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Full Length Research Paper

An integrated inventory location model considering all-unit quantity discount

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One of the recent areas in distribution network design is integrated inventory location problems, which jointly determines the inventory control decisions and facility location decisions of a distribution network. A typical distribution network consists of suppliers, several retailers, and several distribution centers. Distribution centers order products from suppliers to fulfil demands of retailers. To achieve risk pooling benefits, inventory of several retailers is aggregated into one distribution center. This situation has made distribution centers more likely to take the advantage of quantity discount from suppliers. However, most previous models have investigated the problem under the basic economic order quantity EOQ with (Q, r) inventory policy and yet quantity discount has not been considered. This paper shows that considering quantity discount instead of EOQ policy can reduce the total cost and change the optimal configuration of the networks.

Key words: Integrated inventory location problem, distribution network design, all-unit quantity discount, EOQ model, supply chain management.

INTRODUCTION

Distribution network design is one of the important issues in supply chain management (Miranda and Garrido, 2008; Golmohammadi et al., 2010). Two main sub-problems of distribution network design are the location allocation problem and the inventory control problem (Ahmadi Javid and Azad, 2010). Due to the interrelationship of these two problems (ÜSter et al., 2008), recently a number of integrated location-inventory models (Miranda and Garrido, 2004; Shen et al., 2003; Mak and Shen, 2009) have been presented. According to the literature, a typical distribution network is usually composed of suppliers, Distribution Centers (DCs) or warehouses and retailers. In most of integrated inventory location models, it is assumed that Distribution Center act as the only stocking points in the system and orders products from suppliers to fulfill the demand of several retailers. Indeed the inventory of several retailers is centralized into one distribution center. This condition is known as risk pooling. When each distribution center order products for more than one retailer, then the system is more likely to take the advantage of quantity discount from the supplier (Chen, 2009). While majority of models of distribution network design assume that the network operates under the basic Economic Order Quantity (EOQ) and (Q, r) policy (Park et al., 2010; Ozsen et al., 2008; Daskin et al., 2002), yet other policies such as quantity discount have not been considered.

Basic EOQ cannot give any guideline for accepting or ignoring the offered quantity discount (Followill, 1997). On the other hand, Inventory control plays a major role in the supply chain management and therefore, investigation of the network under other types of inventory policies is required.

This work is motivated by real cases of distribution networks that prefer to rent their required warehouses instead of establishing new ones. Especially for those food distributors that work seasonally throughout the year. In these networks, the owner of the chain possesses some wholesalers which are faced with random demand. The products are purchased from suppliers to be sold to

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the wholesalers. A number of warehouses have to be rented to store the purchased products. For each rented warehouse, the distribution network is charged annually a fixed and a variable cost. The fixed part can be interpreted as the fixed setup cost and, the variable part that depends on the amount of the inventory of products is equivalent to the inventory cost. The inventory level of the warehouses is reviewed according to the (Q, r) ordering policy. At the beginning of the year, the supplier offers a quantity discount policy for the whole year. The manager of the supply chain decides the number of the warehouses and their locations in order to minimize the total cost. The problem is formulated as a joint inventory location model in which quantity discount is available and the effect of the pricing policy offered by suppliers on the total cost and optimal configuration of distribution network should be investigated.

Quantity discount is a common pricing policy offered by suppliers to encourage buyers to purchase in larger quantities (Mendoza and Ventura, 2008). The supplier determines the price for different amounts of order quantity and the buyer decides how much to buy. A sample of the quantity discount schedule is presented as follows:

$$p = \begin{cases} p_0 & q_0 \le Q < q_1 \\ & \vdots \\ p_n & q_n \le Q < q_{n+1} \end{cases}$$

Where p is price, Q is the amount of order quantity, $p_0 > p_1 > \ldots > p_n > 0$ and $0 = q_0 < q_1 < \ldots < q_{n+1} = \infty$.

There are two common types of quantity discount; all-unit and incremental (Mendoza and Ventura, 2008). When all-unit quantity discount is offered, the price reduction is applied for all of the units purchased. However, incremental quantity discount offers price reduction only to the units purchased above a specific amount.

