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Full Length Research Paper

Perishable inventory system with joint ordering policy

N. Anbazhagan[1]*, V. Perumal[2] and V. Kumaresan[3]

[1]Department of Mathematics, Alagappa University, Karaikudi, India.
[2]Department of Mathematics, MTN College, Madurai, India.
[3]Department of Mathematics, VHNSN College, Virudhunagar, India.

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In this article we analyze a lost sales two-commodity perishable inventory system, under Poisson demands and exponential lifetimes. The maximum inventory level for the i-th commodity is fixed as $S_i (i = 1,2)$. If the total net inventory level drops to a prefixed level s $[\leq (S_1 - 2)/2 \ or \ (S_2 - 2)/2]$, an order will be placed for $(S_i - s)$ units of i-th commodity $(i = 1,2)$. The lead time for replenishment is assumed to be exponential. The limiting probability distribution for joint inventory level, mean inventory levels and mean reorder rates and mean shortage rates in the steady state are computed. The total cost function is expressed in terms of system performance measures obtained. The results are illustrated numerically.

Key words : Perishable item, Joint replenishment, Stochastic lead time, Markov process.

INTRODUCTION

In many practical multi-item inventory systems concentrated the coordination of replenishment orders for group of items. Now a days it is very much applicable to run a successful Business and Industries. These systems unlike those dealing with single commodity involve more complexities in the reordering procedures. The modelling of multi-item inventory system under joint replenishment has been receiving considerable attention for the past three decades.

Perishable inventory models are of considerable attention in the study of inventory systems over the past three decades. The analysis of perishable inventory systems has been the theme of many articles due to its potential applications in sectors like food, chemicals, pharmaceuticals, photography and blood bank management. Some closely related commodities perishable or technology obsolence character may need joint reordering for the purpose of cost reduction and availability. Central processing unit and monitor, blood and holding container are the right pair of items whose perishable or technology obsolence character leads to the authors choice for the joint replenishment inventory system.

Nahmias (1974) and Fries (1975) have analysed fixed-life perishable inventory problems under various conditions. Weiss (1980) presents the problem in a continuous review framework, considering all costs concerned with ordering, holding, shortage, disposal, penalty and revenue in lost sales and backordering cases. For the lost sales case he identifies a continuous review $(0, S)$ policy and for the backordering case a continuous review (s, S) policy in the case of linear shortage cost, as optimal policies.

Chiu (1995) formulated a continuous review perishable inventory model based on approximations for the expected outdating, the expected shortage quantity and the expected inventory level. He developed a (Q, r) ordering policy under a positive order lead time when the objective function is minimization of the total expected average cost per unit time. Nahmias (1977, 1982) has

*Corresponding author.E-mail: n.anbazhagan.alu@gmail.com; anbazhagan_n@yahoo.co.in.

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formulated approximations for several complex stochastic inventory models. The approximations are formulated for both periodic as well as for continuous review of the inventory.

Anbazhagan and Arivarignan (2000, 2001, 2003) have analyzed two commodity inventory system under various ordering policies. Sivakumar et al. (2006) have considered a two commodity perishable inventory system with coordinated reordering policy. Yadavalli et al. (2004) have analyzed a model with joint ordering policy and varying order quantities. Yadavalli et al. (2006) have considered a two-commodity substitutable inventory system with Poisson demands and arbitrarily distributed lead time.

In this article we consider a two commodity inventory system in which items are perishable in nature. The demand points for each commodity form independent Poisson process and the lead times initiated by joint reorder policy are assumed to be independent and distributed as negative exponential. We have obtained the joint probability distribution for the inventory levels of both commodities in the steady state case. Various system performance measures in the steady state are derived. The total cost function TC(S_1, S_2, s) is obtained in terms of system performance measures. This expected value criterion may be used to take managerial decisions. The results are numerically illustrated.

