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A short range dependence adjusted hurst exponent evaluation for Malaysian and Indonesian financial markets

Chin Wen Cheong^{1*} and Zaidi Isa²

¹Research Centre of Mathematical Sciences, Multimedia University, 63100 Cyberjaya, Selangor, Malaysia.

²DELTA, Faculty of Science and Technology, Universiti Kebangsaan Malaysia, 46100 Bangi, Selangor, Malaysia.

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This study proposed a methodology to measure the Hurst exponent with the adjustment of short-range dependence in the financial markets. The possible short-range dependence is adjusted by heteroscedastic models. Two emerging financial markets have been selected to conduct the adjusted Hurst exponent evaluations for the periods before, during and after the Asian financial crisis. After the short-range dependence adjustment, the empirical results indicated weak and no evidence of long-range dependence in most of the selected markets. As a result, the proposed method is able to handle the possible spurious long range dependence volatility in the financial markets.

Key words: Hurst exponents, long-range dependence, ARCH model, financial time series.

INTRODUCTION

The presence of long-range dependence (LRD) financial markets has important impact in the literature of financial time series analysis. With the inclusion of this statistical property in the model specification, better estimation and forecast can be obtained to help econometricians and researchers in understanding the underlying data generating process of financial time series. The LRD financial time series often referred to market's volatility¹. Ding and Granger (1996) and Granger and Ding (1996) claimed that the volatility proxy using absolute return of S&P500 stock market is more persistence than square return. Similarly, the worldwide stock exchanges are documented with this statistical behaviour by Baillie et al.(1996), Bollerslev and Mikkelsen (1996), Cheong (2010), Hwang (2001), Engle and Lee (1999) and Tse (1998) using the Autoregressive Conditional Heteroscedasticity (ARCH)

dasticity (ARCH) model and its extensions. The application of these studies often related to the risk management analysis especially in quantifying the risk in term of value-at-risk (Jorion, 1997). The related value-at-risk studies can be found in Cheong et al. (2009), Giot and Laurent (2003), Tang and Shieh (2006) and Wu and Shieh (2007).

Besides the contributions to econometric and financial application, the existence of LRD has also provided significant implication to the literature of fractal market hypothesis (Peters, 1994; Dacorogna, 2001). Fractal market hypothesis is evolved from classical efficient market hypothesis (Fama, 1970) by heterogeneous market participants with different endowment, risk profile, degree of information, etc. The presence of LRD have been reported by Cajueiro and Tabak (2004, 2005) who used the Hurst exponent (1951) to rank the global financial markets and found that the developed markets are more efficient than emerging markets. Cheong et al. (2007) on the other hand focused on the regime study of Malaysian stock exchange during the Asian financial crisis. They reported the highest inefficiency during the crisis period, followed by pre-crisis, post-crisis and USD pegged period.

Due to the importance of LRD in the financial studies,

*Corresponding author. E-mail: wcchin@mmu.edu.my, Tel: 603-83125249. Fax: 603-83125264.

¹Since there are studies (Cheung and Lai, 1995; Lo, 1991; Sadiwque and Silvapulle, 2001) reported no evidence of LRD in the return series,

Table 1. Regime selection based on volatility.

Index	Square return	
	F-statistics	Break point
KLSE	105.2767*	28-aug-1998
JSE	26.65230*	27-dec-1996
S&P500	42.73830*	27-mar-1997

* indicated the 5% significance level.

this study aimed to investigate the LRD financial markets with the adjustment of Short-Range Dependence (SRD) using the Ding et al. (1993) asymmetric power ARCH model. The Ding et al. (1993) specification provided a more flexible power form of volatility representation as compared to Taylor (1986) and Bollerslev (1986) with conditional standard deviation and variance respectively. Thus, the suitability representations for volatility are not restricted to the power of one and two only. Later, the Hurst exponents are verified by two time domain heuristic methods and then estimated by frequency domain interval estimations.

Data source

The samples consisted of KLSE (Malaysia) and JSE (Indonesia) from two Asian emerging markets and the S&P500 from the mature market of the United State of America. In order to make the analysis more interesting and meaningful, we divided the data into three regimes namely the *Pre-crisis*, *Crisis* and *Post-crisis* for all the indexes. The impact of Asian Financial crisis to the Hurst exponent can be investigated in the different period of times especially for KLSE and JSE whereas the S&P500 is for the purpose of comparisons. The selections of regimes are determined by Andrews (1993) structural break identification based on the first order autoregressive model in the form of volatility proxy, squared return. Table 1 indicated that the likelihood ratio F-statistics for all the indexes are rejected the null hypothesis of no structural break at 5% significance level under the Hansen's (1995) statistical table and the break point are located around year 1996.