Lots of research has been done on the quantity discount policy. As an example, Tsai (2007) developed some quantity discount models and used a linearization approach to overcome the non-linearity of the model. Mendoza and Ventura (2008) incorporated all-unit and incremental quantity discount into an EOQ model and developed a number of exact algorithms to solve the models. Mirmohammadi et al. (2009) presented an optimal all-unit quantity discount algorithm for a material requirement planning environment considering deter-ministic demand, zero lead time, and constant ordering cost.

On the other hand, there exist a variety of models in the literature for integrated inventory location problems. One of the earliest studies in this area was done by Erlebacher and Meller (2000). This paper formulates the joint inventory location problem as a nonlinear mathematical model and developed a heuristic approach to solve the problem. Daskin et al. (2002) incorporated the inventory and safety stock decisions into the un-capacitated facility location problem. The model which is called Location Model with Risk Pooling (LMRP) captures the risk pooling effects in the problem. A Lagrangian relaxation solution algorithm was developed to solve the models. Shen et al. (2003) also solved the LMRP but solution method was based on a set partitioning approach. Miranda and Garrido (2004) considered an integrated inventory location problem to design a distribution network and presented a Lagrangian relaxation algorithm to solve the problem. Mak and Shen (2009) considered the problem of inventory location to model a distribution network for spare part items. They considered a base stock ($S-1$, S) inventory policy for the stocking points in the network as the demand for spare part is low and the base stock ($S-1$, S) inventory policy is shown to be suitable for items with low demand and high inventory cost. Liao and Hsieh (2009) developed a multi-objective model for an integrated inventory location problem to investigate the trade-off between efficiency and responsiveness of the distribution network. Ahmadi Javid and Azad (2010) incorporated routing decision into an inventory location problem and presented an exact and a hybrid algorithm based on tabu search and simulated annealing to solve the problem. Park et al. (2010) developed an integrated inventory location model that determines the location of two different layers (suppliers and distribution centers).

Finally, the present study formulates the integrated inventory location problem in situations where all-unit quantity discount is available, and compares the total cost and networks configuration under quantity discount and a simpler inventory model that considered EOQ as the inventory policy.

PROBLEM DESCRIPTION

The problem discussed in this paper is a variant of Capacitated Warehouse Location Model With Risk Pooling (CLMRP) that is introduced by Ozsen et al. (2008). CLMRP captures the trade-off between establish-ments of more warehouses (increase of fixed facility cost) versus more frequent order from the supplier (increase of the ordering cost). CLMRP is an extension of the Un-Capacitated Facility Location Problem (UFLP) which is proved to be NP-hard (Krarup and Pruzan, 1983). Accordingly, the proposed model in this paper is also NP-hard.

The problem is described as follows: A distribution network purchases its product supply from a single supplier to meet the demand of several retailers. The owner of the distribution network has to rent some warehouses to store the purchased product. A fixed cost is charged every year for renting a warehouse that can be equivalent to the fixed setup cost of establishing a warehouse. Moreover, a variable cost is charged that depends on the amount of the inventory stored in each warehouse. The supplier offers an all-unit quantity discount at the beginning of every year. The goal of the problem is to determine the network configuration deci-sions (location allocation), and inventory control decision of the network considering quantity discount to minimize the purchasing, ordering, inventory, transportation, and fixed warehouse rental cost.

Model assumptions

The following assumptions are considered in the proposed model:

- Demands of the retailers are independent and follow normal distribution.
- The capacity level and possible locations of distribution centers are known.
- The location of the supplier and the number and location of the retailers are known.
- The quantity discount schedule is offered by the supplier at the beginning of the year and will not change before the end of the year.

Model formulation

The following notations are used in order to model the problem.

