & Gadow (1993)	$H = 1.3 + ae^{\left[b(1 - \frac{dbh}{\bar{d}_q}) \times dbh^c\right]}$ $a = a_1 + a_2 \times BA$
10	Modified from Hui & Gadow (1993)	$H = 1.3 + aBAdbh^c$ $a = a_1 + a_2 \times BA$
11	Schnute (1981)	$H = \left(1.3^b + \left(H_{dom}^b - 1.3^b\right)\frac{1 - e^{-adbh}}{1 - e^{-aD_{dom}}}\right)^{1/b}$

predicted and observed values were tested by student's Paired *t*-test.

RESULTS

Considerable differences were found among the predictive abilities of the height-diameter equations. The *RMSE* values (m) ranged from 1.511, 1.518, 1.510, 1.511 and 1.837 for models 1 through 5, respectively. For models with stand-level attributes, the *RMSE* values (m) ranged from 1.419, 1.459, 1.418, 1.431, 1.460 and 1.226 for models 6 through 11. None of the investigated models were biased. When height-diameter functions were fitted, differences were found among the estimated model parameters and the predictive ability of the height-diameter models. Among the five base models, Model 3 had the lowest *RMSE* value. Among the models tested with stand-level attributes, Model 11 was based on Schnute function that had the lowest *RMSE* value (Table 3).

Judging from the residual plots and the *RMSE* values,

Table 3. Root mean square (*RMSE*; m), coefficient of determination (R^2) and bias (m) of models.

Model	*RMSE*	R^2	Bias
1	1.511	0.790	0.011
2	1.518	0.789	-0.010
3	1.510	0.791	0.001
4	1.511	0.790	0.011
5	1.837	0.790	0.131
6	1.419	0.816	0.015
7	1.458	0.806	-0.005
8	1.418	0.816	0.001
9	1.431	0.799	0.006
10	1.460	0.780	0.050
11	1.226	0.949	0.088

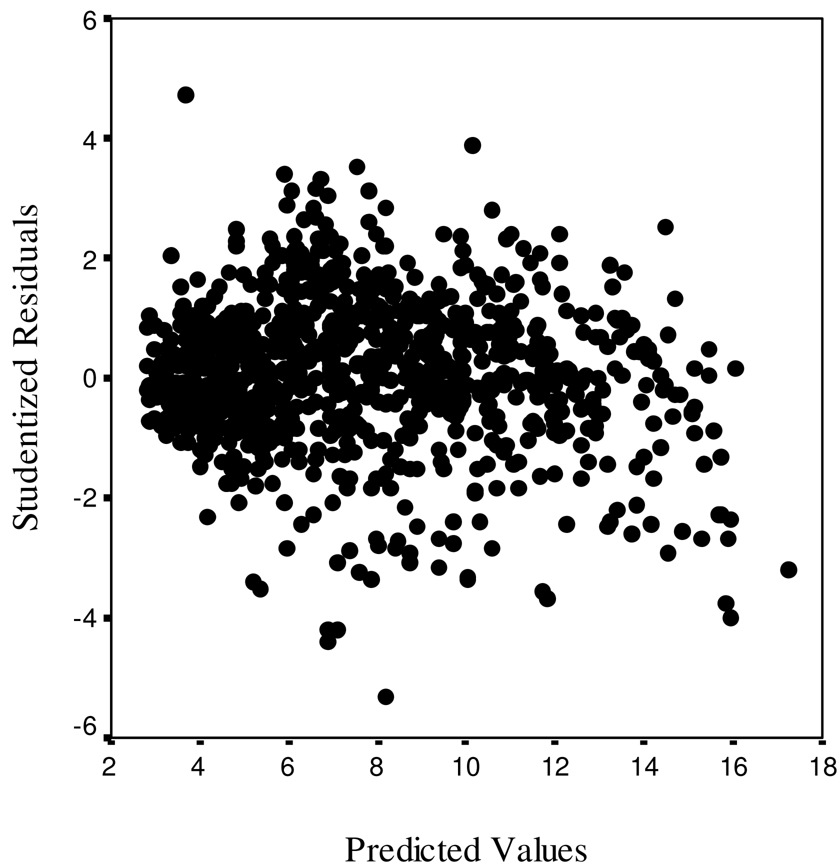

Figure 2. Residuals (actual-predicted) over predicted tree height for trembling aspen using Schnute function).

Model 11 generally performed better than the remaining models. For both Model 3 and Model 11, the confidence intervals for all parameters did not include zero; the Model 11 showed the smallest *RMSE* and approximately homogenous variances over the full range of predicted values, indicating equal variance and reasonable model specification and non systematic pattern in the variation of the residuals. The residual plot also indicated that tree height was well predicted across diameters. The residual plot against the predicted height and diameter for Model 11 clearly show that the function appropriately fits the data (Figure 2).

The parameter estimates obtained for models 3 and 11 widely (Table 4) show significant *t*-statistics. Models 3

Table 4. Parameter estimates (approximated standard error in brackets) and comparison of goodness-of-fit statistics for Model 11 (Schnute function) and Model 3 (base model, using *dbh* only). All estimated parameters were significantly different from zero (p < 0.005).

Parameters		Estimates	
		Model 11 Schnute function (Generalized model)	**Model 3 Base model**
Fixed parameters	*a*	0.06712 (0.005)	2.75399 (0.280)
	b	1.08321 (0.065)	-8.55523 (0.621)
	c		1.460867 (0.338)
Model performance	Adjusted R^2	0.949	0.791
	RMSE	1.226	1.510
	Bias ($\hat{E}$)	0.088	0.001

a, b and *c* are variables.

and 11 have the flexibility to assume various shapes with different parameter values and produce satisfactory relationships under most circumstances. The relationship is biologically reasonable; unrealistic height predictions do not occur beyond the range of the empirical observations. The base model (Model 3) and the basic generalized height-diameter model (Model 11) were tested using Student's Paired *t*-test by an independent control data set (9 sample plots). The models presented in this study were considered to have an appropriate level (*α* = 0.05) of reliability ($t_{Model3} = -2.012$ and $t_{Model11} = 1.917 >$ $P = 0.05$).