Specifically, the implementation of currency control (pegged USD to RM) in September 1998 by Malaysian government had somehow prevented the RM from further depreciated to approximated 40% according to June 1996 currency exchange. Besides this, the volatility in JSE and S&P500 are also influenced by the crisis. For comparison purposes, the selection periods for pre-crisis, crisis and post-crisis are from January 1990 to August 1996, September 1996 to December 1998 and January 1999 to December 2007 respectively. The selected regimes are three months before the earliest break point in JSE and after the latest in KLSE. This is to ensure the

preceding and following impacts encountered by the structural break are taken into account in the respective periods. The total observations for each indexes are 4213, 4269 and 4536 for KLSE, JSE and S&P500 respectively. The numbers of exact trading days during this period are different according to their public holidays and the percentage continuously compounded daily return is defined as $r_t = 100 \times (\ln P_t - \ln P_{t-1})$.

METHODOLOGY

Figure 1 presented the flow of the computation and analysis of this study. The details of each level are explained as follows:

Short range dependence (SRD) adjustment

The main goal of the adjustment is to eliminate the possible SRD conditional volatility that might existed in the form of moving Average Autoregressive (ARMA). Figure 1 illustrated that the SRD adjustment consisted of procedures started from preliminary analysis, model identification, estimation and finally diagnostic that followed a standard Box-Jenkins (1994) framework before stationary standardized residual series are produced:

Step 1

Preliminary analysis focused on graphical illustration, descriptive statistics and normality tests;

Step 2

Let r_t be a general univariate asset return which is serially uncorrelated but dependent in the ARCH specification. For a given information set I_{t-1} available at time $t-1$, the conditional mean of r_t is defined as

$$E(r_t | I_{t-1}) = E_{t-1}(r_t) = \mu_t \quad (1)$$

with the innovation process $a_t = r_t - \mu_t$ with the conditional variance $Var(r_t | I_{t-1}) = Var_{t-1}(a_t^2) = \sigma_t^2$. In financial time series, the conditional mean often captured by a stationary ARMA(m,n) model² under the non-vector form:

$$\mu_t = \phi_0 + \sum_{i=1}^m \phi_i r_{t-i} + \sum_{i=1}^n \theta_i a_{t-i} \quad (2)$$

The corresponding unconditional variance can be expressed as $Var(a_t(\theta)) = E(a_t^2(\theta)) = \sigma_t^2(\theta)$ where $E(a_t) = 0$ and $E(a_k a_h) = 0$ for all $k \neq h$. Further, the conditional variance begun with the relationship $a_t = \sigma_t z_t$ where for standardized process of z_t , $E(z_t | I_{t-1}) = 0$ and $Var(z_t | I_{t-1}) = 1$ for all t under the normality assumption. Now, considered an asymmetric power Ding, Granger

² An ARCH model frequently represented by a regression model in the form of $r_t = x_t' \beta + a_t$, where x_t' is a column vector.

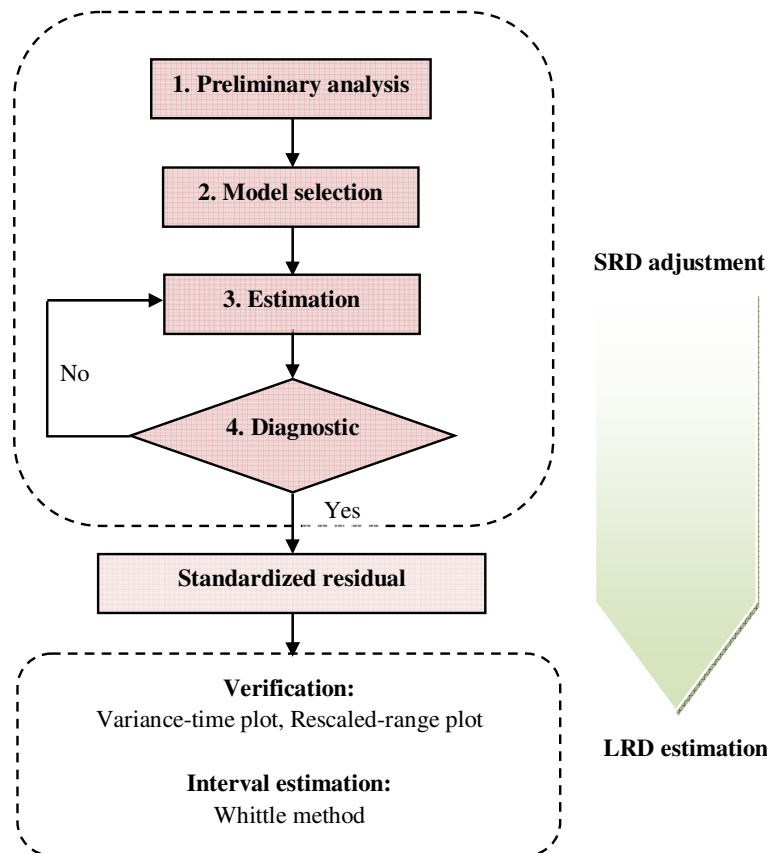


Figure 1. Flowchart for computation procedures.