1. Sets: *J*, set of retailers; I, set of candidate locations for DCs; *K*, set of intervals in the price function.
2. Indices: *i:* Index for DCs; *j:* Index for retailers; *k:* index for intervals in the price function.
3. Input parameters: F_i: Annual fixed setup cost for DC_i ; *T:* Transportation cost per unit of product per unit of distance between DCs and retailers; *g:* Fixed transportation cost per vehicle dispatched from supplier to the each *DC*; h_i: Inventory holding cost at DC_i per unit of product per year; O_i: Fixed ordering cost per order placed by DC_i to the supplier; C_i: Capacity of DC_i; d_j: Mean monthly demand of retailer$_j$; v_j: Variance of monthly demand for retailer$_j$; dis_{ij}: Distance between DC_i and Retailer$_j$; lt_i: Lead time in months from the supplier to DC_i; P_k: Price for one item of product that is purchased with the quantity that place in the *k*th interval; L_k: Lower bound of order quantity in the *k*th Interval of discount schedule; U_k: Upper bound of order quantity in the *k*th Interval of discount schedule; 1-α: Risk of stock out; 1-β: Risk of exceeding the inventory level of each warehouse from its capacity level;
Z_0: Standard Normal deviate such that P (z ≤ Z_0) = θ.
4. Decision variables: Q_i: Order quantity of DC_i; y_{ij}: Binary variable, taking the value 1 if Retailer$_j$ is assigned to DC_i and 0 otherwise; x_i: Binary variable, taking the value 1 if DC_i is open and 0 otherwise uik: Binary variable, taking the value 1 if the order quantity of DCi falls within the kth interval of the discount schedule and 0 otherwise.

For the rest of this section, the components of objective function are described thus:

Holding cost

Q3. According to the above notation the annual inventory cost of DC_i can be written as:

$$\sum_i h_i(Q_i/2+Z_\alpha\sqrt{lt_i}\sqrt{\sum_j(v_j y_{ij})} \quad (1)$$

Where the first term represents the average working inventory cost and the second term is the safety stock holding cost.

Ordering cost

Ordering cost can be formulated as:

$$NO_i\sum (d_i y_{ij})/Q_i \qquad \forall i\in I,\ \forall j\in J \quad (2)$$

Where $N\sum (d_i y_{ij})/Q_i$ .N)/Q_i represents the number of the orders that will be placed by each DC per year.

Transportation cost

Transportation cost contains two parts which are calculated according to the following formula.

$$Ng\sum_j (d_j y_{ij})/Q_i + T\sum_j (d_j y_{ij})dis_{ij} \quad \forall i\in I \quad (3)$$

The first part is the fixed transportation cost that is charged per vehicle dispatched from the supplier to the DCs assuming that the capacity of the transportation vehicles equals to the order quantity (Q_i). The second part is the variable transportation cost which depends on the number of the items shipped from DC_i to *Retailer$_j$*.

Fixed setup cost

Fixed setup cost, the cost of renting DC_i, is calculated using the following formula:

$$\sum_i F_i x_i \qquad \forall i\in I \quad (4)$$

Where x_i is a binary variable that is equal to 1 for open DCs and 0 for close DCs.

Purchasing cost

Where quantity discount is allowed, the purchasing cost would be variable depending on the amount of the order quantity. If P_k is the price of a specific order quantity that falls in the *k*th interval, then the total purchasing cost is calculated by the following formula:

$$N\sum_i\sum_j\sum_k P_k u_{ik}(d_j y_{ij}) \qquad (5)$$

Total annual cost

The objective function which is the summation of the above-mentioned cost is written as follows.

Min

$$\sum_i h_i(Q_i/2+Z_\alpha\sqrt{lt_i}\sqrt{\sum_j(v_j y_{ij})}) + \sum_i N(O_i+g)\sum_j (d_i y_{ij})/Q_i + \sum_i F_i x_i + N\sum_i\sum_j\sum_k P_k u_{ik}(d_j y_{ij}) + T.N\sum_i\sum_j (d_j y_{ij})dis_{ij}$$

(6)

Subject to:

$$\sum_i y_{ij} = 1 \qquad \forall j\in J \qquad (7)$$

$$x_i \ge y_{ij} \qquad \forall i\in I,\ \forall j\in J \qquad (8)$$

$$Q_i + (Z_\alpha + Z_\beta)\sqrt{\sum_j v_j y_{ij} lt_i} \le C_i x_i \quad \forall i\in I \quad (9)$$

$$Q_i \ge L_k u_{ik} \qquad \forall i\in I,\ \forall k\in K \qquad (10)$$

$$Q_i \le U_k u_{ik} \qquad \forall i\in I,\ \forall k\in K \qquad (11)$$

$$\sum_k u_{ik} = x_i \qquad \forall i\in I \qquad (12)$$

Table 1. All unit quantity discount scheme.