DISCUSSION

A large number of both local and generalized height-diameter equations are available in the forestry literature (Ek, 1973; Huang et al., 1992b; Fang and Bailey, 1998; Peng, 1999; Huang et al., 2000; Gadow et al., 2001; Soares and Tomé, 2002; Lopéz Sanchéz et al., 2003; Temesgen and Gadow, 2004). According to Lei and Parresol (2001), when selecting a functional form for the height-diameter relationship, the following mathematical properties should be considered: (i) Monotonic ascent, (ii) inflection point and (iii) horizontal asymptote. The number of parameters and their biological interpretation (e.g., asymptote, maximum or minimum growth rate) and satisfactory predictions of the height-diameter relationships are also important features (Peng, 2001).

In Huang et al. (1992), Model 1 is a Weibull-type function was consistently the best among the 19 height-diameter functions they tested. Flewelling and de Jang (1994) also used Ratkowsky' (1990) model to estimate missing heights in the British Columbia Permanent Sample Plot data sets. In Temesgen and Gadow (2004), Models 2 and 7 are most suitable for predicting tree heights from a diameter-stand table. These models could be recommended for unthinned stands in interior British Columbia. The Chapman-Richards function has been extensively used in describing height-diameter relationship. Huang et al. (1992) gave a cautionary note, however, stating that this function approaches the asymptote too quickly when there is a weak relationship between the dependent and independent variables. Accordingly, this model was not selected in this study.

The height-diameter model developed in the present study was based on the Schnute (1981) function. According to Lei and Parresol (2001), the Schnute function together with the Bertalanffy-Richards function (Bertalanffy, 1949, 1957; Richards, 1959) are probably the most flexible and versatile functions available for modeling height-diameter relationships. The Korf function (Lundqvist, 1957; Larsen and Hann, 1987; Lappi, 1991; Mehtätalo, 2004; Lynch et al., 2005) and the logistic function (Pearl and Reed, 1920; Ratkowsky and Reedy, 1986; Seber and Wild, 1989; Huang et al., 1992) have also been widely used. One of the important advantage of the Schnute function that is, easy to fit and quick to achieve convergence for any database (Bredenkamp and Gregoire, 1988; Lei and Parresol, 2001), even with small datasets (Castedo et al., 2005). This was particularly true in a preliminary analysis of our database in which convergence in the parameter estimates for all the plots was not achieved using the functions of Bertalanffy-Richards and Korf.

Tree height is an important variable which is used for estimating stand volume and site quality and for describing stand vertical structure. Measuring tree heights is costly however, and foresters usually welcome an opportunity to estimate this variable with an acceptable accuracy. Missing heights may be estimated using a height-diameter function. Based on a comprehensive data set which includes very small diameters, such height-diameter functions were fitted for trembling aspen natural stands of Turkey. The inclusion of relative position of a tree and stand variables into the base height-diameter function increased the accuracy of prediction. The fit statistics indicated that models 3 and 11 are most suitable for predicting tree heights from a diameter-stand table. The parameter

Abbreviations:

Dbh	Observed diameter outside bark at breast height (cm)
H	Observed total tree height (m).
H	Estimated total tree height (m).
$\bar{d}_q$	Quadratic mean diameter (cm).
D_{dom}	Dominant diameter (the mean diameter of the 100 thickest trees per hectare, cm).
H_{dom}	Dominant height (the mean height of the 100 thickest trees per hectare, m).
BA	Stand basal area (m^2/ha).
N	Number of trees per hectare (trees/ha).
BAL	The summarized basal area for all greater than the subject tree (m^2/ha).
SI	Site index.
E	Naperian constant (2.718281828).
a, b, c, a_1, a_2, a_3, a_4, a_5, a_6	Regression coefficients.
S.D.	Standard deviation.

estimates using Model 3 and 11 will provide reasonable precision and therefore these models can be recommended for thinned stands in Turkey. Due to the kind of data used, the suggested height curves should not be used in unthinned stands and plantations.

The inclusion of relative position of a tree and stand density variables improved the predictive abilities of models 6 through 11. On average, the addition of these variables to the height-diameter functions reduced the RMSE by 21 cm. Model 11 based on Schnute function also includes stand-level attributes. This model has the lowest RMSE value. On average, this model (Schnute model) reduced the RMSE by 35 cm (Table 3). The relative position and stand measures used in this study are easily obtained and are available in most growth and yield models. Where possible, the use of the height-diameter function with these attributes is suggested. In summary, the suggested model improves the accuracy of height prediction that ensures compatibility among the various estimates in a growth and yield model and maintains projections within reasonable biological limits.

The testes (controlled) generalized height-diameter model allowed accurate results, making this approach highly effective and useful. The suggested approach allowed the natural variability in heights within diameter classes to be mimicked and therefore provided more realistic height predictions at stand level. This feature is considered very important, since the height-diameter model developed in the present study will be used to fill in the missing heights corresponding to trees that were not measured. The inclusion of the dominant diameter and dominant height as an explanatory variable in Model 11 appeared to take into account the competition level within the stand, as there was a close relationship between these variables and the number of trees per hectare. The absence of age was also surprising, because in even-aged, uniform stands, age is a good indicator of the mean size of the individual trees. Once again, this result may be explained by the high correlation between age and dominant height, being the age of the plots implicitly included in the model.

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