and Engle³ (1993) GARCH(1,1) model with the following specifications:

$$\sigma_t^\delta = \alpha_0 + \alpha_1 [k_\gamma(a_{t-1})]^\delta + \beta_1 \sigma_{t-1}^\delta \quad (3)$$

where, $k_\gamma(a_{t-1}) = |a_{t-1}| - \gamma a_{t-1}$ and δ is the flexible volatility transformation parameter. Specifically, when the conditional volatility representation restricted to $\delta=2$ (conditional variance), the model changed Glosten, Jagannathan and Runkle⁴ (1993) model with leverage effect (dummy variable) as follows:

$$\text{GJR: } \sigma_t^2 = \alpha_0 + \alpha_1 a_{t-1}^2 + \gamma d_{t-1} a_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (4)$$

$$\text{where } d_{t-1} = \begin{cases} 1 & \text{if } a_{t-1} < 0. \\ 0 & \text{if } a_{t-1} > 0 \end{cases}$$

The GJR model is also considered for the purpose of comparison. It is worth noting that the GJR asymmetric coefficient initiated with positive sign whereas DGE started with negative sign. This is to make sure that the interpretation of news impact is consistent across the models. For example, $\gamma > 0$ indicated the presence of leverage effect with additional impact (γ) as compared to good

news. From the economic point of view, the leverage effect can be explained based on the debt-equity ratio. Market equity values often determined by the stock price where a drop in stock price would increase the ratio and consequently increase the risk from the investor perspectives. Thus negative news has a deeper impact to future volatility than positive news.

Step 3

Under the assumption of $z_t \sim N(0,1)$, the density function $f(a_t | \Omega_{t-1}) = \left(\frac{1}{2\pi\sigma_t^2}\right)^{\frac{1}{2}} \exp\left(-\frac{1}{2} \frac{a_t^2}{\sigma_t^2}\right)$ with the log-likelihood function

$$L_T(\eta) = \sum_{t=1}^T l_t(\eta) = \ln f_a(a_1) + \left\{ -\frac{T-1}{2} \ln(2\pi) - \frac{1}{2} \sum_{t=2}^T \ln(\sigma_t^2) - \frac{1}{2} \sum_{t=2}^T \frac{a_t^2}{\sigma_t^2} \right\} \quad (5)$$

where $\eta = (\alpha_0, \alpha_1, \beta_1, \phi, \delta)$ represented the vector of unknown parameter for conditional dispersion equation all set at time t .

For large sample size, the unknown marginal density $\log f_a(a_1)$ can be ignored under the following derivation:

$$L_T(\eta) = \sum_{t=1}^T \ln f(a_T, \dots, a_2 | a_1) = \sum_{t=2}^T \ln f(a_t | \Omega_{t-1}). \quad (6)$$

³ DGE henceforth.

⁴ GJR henceforth.

Apart from the constants, $l_t(\eta) = -\frac{1}{2}\ln(\sigma_t^2) - \frac{1}{2}\frac{a_t^2}{\sigma_t^2}$.

Differentiating with respect to the vector parameter yielded

$$\frac{\partial l_t}{\partial \eta} = -\frac{a_t}{\sigma_t^2} \frac{\partial a_t}{\partial \eta} - \frac{1}{2} \left(\frac{1}{\sigma_t^2} - \frac{a_t^2}{\sigma_t^4} \right) \frac{\partial \sigma_t^2}{\partial \eta}. \quad (7)$$

However, the DGE ARCH is computed under the representation of σ_t^δ , therefore the additional separated analytical derivatives for conditional dispersion are

$$\frac{\partial \sigma_t^2}{\partial \theta} = -\frac{2\sigma_t^2}{\delta \sigma_t^\delta} \frac{\partial \sigma_t^\delta}{\partial \theta} \quad \text{and} \quad \frac{\partial \sigma_t^2}{\partial \delta} = -\frac{2\sigma_t^2}{\sigma_t^\delta} \left[\frac{1}{\delta} \frac{\partial \sigma_t^\delta}{\partial \delta} - \frac{\sigma_t^\delta}{\delta^2} \ln \sigma_t^\delta \right] \quad (8)$$

where $\theta = (\alpha_0, \alpha_1, \beta_1, \phi)$. The vector gradients with respect to the conditional dispersion parameter can be obtained in the following equations:

$$\begin{aligned} \frac{\partial \sigma_t^\delta}{\partial \alpha_1} &= k_\phi (a_{t-1})^\delta + \beta_1 \frac{\partial \sigma_{t-1}^\delta}{\partial \alpha_1}; \\ \frac{\partial \sigma_t^\delta}{\partial \phi} &= -\alpha_1 \delta k_\phi (a_{t-1})^{\delta-1} a_{t-1} + \beta_1 \frac{\partial \sigma_{t-1}^\delta}{\partial \phi}; \\ \frac{\partial \sigma_t^\delta}{\partial \delta} &= \alpha_1 \left[k_\phi (a_{t-1})^\delta \ln k_\phi (a_{t-1}) \right] + \beta_1 \frac{\partial \sigma_{t-1}^\delta}{\partial \delta}. \end{aligned} \quad (9)$$

A more comprehensive analytic derivatives of DGE ARCH(p,q) can be found in Laurent (2004) and He and Terasvirta (1997). Due to the nonlinearity condition, the iterative optimization algorithm is used instead of analytical derivative approach with the log-likelihood function L_N as follows:

$$\frac{\partial L_N}{\partial \psi} \approx \frac{\partial L_N}{\partial \psi^{(0)}} + (\psi - \psi^{(0)}) \frac{\partial^2 L_N}{\partial \psi^{(0)} \partial \psi^{(0)}}, \quad (10)$$

where $\psi^{(0)}$ denoted the trial values of the estimates and

$\frac{\partial^2 L_N}{\partial \psi^{(0)} \partial \psi^{(0)}}$ represented the Hessian matrix. Rearranging the terms in the form Newton-Raphson algorithm, the $(k+1)^{\text{th}}$ vector set of parameters values is defined as

$$\psi^{(k+1)} = \psi^{(k)} - \left(\frac{\partial^2 L_N}{\partial \psi^{(0)} \partial \psi^{(0)}} \right)^{-1} \frac{\partial L_N^{(k)}}{\partial \psi}. \quad (11)$$

For heavy-tailed ε_t , we used the standardized student-t distribution (Bollerslev, 1987) with the degree freedoms exceeded 3 with the following representation:

$$f(\varepsilon_t; \nu) = \frac{\Gamma\left[\frac{\nu+1}{2}\right]}{\Gamma[\nu/2] \sqrt{\pi(\nu-2)}} \left(1 + \frac{\varepsilon_t^2}{\nu-2}\right)^{-(\nu+1)/2} \quad (12)$$

where $\Gamma[\bullet]$ is the gamma function. By replacing the log-likelihood function of normal distribution in equation 6, the student-t log-likelihood is defined as

$$L_N = N \left(\ln \Gamma\left[\frac{\nu+1}{2}\right] - \ln \Gamma\left[\frac{\nu}{2}\right] - \frac{1}{2} \ln(\pi(\nu-2)) \right) - \frac{1}{2} \left(\sum_{t=1}^N \ln(\sigma_t^2) + (1+\nu) \ln \left(1 + \frac{\varepsilon_t^2}{\nu-2}\right) \right) \quad (13)$$

Step 4

All the models are diagnosed using the Ljung-Box statistics for both standardized and squared residuals. The acceptance of the test statistics indicated no significant autocorrelation in the conditional mean and variance equations. For heteroskedastic effect, the Engle LM ARCH test (Engle, 1982) is used upon the squared standardized residuals.

Hurst exponent LRD estimation

When the stationary power standardized residuals $(\bar{\varepsilon}_t)^\delta$ and $\bar{\varepsilon}_t^2$ are free from SRD, the presence of LRD is verified using two heuristic methods (Variance-time plot and rescaled range method) and then under the assumption of self-similar process, the Whittle maximum likelihood estimation is employed to produce interval estimations. Other domain and frequency domain estimators can be found in Beran (1994) and Mandelbrot (1997). The Hurst exponent is interpreted as Brownian motion (random process) if the value is exactly 0.50 whereas if the value range from 0.50 to 1.00, the time series is LRD. Thus, the LRD become more intense when the Hurst exponent is closer to 1.00. For interval estimation, we have selected the Whittle's estimation which based on the determination of periodogram, $I(\lambda) = \frac{1}{2\pi T} \left| \sum_{k=1}^T y_k e^{ik\lambda} \right|^2$. The advantage of this

method is that it provided interval estimations compared to the previous two point estimators. Suppose a log-likelihood function is divided by the sample size (T) is given by

$$L(H) = \frac{1}{4\pi} \left[\int_{-\pi}^{\pi} \ln f(\lambda; H) d\lambda + \int_{-\pi}^{\pi} \frac{I(\lambda)}{f(\lambda; H)} d\lambda \right]. \quad (14)$$