Order quantity	Unit cost
1 - 1999	10
2000 - 3499	9.5
3500 - 4699	9.0
4700 - 5999	8.5
6000 and more	8

$$Q_i \geq 0 \qquad \forall i \in I \qquad (13)$$

$$x_i, y_{ij}, u_{ik} \qquad \forall i \in I,\ \forall j \in J,\ \forall k \in K \qquad (14)$$

Constraint set (7) ensures the single sourcing strategy for retailers. Constraint set (8) makes sure that retailers are not assigned to close DCs.

Constraint set (9) is the stochastic inventory capacity constraint. This expression ensures that the inventory capacity of each warehouse is satisfied at least with a probability of β. More details can be found in Miranda and Garrido (2008), Miranda and Garrido (2006) and Ozsen et al. (2008). Constraint set (10) and (11) guarantee that if an interval k is selected, the amount of the order quantity falls between the bound of the selected interval. Constraint set (12) makes sure that for each open DC, only one interval of order quantity will be selected from the price function. Constraint set (13) implies that the order quantity is a non-negative value and Constraint set (14) specifies that x_i,y_{ij},u_{ik} are all binary variables.

SOLUTION APPROACH

As discussed earlier, this paper assumes that the distribution network is operating under quantity discount and continuous review (Q, r) policy. In (Q, r), when the inventory level drops below the reorder point (r), an order of Q will be placed (Al-Harkan and Hariga, 2007). To apply the quantity discount with (Q, r) ordering policy into the model of this article, the same procedure as used by (Park et al., 2010; Ozsen et al., 2008; Shen et al., 2003) has been applied in order to approximate the economic order quantity (EOQ), and (Q, r). In this procedure, at first, the amount of order quantity is determined under basic EOQ inventory policy, and, based on the order quantity the reorder point (r) is calculated. To do so, in this paper, the amount of the order quantity is determined based on quantity discount, and afterward the reorder point is calculated. Park et al. (2010); Ozsen et al. (2008) and Shen et al. (2003) approximated the (Q, r) model in two steps. At first they found the order quantity by EOQ model, and then determined the reorder point. This paper applied the same approach but using quantity discount in order to find the optimal order quantity.

An enumeration algorithm has been developed to solve the problem optimally. The algorithm is able to solve problems with multiple discount level. To determine the amount of order quantity, based on the quantity discount policy, the following steps are included in the algorithm.

i. Step 1: At first the order quantity (Q) is calculated based on the EOQ inventory policy.
ii. Step 2: Based on the interval in which Q is placed, the price will be identified.
iii. Step 3: Knowing the amount of Q and the price, the inventory, ordering, and purchasing cost will be computed.
iv. Step 4: While a lower price or a greater level of Q exists, Q will be replaced by the minimum amount of that interval and the algorithm will continue from Step 2 or otherwise from Step 5.
v. Step 5: The amount of Q that results in the lowest cost is chosen as the optimal order quantity.
vi. Step 6: Constraint set (9) will be taken into account.

The two

9). In this case, order quantity does not change.

9). In this situation, the amount of order quantity will change to the maximum amount that the violated constraint allows.

RESULTS AND DISCUSSION

The computational experiment involves solving 14 test problems on a T2350, 1.86 GHZ with 1 GB RAM. The parameters of the problems are constructed as follows.

The location of the retailers and warehouses are uniformly distributed over the square of [0, 10].

The average monthly demands of the retailers are uniformly drawn between [1000, 3000]. The variances of the retailers' demands are uniformly drawn between [900, 1500]. The Capacity of distribution centers are selected uniformly between [3000, 9000], and the fixed cost of renting each DC is selected as a proportion of its capacity. The pricing policy offered by supplier assumed to be as Table 1 and the rest of the input parameters are shown in Table 2. In order to show that the network configuration, that is, number of warehouses and assignment of retailers to warehouses, is sensitive to the pricing policy offered by the supplier, several test problems are solved under EOQ and Quantity discount inventory policies. Obviously, in order to solve the problems under EOQ, the purchasing price for any amount of order quantity is constant and is equal to the price of the first level of the quantity discount scheme, that is the highest price.

