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Full Length Research Paper

Modelling volatility and financial market risk of shares on the Johannesburg stock exchange

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In this paper, we develop ARMA-GARCH type models for modelling volatility and financial market risk of shares on the Johannesburg Stock Exchange under the assumption of a skewed Student-t distribution. Daily data is used for the period 2002 to 2010. Several GARCH type models are used including threshold GARCH, GARCH-in mean and exponential GARCH. The results suggest that daily returns can be characterized by an ARMA (0, 1) process. This means that shocks to conditional mean dissipate after one period. Empirical results show that ARMA (0,1)-GARCH(1, 1) model achieves the most accurate volatility forecast. These results are useful to financial managers and modellers in both emerging and developed economies.

Key words: GARCH, volatility clustering, risk, forecasting.

INTRODUCTION

Financial time series data such as stock prices and returns are characterized by high volatility and fat tails. They exhibit high frequency, non-constant mean and variance. Models for volatility modelling were first developed by Engle (1982). These models known as the autoregressive conditional heteroskedasticity (ARCH) models were developed to capture the non-constant variance. ARCH models were later extended to generalized ARCH (GARCH) models by Bollerslev (1986) and Nelson (1991). This paper investigates the behaviour of ARMA-GARCH type models for modelling volatility and financial market risk of shares on the Johannesburg Stock Exchange (JSE). The JSE is licensed as an exchange under the Securities Services Act of 2004 and is Africa's premier exchange (JSE website). It has operated as a market place for the trading of financial products for nearly 120 years. The JSE is also a major provider of financial information. In everything it does, the JSE strives to be a responsible corporate citizen. The JSE does not only channel funds into the economy, but also provides investors with returns on investments in the form of dividends. The JSE analyzes business information to identify areas of risk and make recommendations on profitability models. The exchange is fulfilling its main function by rechanneling cash resources into the productive economy activity, thus building the economy while enhancing job opportunities and wealth creation. In this time, the JSE has evolved from a traditional floor based equities trading market to a modern securities exchange providing fully electronic trading, clearing and settlement in equities, financial and agricultural derivatives and other associated instruments and has extensive surveillance capabilities. Volatility is defined as the statistical measure of the dispersion of returns for a security or market index within a specific time horizon. Volatility can either be measured by using the standard deviation or variance between returns from that same security or market index. It is used to quantify the risk of the financial instrument over the specified time period. Investors and financial analysts are concerned about the uncertainty of the returns on their investment assets caused by market risk and instability of business performance (Alexander, 1999).

DATA

Daily data of stock prices of all counters listed on the JSE

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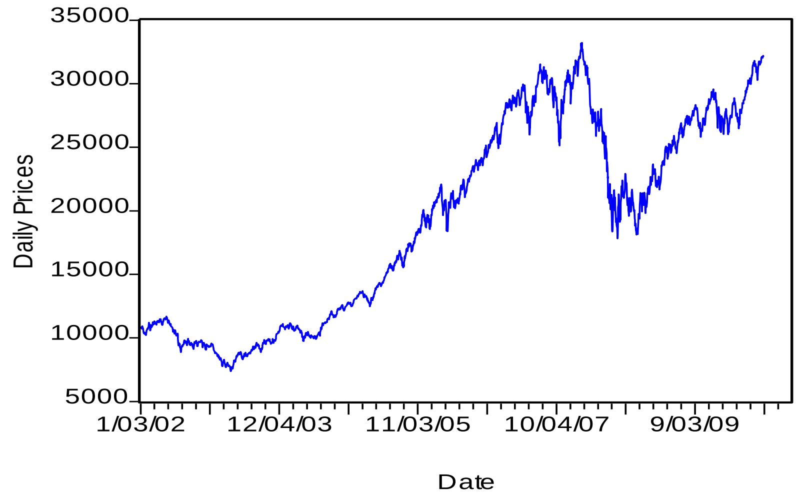

Figure 1. Plot of daily prices for JSE stock index (2002 - 2010).