THE METHODOLOGY MODEL

Consider a two-commodity stochastic inventory of perishable items with the maximum capacity S_i units for i -th commodity $(i=1,2)$. The demand for i -th commodity is of unit size and the time points of demand occurrences form independent Poisson processes each with parameter λ_i $(i=1,2)$. The life time of each commodity is exponential with parameter γ_i $(i=1,2)$. If the total net inventory level drops to a prefixed level s $[\leq (S_1-2)/2 \; or \; (S_2-2)/2]$, an order will be placed for (S_i-s) units of i -th commodity $(i=1,2)$. The condition $S_i-s>s+1$, $(i=1,2)$ ensures that after a replenishment the inventory levels of both commodities will be always above the respective reorder levels. Otherwise it may not be possible to place reorder(according to this policy) which leads to perpetual shortage. The lead time is assumed to be distributed as negative exponential with parameter $\mu(>0)$ and that the demands that occur during stock-out periods are lost.

Notations:

0 : zero matrix

$1_N'$: $(1,1,\cdots,1)_{1\times N}$

I_N : identity matrix of order N

δ_{ij} : Kronecker delta

$\overline{\delta_{ij}}$: $(1-\delta_{ij})$

A_{ij} : $(i,j)-$th element of the matrix A

$$\Omega_{i=j}^{k} c_i = \begin{cases} c_j c_{j-1} \cdots c_k & if \;\; j \geq k \\ 1 & if \;\; j < k \end{cases}$$

E_1 : $\{0,1,\cdots,S_1\}$.

E_2 : $\{0,1,\cdots,S_2\}$.

$E = E_1 \times E_2$

Analysis

Let $L_i(t)$ denote the inventory level of i -th commodity in the system at time t. From the assumptions made on the input and output processes, it can be shown that $\{(L_1(t),L_2(t)),t\geq 0\}$ is a Markov process with the state space E. The infinitesimal generator of this process,

$$A = ((a((i,m),(j,n)))), \qquad (i,m),(j,n)\in E$$

can be obtained by using the following arguments:

1. A transition from (i,m) to $(i-1,m)$ will take place

i) when a demand for first commodity occurs. The intensity of transition is λ_1 .

or

ii) when any one of i items perish and this occurs at a rate γ_1 . Thus intensity for this transition is $i\gamma_1$, $i=1,2,\ldots,S_1$.

For this case, $a((i,m),((i-1),m)) = \lambda_1 + i\gamma_1,$ $i=1,2,\ldots,S_1; \quad m=0,1,\ldots,S_2.$

2. A transition from (i,m) to $(i,m-1)$ will take place

i. when a demand for second commodity occurs. The intensity of transition is λ_2 .

or

ii. when any one of m items perish and this occurs at a rate γ_2 . Thus intensity for this transition is $m\gamma_2$, $m=1,2,\ldots,S_2$.

For this case, $a((i,m),(i,(m-1))) = \lambda_2 + m\gamma_2,$ $i=0,1,\ldots,S_1, m=1,2,\ldots,S_2$.

3. From the state (i,m), for which $i+m=s$, a replenishment takes it to $(i+Q_1,m+Q_2)$ and the intensity of transition is given by μ .

In this case, $a((i,m),(i+Q_1,m+Q_2)) = \mu,$ $i=0,1,\ldots,s, m=0,1,\ldots,s$

4. No other transitions from (i,m) to (j,n), except $(j,n)\neq(i,m)$ are possible. Hence their rates are zero.

5. To obtain the intensity of passage, $a((i,m),(i,m))$ of state (i,m), we note that the entries in any row of this matrix add to zero. Hence the diagonal entry is equal to the negative of the sum of the other entries in that row. More explicitly

$$a((i,m),(i,m)) = -\sum_{\substack{j \\ (j,n)\neq(i,m)}}\sum_{n} a((i,m),(j,n)).$$