In order to simplify the integration computation, a simple Riemann sums is used in the following way

$$L_{\text{Riemann}}(H) \approx \frac{1}{2T} \left[\sum_{k=1}^T \log f(\lambda_k; H) + \sum_{k=1}^T \frac{I(\lambda_k)}{f(\lambda_k; H)} \right] \quad (15)$$

where $\lambda_k = 2\pi k/T$ are the Fourier frequencies. For further simplicity, we assumed that the spectral density function is normalized with

$\int_{-\pi}^{\pi} \ln f(\lambda; H) d\lambda = 0$. Thus the Whittle log-likelihood function is

reduced to $L_{\text{Riemann}}(H) \approx \frac{1}{2T} \sum_{k=1}^T \frac{I(\lambda_k)}{f(\lambda_k; H)}$. By minimizing

the sum of these ratios with respect to H , the value of H can be estimated. Under the asymptotically normal assumption, the sample variance is computed as:

Table 2. Descriptive statistics.

Statistic	Index	KLSE			JSE			S&P500		
		Pre-crisis	Crisis	Post-crisis	Pre-crisis	Crisis	Post-crisis	Pre-crisis	Crisis	Post-crisis
Mean		0.048574	-0.111233	0.040719	0.019364	-0.069350	0.088836	0.035453	0.107659	0.007858
Std. Dev.		1.761661	2.945416	1.047450	1.046618	2.800983	1.508177	0.726508	1.150083	1.117465
Skewness		12.82542	0.605720	-0.237202	1.541135	0.409928	-0.146884	-0.175244	-0.669868	0.050716
Kurtosis		340.8189	21.41869	8.591160	21.26730	7.452576	8.073822	5.349084	9.135545	5.229339
Jarque-Bera		6771996*	8248.162*	2907.219*	23380.14*	392.8706*	2339.765*	396.0479*	967.9187*	469.3878*
Observations		1416	581	2216	1635	460	2174	1685	589	2262

* indicated the 5% significance level.

$$\text{Variance}(\hat{H}) = \frac{1}{2T} \sum_{k=1}^T \left(\frac{\partial f(\lambda_k; H)}{\partial H} \right)^{-1}$$
. Thus the estimator can be expressed in the form of confidence interval.

RESULTS

Table 2 reported the descriptive statistics for the unconditional return. For Asian countries, both the means are negatives during the crisis period whereas the S&P500 remained positive for all the regimes. Relatively, the standard deviation across the crisis periods indicated highest values than other periods. This implied that the unconditional volatilities are greater than other periods. Next, the non-zero skewness and kurtosis exceeded three indicated the presence of non-normal distribution for all the series. After the Jacque-Bera normality tests, all the indexes rejected the null hypothesis of normal distribution.

SRD adjustment results

The representations of conditional volatility are

considered in the form of σ^2 and $|\sigma|^\delta$ using GJR and DGE models. Each of the indexes is adjusted according these models before stationary standardized residuals are generated. For the sake of space scarcity, Table 3 only illustrated the maximum likelihood estimation for KLSE under the GJR and DGE specifications. For the conditional mean equation, an autoregressive AR(1) model is sufficient to adjust the serial correlation for the stock markets. According to Miller (1994), similar correction can be adjusted by using a moving average model. Across the periods, the shocks are all t-distributed at 5% significance level. In addition, there is a fading tendency in the degree of freedom from pre-crisis to post-crisis period. In other words, the tail distributions become heavier with small degree of freedom.

In conditional variance estimation, the GARCH coefficient β_1 is less persistent when the power transformation decreased from σ^δ to σ^2 . For example in the pre-crisis period, the persistence of shocks reduced from 0.807912 to 0.749577 when the volatility switched from σ^δ to σ^2 . Similar results are also found in other stock markets. These findings are similar to Ding et al. (1993)

where they claimed that the absolute return exhibited longer memory than the squared returns. In short, higher persistence implied higher correlation between the current and historical volatility. Another interesting stylized fact is captured by the news impact coefficient γ where all the indexes are positive and statistically different from zero at 5% significant level. These findings implied that downward movements (shock) in the global stock markets are followed by greater volatilities than upward movements of the same magnitude. Under the ordinary market condition, this can be easily explained by using the leverage ratio⁵ of a particular listed company in the stock exchange where a crash in stock price can lead to an increase in equity risk and thus triggered a more intense volatility. Next, the power coefficient δ is determined as an endogenous variable where the optimal power transformation is in the range 1.184910 to 1.352293. From the statistical tests, all the δ s are failed to reject the null hypothesis that δ is equivalent to one.

⁵ Similar to debt-equity ratio (Black, 1976).

Table 3. Maximum likelihood estimation for KLSE.