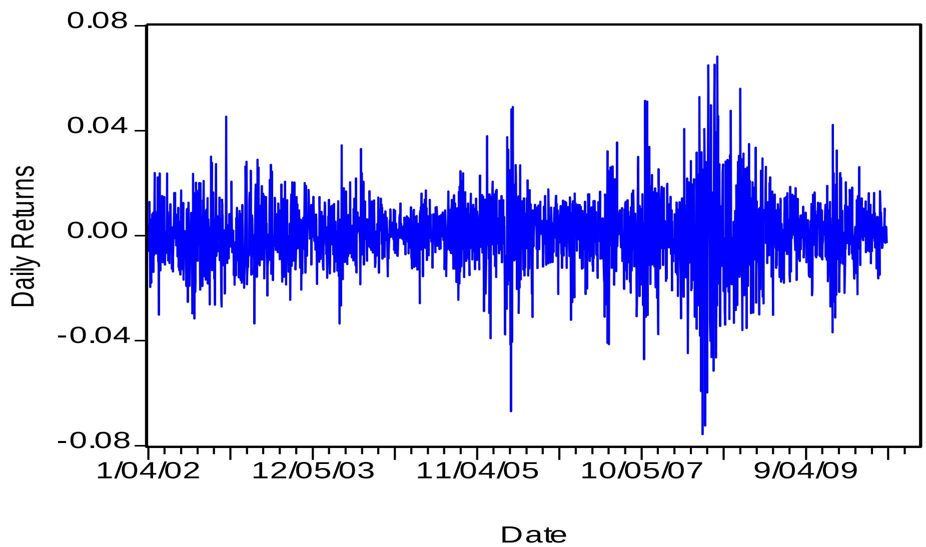

Figure 2. Plot of daily returns for JSE stock index (2002-2010).

Table 1. Descriptive statistics for indices and returns.

	Mean	Median	Max	Min	Std. dev.	Skew.	Kurtosis	Jarque-Bera
Prices	19249	20212	33233	7361	7876	0.02883	1.4776	217.5 (0.000)
Returns	0.00049	0.00094	0.06834	-0.07581	0.01348	-0.11421	6.105	910.4 (0.000)

for the period 2002 to 2010 is used. Graphical plots of price indices and returns are shown in Figures 1 and 2, respectively. Figure 1 shows that the daily stock prices are not stationary while Figure 2 for the returns shows that volatility occurs in bursts and shows volatility clustering.

Formal unit root tests were carried out using the Augmented-Dickey Fuller tests. The results indicated that the logs of stock prices are stationary after taking the first difference. Based on the stationarity requirements, we calculated the daily returns (r_t) as shown in Equation 1:

[1] $$r_t = \ln P_t - \ln P_{t-1} \quad (1)$$

where P_t, P_{t-1} are the current and one period lagged prices, respectively.

Table 1 shows descriptive statistics for stock indices and returns. The skewness of the indices is positive showing that the distribution of these indices has a long right tail. The kurtosis of the indices is

[1] $r_t = \nabla lnP_t = (1-B)lnP_t = \ln P_t - \ln P_{t-1} = ln\frac{P_t}{P_{t-1}}$

1.4776. The skewness of the returns is negative. This is an indication that the distribution of the returns has a long left tail. The kurtosis of returns is very high and greater than three indicating that the distribution of the returns is leptokurtic, that is it is fat tailed. This shows that the sample data exhibits financial characteristics of leptokurtosis and volatility clustering. The negative value of skewness suggests greater probability of large decreases in stock returns during the sample period.

The high value of kurtosis for the returns also suggests that extreme price changes occurred more frequently. For both prices and returns, the null hypothesis of normality was rejected as shown by the Jarque-Bera test statistics which were all significant at the 5% level.

THE MODELS

Modelling volatility in time series data using GARCH-type models has been studied extensively over the past three decades. More recent work includes that of (Lee and Kim, 2008; Horv´ath et al., 2008; Kallsen and Vesenmayer, 2009; He and Maheu, 2010; Kim et al., 2010; Matías et al., 2010; Mohammadi and Su, 2010; Zanotti et al., 2010). Our modelling consists of two steps. Initially, we specify the ARMA (p, q) model for the mean returns; this is followed by fitting GARCH (p, q) models for conditional volatility. In both steps residual analysis is carried out. The results from this analysis suggest that the returns may be modelled as an ARMA (0, 1) process. That is:

$$r_t = \varepsilon_t - \theta\varepsilon_{t-1} + \mu \quad (2)$$

where r_t are returns, μ is the mean value of the returns and ε_t represents the error term with mean zero and potentially subject to conditional heteroskedasticity.