The results of solving test problems are presented in Table 3. This table shows in most cases, the optimal configuration of the network varies under EOQ and quantity discount policies. This phenomenon can be interpreted in this way: when the quantity discount leads to larger quantity for a particular warehouse, then that warehouse may no longer have enough capacity to receive that amount of order quantity and another warehouse with sufficient capacity may result in less total cost for the network.

However, if the break points of discount schedule offered by the supplier are much more or much less than the optimal order quantity of the warehouses using EOQ model. Then pricing policy may not affect the total cost or network configuration.

Conclusion

Majority of models on inventory location problems assume that the system is operating under approximation

Table 2. Parameters of generated test problems.

Generated test problem	Value
Per unit per mile transpiration cost from DCs to Retailers	0.5
Fixed transportation cost from supplier to each DC	3
Ordering cost per order	500
Per unit per year average holding cost	2
Lead time in terms of month	0.1
Level of service α, β	2.5%

Table 3. Comparison of network configuration and total cost under EOQ and Quantity Discount.

Test problem	# of DCs	# of Retailer	Quantity discount			EOQ			Gap (%)
			Open DCs	Assignment of retailer	Total cost	Open DCs	Assignment of retailer	Total cost	
1	3	6	1 3	3,5 1,2,4,6	1702830	3 -	1,2,3,4,5,6	1721810	1.10
2	4	6	1 3	1,3 2,4,5,6	1676900	1 3	3 1,2,4,5,6	1707960	1.82
3	4	8	2 3	2,4,5,6,7,8 1,3	1984120	2 3	2,4,7,5,6,8 1,3	2003420	0.96
4	4	9	1 2 3	3,6,8 2 1,4,5,7,9	2336370	1 2 3	3,6,8 2 1,4,5,7,9	2336370	0
5	7	7	4	1,2,3,4,5,6,7	868032	4	1,2,3,4,5,6,7	868032	0
6	6	8	1 2 3	1,4,6 8 2,3,5,7	1836300	1 2 3	1,4,6 8 2,3,5,7	1870060	1.80
7	4	10	2 3	6,7,8,9,10 1,2,3,4,5	2461700	2 3	6,7,8,9,10 1,2,3,4,5	2461700	0
8	6	9	3 4 5	8 3,5,9 1,2,4,6,7	2270740	3 4 5	7,8 3 1,2,4,6	2291600	0.91

Table 3. Contd.

9	8	8	1	5	1935860	1	5	1943940	0.42
			2	4,6,8		2	4,6,8		
			6	1,7		7	1,2,3,7		
			7	2,3		-			
10	6	10	2	1,2,3,4,5,8,9,10	2425260	1	5,6,10	2449060	0.97
			6	6,7		3	2,4,8		
			-	-		4	1,3,7,9		
1	6	11	1	5,6,10	2706560	1	5,6,10	2713500	0.26
			2	11		3	2,4,8		
			3	2,4,8		4	1,3,7,9,11		
			4	1,3,7,9		-			
12	9	9	1	5	2155240	2	8	2167930	0.59
			2	8		4	3,4,6		
			4	6,3,4		5	1,2,5,7,9		
			5	1,7,9		-			
			6	2		-			
13	8	10	1	4,8	2433650	1	4,8	2441340	0.31
			3	3,10		3	3,10		
			4	5,7		4	5,7		
			5	1,2,6,9		5	1,2,6,9		
14	10	10	4	6,9	2410410	4	5,6,7,9	2424660	0.59
			8	1,2,3,4,5,7,8,10		8	1,2,3,4,8,10		

Gap= (Total cost under EOQ- Total cost under Quantity Discount)/ Total cost under EOQ *100.

of basic economic order quantity (EOQ) with (Q, r) inventory policy and other types of inventory policies are rarely considered.

This paper introduces a mathematical model and its solution approach for the distribution network design problem when quantity discount inventory policy is available. The model jointly determines the location allocation and inventory control decisions of a network. The paper formulates the problem based on a nonlinear mixed integer mathematical model, and proposes an enumeration algorithm to solve the problem.

The algorithm can be applied for both quantity discount with multiple discount level and basic EOQ inventory policy. Several test problems have been solved separately under both EOQ and quantity discount policies, and the results have shown that the pricing policy can affect the optimal cost and configuration of the distribution networks. Since the model is NP-Hard, for future works, it would be interesting to solve the model using heuristic algorithms to be able to solve larger test problems.

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