Estimation		GJR			DGE		
Mean		Pre-crisis	Crisis	Post-crisis	Pre-crisis	Crisis	Post-crisis
Constant:	ϕ_0	-0.023569	-0.031118	0.031190	-0.016969	-0.023520	0.026045
MA(1):	ϕ_1	0.207559*	0.162321*	0.155474*	0.200968*	0.161435*	0.149560*
Variance							
Constant:	α_0	0.124223*	0.021661*	0.013797*	0.085644*	0.021393*	0.018134*
ARCH:	α_1	0.064380*	0.025874*	0.092852*	0.163711*	0.125793*	0.135331*
GARCH:	β_1	0.749577*	0.877377*	0.869546*	0.807912*	0.893268*	0.877450*
News impact:	γ	0.265717*	0.222290*	0.065992*	0.398249*	0.545203*	0.179681*
Power:	δ				1.352293*	1.184910*	1.210909*
$H_0: \delta=1$					1.31158	0.488432	0.862921
$H_0: \delta=2$					-2.41140*	-2.15302*	-3.22851*
Tail:	ν	4.122349*	5.174973*	5.653057*	4.120701*	5.055660*	5.675102*
Selection							
AIC		2.927623	3.983183	2.494411	2.926794	3.977782	2.491995
SIC		2.953619	4.035770	2.512427	2.956504	4.037882	2.512585
Diagnostic							
Q(6) for \tilde{a}_t^2		0.0107 (0.996)	1.3415 (0.931)	5.5604 (0.351)	0.0088 (0.999)	1.2849 (0.936)	9.0059 (0.109)
LM(6) for \tilde{a}_t^2		0.00177 (0.999)	0.212401 (0.9729)	0.901760 (0.4925)	0.001456 (0.999)	0.202094 (0.9761)	1.462696 (0.1871)

1. \tilde{a}_t represents the standardized residual. Ljung Box Serial Correlation Test (Q-statistics) on \tilde{a}_t and \tilde{a}_t^2 : Null hypothesis – No serial correlation; LM ARCH test: Null hypothesis - No ARCH effect; 2. The values in parentheses represent the p-value. 3.* denotes significance at 5% level.

Table 4. Summary estimation results for JSE and S&P500.

Estimation	JSE			S&P500		
	Pre-crisis	Crisis	Post-crisis	Pre-crisis	Crisis	Post-crisis
GARCH: β_1	0.538394*	0.954712*	0.727583*	0.953291*	0.896952*	0.941994*
News impact: γ	0.036626*	0.999669*	0.255962*	0.477139*	1.000000*	1.000000*
Power: δ	1.131871*	1.470831*	2.085031*	1.457916*	1.080403*	1.186539*
$H_0: \delta=1$	0.593241	1.309563	2.514434*	1.146507	0.195519	0.960358
$H_0: \delta=2$	-3.90541*	-1.47182	0.19705	-1.35724	-2.23622*	-4.18794*

* denoted significance at 5% level.

However contrary results are observed for null hypothesis for δ is equivalent to two. These findings implied that the KLSE volatility is statistically preferable in the representation of conditional standard deviations.

Table 4 summarized the DGE estimations where there is a mixture of conditional variance and standard deviation representation in the market volatility in JSE and S & P500. The most appropriate models are selected

based on the Akaike information (AI) and Schwarz information (SI) criteria which evaluated from the adjusted (penalty function) average log likelihood function. Both the information criteria reported smaller AIC for DGE, however the adjusted penalty function using SIC has caused additional values to this extra parameter. Overall the AIC and SIC are quite similar in all the models. For diagnostic analysis, the Ljung-Box statistic

Table 5. Preliminary LRD evaluations before and after the SRD adjustment.

Method	After GJR SRD adjustment				Before SRD adjustment			
	VT plot		R/S plot		VT plot		R/S plot	
	H	R ²	H	R ²	H	R ²	H	R ²
KLSE								
Pre-crisis	0.497	0.999	0.511	0.998	0.495	0.999	0.620	0.997
Crisis	0.409	0.989	0.511	0.994	0.716	0.982	0.671	0.997
Post-crisis	0.593	0.968	0.533	0.997	0.820	0.956	0.714	0.995
JSE								
Pre-crisis	0.527	0.999	0.568	0.999	0.637	0.994	0.637	0.998
Crisis	0.463	0.999	0.525	0.997	0.739	0.946	0.628	0.999
Post-crisis	0.663	0.962	0.555	0.999	0.727	0.985	0.633	0.999
S&P500								
Pre-crisis	0.781	0.896	0.613	0.997	0.806	0.926	0.655	0.996
Crisis	0.380	0.954	0.603	0.996	0.640	0.989	0.678	0.994
Post-crisis	0.611	0.981	0.575	0.998	0.849	0.930	0.718	0.996

R² denoted the coefficient of determinant.