The generalized ARCH (GARCH) process

Following the natural extension of the ARMA process as a parsimonious representation of a higher order AR process, Bollerslev (1986) extended the work of Engle to the generalized ARCH or GARCH process. The GARCH (p, q) process is defined as,

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i\, \varepsilon_{t-i}^2 + \sum_{j=1}^{p} \beta_j\, \sigma_{t-j}^2$$
$$\omega > 0, \alpha_i \geq 0, \beta_j \geq 0\ . \quad (3)$$

where σ_t^2 is the conditional variance, which is a linear function of q lags of the squares of the error terms ε_t^2 or the ARCH terms and p lags of the past value of the conditional variances σ_t^2 or the GARCH terms, and the constants α_i, β_j and ω. The inequality restrictions are imposed to guarantee a positive conditional variance, almost surely. Often, the GARCH (1, 1) process, that is:

$$\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2,$$

is sufficient enough to explain the characteristics of the time series and is a popular model in econometric and financial time series, (Hansen and Lunde, 2001).

GARCH-in-mean (GARCH-M)

Financial theory suggests that an increase in variance (that is, an increase in risk) results in a higher expected return. To account for this, GARCH-in-mean models are also considered Floros (2008). The standard GARCH-M model is given by:

$$r_t = \mu + \beta_1\sigma_t^2 + \varepsilon_t$$
$$\varepsilon_t \sim N(0, \sigma_t^2)$$
$$\sigma_t^2 = \omega + \alpha\varepsilon_{t-1}^2 + \beta\sigma_{t-1}^2 \quad (4)$$

If β_1 is positive (and significant), then increased risk leads to a rise in the mean return ($\beta_1\sigma_t^2$ can be interpreted as a risk premium).

Exponential-GARCH (EGARCH)

EGARCH models were designed to capture the leverage effect noted in Black (1976) and French et al. (1987). The EGARCH model was developed by Nelson (1991). A simple variance specification of EGARCH is given by:

$$\log\sigma_t^2 = \omega + \beta\log\sigma_{t-1}^2 + \alpha\left|\frac{\varepsilon_{t-1}}{\sigma_{t-1}}\right| + \gamma\frac{\varepsilon_{t-1}}{\sigma_{t-1}} \quad (5)$$

The EGARCH models were developed to capture the leverage effect, Floros (2008). For stock prices, negative shocks (bad news) generally have large impacts on their volatility than positive shocks (good news). The presence of leverage effect can be tested by the hypothesis that $\gamma < 0$. If $\gamma \neq 0$, then the impact is asymmetric.

Threshold-GARCH (TGARCH)

The TGARCH model was introduced by Zakoian (1994) and Glosten et al. (1993). The TGARCH specification for the conditional variance is given by:

$$\sigma_t^2 = \alpha_0 + \sum_{i=1}^{q}\alpha_i\varepsilon_{t-i}^2 + \gamma\varepsilon_{t-1}^2 d_{t-1} + \sum_{j=1}^{p}\beta_j\sigma_{t-j}^2 \quad (6)$$

where $d_t = 1$ if $\varepsilon_t < 0$ and $d_t = 0$ otherwise. In this model, good news ($\varepsilon_t > 0$) and bad news ($\varepsilon_t < 0$) have differential effects on the conditional variance. Good news has impact of α, while bad news has impact of $\alpha + \gamma$. If $\gamma > 0$ then the leverage effect exists and bad news increases volatility, while if $\gamma \neq 0$ the news impact is asymmetric.

EMPIRICAL RESULTS AND DISCUSSION

The models were tested on JSE data. We first estimate ARMA (p, q) models with r_t as the dependent variable. The ARMA (0, 1) model is selected for all the indices. We then estimate GARCH (p, q) type models.

The QQ-plot for the returns shown in Figure 3 falls nearly on a straight line except at the beginning, where the plot goes up marginally. QQ-plots that fall on a straight line in the middle but curve upward at the beginning indicate that the distribution is leptokurtic and has a thicker tail than the normal distribution.