Hence we get,

$$a((i,m),(j,n)) = \begin{cases}
\lambda_1+i\gamma_1, & j=i-1, & n=m, \\
 & i=1,2,\cdots,S_1, & m=0,1,\cdots,S_2 \\
\lambda_2+m\gamma_2, & j=i, & n=m-1, \\
 & i=0,1,\cdots,S_1, & m=1,2,\cdots,S_2 \\
\mu, & j=i+Q_1, & n=m+Q_2, \\
 & i=0,1,\cdots,s, & m=0,1,\cdots,s-i \\
-(\lambda_1+\lambda_2+i\gamma_1+m\gamma_2), & j=i, & n=m, \\
 & i=s+1,\cdots,S_1, & m=1,2,\cdots,S_2 \\
-(\lambda_1+i\gamma_1), & j=i, & n=m, \\
 & i=s+1,\cdots,S_1, & m=0 \\
-(\lambda_1+\lambda_2+i\gamma_1+m\gamma_2), & j=i, & n=m, \\
 & i=1,2,\cdots,s, & m=s-i+1,\cdots,S_2 \\
-(\lambda_1+\lambda_2+i\gamma_1+m\gamma_2+\mu), & j=i, & n=m, \\
 & i=1,2,\cdots,s-1, & m=1,2,\cdots,s-i \\
-(\lambda_1+i\gamma_1+\mu), & j=i, & n=m, \\
 & i=1,2,\cdots,s, & m=0 \\
-(\lambda_2+m\gamma_2), & j=i, & n=m, \\
 & i=0, & m=s+1,\cdots,S_2 \\
-(\lambda_2+m\gamma_2+\mu), & j=i, & n=m, \\
 & i=0, & m=1,2,\cdots,s \\
-\mu, & j=i, & n=m, \\
 & i=0, & m=0 \\
0, & Otherwise. &
\end{cases}$$

Denoting $\mathbf{q} = (\,(q,S_2),(q,S_2-1),\cdots,(q,1),(q,0)\,)$ for $q=S_1,S_1-1,\cdots,1,0$, the infinitesimal generator A can be conveniently expressed as a partitioned matrix:

$$A = ((\,A_{ij}\,))$$

where A_{ij} is a $(S_2+1)\times(S_2+1)$ submatrix, and is given by

$$A_{ij} = \begin{cases}
B_i & if \quad j=i-1, \quad i=S_1,S_1-1,\cdots,1, \\
M_{s-i+1} & if \quad j=i+Q_1, \quad i=s,s-1,\cdots,0, \\
A_i & if \quad j=i, \quad i=S_1,S_1-1,\cdots,1,0, \\
0 & otherwise.
\end{cases}$$

More explicitly,

$$A = \begin{array}{c} S_1 \\ S_1-1 \\ \vdots \\ s+1 \\ s \\ s-1 \\ \vdots \\ 1 \\ 0 \end{array}
\begin{pmatrix}
A_{S_1} & B_{S_1} & & & & & & & \\
 & A_{S_1-1} & B_{S_1-1} & & & & & & \\
 & & \cdots & & & & & & \\
 & & & \cdots & A_{s+1} & B_{s+1} & & & \\
M_1 & & & & & A_s & B_s & & \\
 & M_2 & & & & & A_{s-1} & & \\
 & & \cdots & & & & \cdots & & \\
 & & M_s & & & & \cdots & A_1 & B_1 \\
 & & & M_{s+1} & & & & & A_0
\end{pmatrix}$$

where,

$$B_i = (\lambda_1+i\gamma_1)I_{(S_2+1)\times(S_2+1)}, \qquad i=1,2,\ldots,S_1,$$

$$M_i = L_1+\cdots+L_i, \quad i=1,2,\cdots,s+1, with$$

$$L_i = \begin{array}{c} S_2 \\ \vdots \\ i-1 \\ \vdots \\ 0 \end{array}
\begin{pmatrix}
0 & \cdots & \cdots & 0 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & \mu & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & \cdots & \cdots & 0 & \cdots & \cdots & 0
\end{pmatrix}$$