(squared-standardized residuals) and the Engle LM (1982) statistics are implemented in all the models. Overall, all the models in the selected global markets are free from heteroskedastic effect at 5% significance level. Based on the diagnostic results, both the GJR and DGE standardized residuals ($\hat{\epsilon}_t$) are generated for the LRD evaluations.

LRD Hurst exponent estimation

This analysis begins with the examination of volatility proxies using raw unconditional return (without any adjustment) in the form of square values. Between the two heuristic methods, R/S method provided higher R² (coefficient of determinant) as compared to variance-time plot under the ordinary least squared estimation in a simple log-log regression. In other words, the R/S method indicated higher proportion of variability (99% and above) explained by the regression model. These findings are inline with Mandelbrot and Taqqu (1979) who suggested R/S is more superior to more conventional methods (including variance-time method) of determining long-range dependence. However, the R/S method is sensitive to SRD. Although Lo (1991) introduced a modified version of R/S, Willinger et al (1999) and Teverovsky et al.(1999) claimed that this method has an issue in inference power and poor performance in detecting long-range dependence.

Table 5 showed that all the indexes indicated strong LRD with the Hurst exponent exceeded 0.600 in the R/S estimation for all three periods. It is also worth noting that

the LRD after the crisis is more intense in all the three markets. However, there is a possibility that the LRD is caused by the underlying SRD. Due to this we conducted SRD adjustment to generate stationary standardized residuals ($\hat{\epsilon}_t$) in the power form of σ^2 and $|\sigma|^\delta$ using the GJR and DGE models. Again, Table 5 reported significant lower Hurst exponent in all the periods for three markets. The R/S method indicated weakest LRD in KLSE (0.511-5.33), followed by JSE (0.525-0.568) and lastly S&P500 (0.575-0.613). In order to obtain a better estimation, now we turned to Whittle interval estimation.

Table 6 and 7 reported the variance of Hurst exponent for each market. With the estimated Hurst exponent variance, the estimations are presented with the confidence interval for 95% and 99%. Interesting results have been observed where almost all the estimators indicated Hurst exponent close to 0.500 which implied that the volatility series are stationary random processes with no LRD. Moreover, the lower bounds for all the interval estimations are below 0.500 and suggested that there is no evidence of LRD. In other words, this contrary results compared to the raw volatility proxies in the previous analysis provided spurious LRD in all the indexes with $H > 0.600$. As a conclusion, there is no or mostly weak LRD in the volatility series after the adjustment of SRD.

From the finance point of view, the SRD time series or weakly LRD implied that the studied financial markets are supporting the random walk hypothesis and in accordance with weak-form informational efficiency. The Hurst

Table 6. Adjusted Hurst exponent using GJR model (σ^2).

Estimation	Variance (H)	H	95% C.I.		99% C.I.	
			Lower bound	Upper bound	Lower bound	Upper bound
KLSE						
Pre-crisis	0.019643	0.500	0.461	0.538	0.449	0.550
Crisis	0.027807	0.500	0.445	0.554	0.428	0.571
Post-crisis	0.014031	0.510	0.482	0.537	0.473	0.546
JSE						
Pre-crisis	0.019643	0.500	0.461	0.538	0.449	0.550
Crisis	0.039031	0.503	0.426	0.579	0.402	0.603
Post-crisis	0.013776	0.510	0.483	0.537	0.475	0.545
S&P500						
Pre-crisis	0.019643	0.524	0.485	0.562	0.473	0.574
Crisis	0.028062	0.540	0.485	0.595	0.468	0.612
Post-crisis	0.014031	0.500	0.472	0.527	0.463	0.536

Table 7. Adjusted Hurst exponent using DGE model ($|\sigma^{\beta}|$).

Estimation	Variance (H)	H	95% C.I.		99% C.I.	
			Lower bound	Upper bound	Lower bound	Upper bound
KLSE						
Pre-crisis	0.019643	0.523	0.461	0.538	0.449	0.550
Crisis	0.027807	0.551	0.445	0.554	0.428	0.571
Post-crisis	0.014031	0.509	0.472	0.527	0.463	0.536
JSE						
Pre-crisis	0.019643	0.500	0.484	0.561	0.472	0.573
Crisis	0.039797	0.521	0.473	0.629	0.448	0.654
Post-crisis	0.090308	0.526	0.482	0.536	0.426	0.544
S&P500						
Pre-crisis	0.019643	0.500	0.461	0.538	0.449	0.550
Crisis	0.027807	0.500	0.466	0.575	0.449	0.592
Post-crisis	0.014031	0.500	0.498	0.553	0.489	0.562

estimations with tendency toward SRD indicated the elimination of predictability in these studied financial markets and consequently provided little chances for investors, portfolio managers and practitioners to excel in these markets.