Results of the ARMA (0, 1)-GARCH (1, 1) model for returns are shown in Table 2. The estimate of θ is significant supporting the use of ARMA (0, 1) model for the returns. Volatility shocks are persistent since the sum of the ARCH and GARCH coefficients are very close to

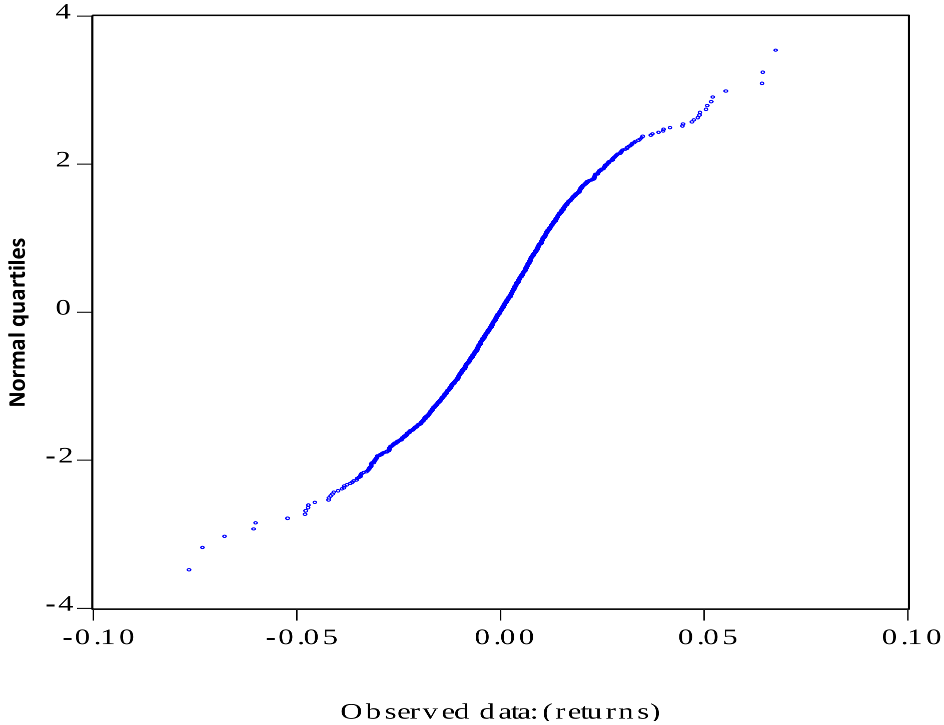

Figure 3. Q-Q plot for daily returns.

Table 2. ARMA (0, 1)-GARCH (1, 1) models for returns.

Mean equation	Variance equation	Model diagnostics
μ = 0.000699 (0.0025)	ω = 0.00000275 (0.0002)	$\alpha+\beta$ = 0.981264
θ = 0.052105 (0.0157)	α = 0.100866 (0.0000)	$Q(20)$ = 16.591 (0.618)
	β = 0.880398 (0.0000)	$Q^2(20)$ = 110.74 (0.000)
		$\text{ARCH}(10)$ = 0.098048(0.0002)

$Q(20)$ and $Q^2(20)$ are the Box-Pierce Q statistics tests for serial correlations in the standardized residuals and the standardized residuals squared respectively with 20 lags while ARCH (10) is Engle's LM test of ARCH effects up to the 10th order. P-values are shown in parentheses. In all cases 5% level of significance is used.

one. The estimates for α and β are highly significant. The Box-Pierce Q statistics is insignificant up to lag 20 indicating that there is no excessive autocorrelation left in the residuals. Engle's LM test indicate that there are no more ARCH effects left up to lag 10.

Table 3 summarizes the parameter estimates for the ARMA (0, 1)-GARCH (1, 1)-M model. The coefficient in Equation 4 denoted by β_1 is positive and insignificant meaning that increased risk does not necessarily imply higher returns. Volatility shocks are persistent. The coefficient of α is significantly positive and less than one, indicating that the impact of "old" news on volatility is significant. The box-pierce Q statistics up to lag 20 are insignificant showing that there is no correlation left in the residuals. Engle's LM test indicate that there is no more ARCH effects left up to lag 10.

Table 4 presents results for the ARMA (0, 1)-EGARCH (1, 1) model. The leverage effect term, γ is negative, indicating the existence of the leverage effect in future returns during the sampling period. Since $\gamma \neq 0$ the news impact is asymmetric, supporting the use of the skewed Student-t distribution for z_t (the standardized residuals). Engle's LM test indicates that there are no more ARCH effects left up to lag 10.

The ARMA (0, 1)-TGARCH (1, 1) model for returns is

Table 3. ARMA (0, 1)-GARCH (1, 1)-M model for returns.