Conclusion

This study contributed to the literature of long-range dependence financial time series by showing the importance of eliminating the possible SRD using ARCH-

family models. From the case studies of KLSE, JSE and S&P500, the SRD adjustment procedures shown that they are able to eliminate the spurious dependence behaviours which have been found using the proxy volatility directly from the raw data. The presence of SRD or weakly LRD, implied that the studied financial markets predictability are somewhat weak and might only provided little information for forecasting. Thus for other LRD analysis, the empirical findings of this study suggested that one should aware of the possible spurious long-range dependence volatility in the financial time series. Because misspecification of modelling may end up with

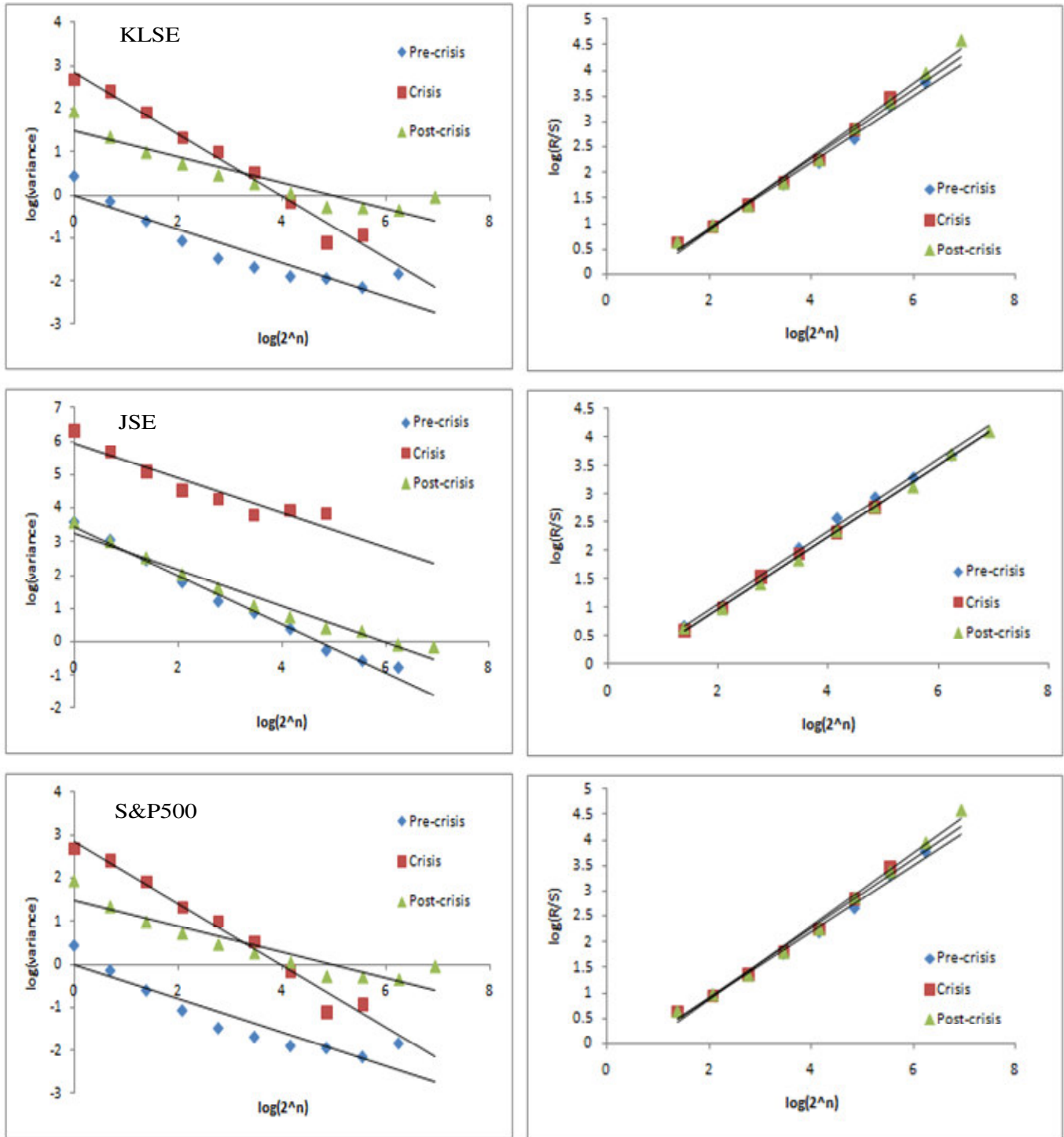


Figure 2. Variance-time plot and R/S plot for square return.

spurious inferences and forecasts. For future study, other alternatives such as time and frequency domain Hurst parameter estimators can be considered in order to obtain a more accurate result.

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