Mean equation	Variance equation	Model diagnostics
μ = 0.000700 (0.0666)	ω = 3.51E-06 (0.0000)	$\alpha + \beta$ = 0.973041
θ = 0.038229 (0.0771)	α = 0.087248 (0.0000)	$Q(20)$ = 17.364 (0.565)
β_1= 1.992545 (0.4863)	β = 0.885793(0.0000)	$Q^2(20)$ = 20.185 (0.384)
		$\mathrm{ARCH}(10)$ = 0.051124 (0.0339)

Note: See Table 2.

Table 4. ARMA (0, 1)-EGARCH (1, 1) model for returns.

Mean equation	Variance equation	Model diagnostics
μ = 0.000585 (0.0117)	ω = -1.283838(0.0000)	$\alpha + \beta$ = 0.98663
θ = 0.043173 (0.0517)	α = 0.104387 (0.0000)	$Q(20)$ = 18.990 (0.457)
	β = 0.882243 (0.0000)	$Q^2(20)$ = 110.74 (0.000)
	γ = -0.148124 (0.0000)	$\mathrm{ARCH}(10)$ = 0.098048(0.0002)

See Table 2.

Table 5. ARMA (0, 1)-TARCH (1, 1) model for returns.

Mean equation	Variance equation	Model diagnostics
μ = 0.000111 (0.6278)	ω = 0.0000024 (0.0000)	$\alpha + \beta$ = 0.9253794
θ = 0.046135 (0.0296)	α = -0.002896 (0.8408)	$Q(20)$ = 16.310 (0.636)
	β = 0.925377 (0.0000)	$Q^2(20)$ = 25.018 (0.160)
	γ = 0.126200 (0.0000)	$\mathrm{ARCH}(10)$ = 0.04526 (0.067)

See Table 2.

Table 6. Out of sample forecast evaluation for conditional variance.

	ARMA(0, 1)-GARCH(1, 1)	ARMA (0, 1)-GARCH (1, 1)-M	ARMA (0, 1)-EGARCH (1, 1)	ARMA(0, 1)-TGARCH (1, 1)
RMSE	0.013475	0.013483	0.013480	0.013476

shown in Table 5. Since $\gamma \neq 0$ the news impact is asymmetric. The leverage effect exists since γ is significantly positive. This implies that bad news will increase volatility. Persistence in volatility shocks is evident as the sum of the ARCH and GARCH terms is close to one.

Out of sample predictions

The root mean squared error (RMSE) is used in the evaluation of the out of sample predictions for the period January to March 2011. The RMSE for the conditional mean are calculated as follows:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}\left(r_{at} - r_{ft}\right)^2}{n}}, \quad (7)$$

where n is the number of out of sample forecast data points, with $r_{at} - r_{ft}$ being the forecast errors. The terms

r_{at} and r_{ft} are the actual return and its future forecast, respectively.

For the conditional volatility models RMSE is calculated as follows:

$$\mathrm{RMSE} = \sqrt{\frac{\sum_{t=1}^{n}\left(\sigma_{at}^{2} - \sigma_{ft}^{2}\right)^{2}}{n}}, \quad (8)$$

where σ_{at}^{2} and σ_{ft}^{2} are realized and forecasts of volatility, respectively. Table 6 presents the RMSE for the out of sample forecast evaluation for conditional variance. The RMSE statistics is used to rank the models based on their out of sample forecasting accuracy. The ARMA (0, 1)-GARCH (1, 1) model achieve the most accurate volatility forecasts.

Conclusion

We used GARCH type models for modeling daily returns on the Johannesburg Stock Exchange. Empirical results show that returns are characterized by an ARMA (0, 1) process.

This implies that shocks to conditional mean dissipate after one period. The results indicate that the daily returns can be characterized by the GARCH type models. The out of sample forecasting evaluations indicate that the ARMA (0, 1)-GARCH (1,1) model achieve the most accurate volatility forecasts. Our results show that increased risk does not necessarily imply an increase in returns. The high values of kurtosis for the returns suggest that extreme price changes occurred more frequently during the sample period, 2002 to 2010.

Future research should look at forecasting volatility of daily data using Markov regime switching GARCH models, extreme value theory in predicting the probabilities of extreme returns and the use of the Bayesian GARCH approach in the estimation of the volatility of the residual returns. These will be considered elsewhere.

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