$$S_2 \quad \cdots \quad Q_2+i-1 \quad \cdots \quad \cdots \quad 0$$

$$For\ i = S_1, S_1-1,\ldots,s+1$$

$$A_i = \begin{pmatrix}
d_{S_2} & e_{S_2} & 0 & \cdots & 0 & 0 \\
0 & d_{S_2-1} & e_{S_2-1} & \cdots & 0 & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & 0 & \cdots & d_1 & e_1 \\
0 & 0 & 0 & \cdots & 0 & d_0
\end{pmatrix}$$

$$with\ d_j = -(\lambda_1+\lambda_2+i\gamma_1+j\gamma_2), e_j = (\lambda_2+j\gamma_2),\ j=1,2\ldots,S_2$$
$$and\ d_0 = -(\lambda_1+i\gamma_1)$$

$$For\ i = s, s-1,\ldots,1,0,$$

$$A_i = \begin{array}{c} S_2 \\ S_2-1 \\ \vdots \\ s-i+1 \\ s-i \\ \vdots \\ 1 \\ 0 \end{array}
\begin{pmatrix}
u_{S_2} & v_{S_2} & 0 & \cdots & \cdots & \cdots & 0 \\
0 & u_{S_2-1} & v_{S_2-1} & 0 & \cdots & \cdots & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & u_{s-i+1} & v_{s-i+1} & \cdots & 0 \\
0 & 0 & \cdots & & u_{s-i} & v_{s-i} & 0 \\
\cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\
0 & 0 & \cdots & \cdots & 0 & u_1 & v_1 \\
0 & 0 & \cdots & \cdots & 0 & 0 & u_0
\end{pmatrix}$$

$$with\ u_j = \begin{cases} -(\overline{\delta_{i0}}(\lambda_1 + i\gamma_1) + (\lambda_2 + j\gamma_2)), & j = S_2, S_2 - 1, \ldots, s - i + 1, \\ -(\overline{\delta_{i0}}(\lambda_1 + i\gamma_1) + (\lambda_2 + j\gamma_2) + \mu), & j = s - i, s - i - 1, \ldots, 1, \end{cases}$$

$$u_0 = -(\overline{\delta_{i0}}(\lambda_1 + i\gamma_1) + \mu)$$ and

$$v_j = (\lambda_2 + j\gamma_2), \qquad j = S_2, \ldots, 2, 1,$$

The rate matrix, as partitioned above will be used to find the steady state probability distribution. This is discussed in the next section.

RESULTS

It can be seen from the structure of A that the homogeneous Markov process $(L_1(t), L_2(t)), t \geq 0$ on the finite state space E is irreducible, aperiodic and persistent non-null. Hence the limiting probability $\phi_{(i,k)}$ exists, where

$$\phi_{(i,k)} = \lim_{t \to \infty} Pr\left[L_1(t) = i, L_1(t) = k \mid L_1(0), L_2(0)\right], \qquad (i,k) \in E.$$

Let $\phi_q = (\phi_{(q,S_2)}, \phi_{(q,S_2-1)}, \cdots, \phi_{(q,0)}), \qquad q = 0, 1, \cdots, S_1$ be the steady probability vector with the structure:

$$\Phi = (\phi_{S_1}, \phi_{S_1-1}, \cdots, \phi_1, \phi_0).$$

Then the vector of limiting probabilities Φ satisfies

$$\Phi A = 0$$

$$and\ \Phi e = 1.$$

Expanding the matrix equation (1) we get the following set of equations:

$$\phi_{i+1} B_{i+1} + \phi_i A_i = 0, \qquad i = 0, 1, \cdots, Q_1 - 1, \tag{3}$$

$$\phi_{i+1} B_{i+1} + \phi_i A_i + \phi_{i-Q_1} M_{S_1-i+1} = 0, \qquad i = Q_1, \tag{4}$$

$$\phi_{i+1} B_{i+1} + \phi_i A_i + \phi_{i-Q_1} M_{S_1-i+1} = 0, \qquad i = Q_1 + 1, Q_1 + 2, \cdots, S_1 - 1, \tag{5}$$

$$\phi_i A_i + \phi_{i-Q_1} M_{S_1-i+1} = 0, \qquad i = S_1. \tag{6}$$

After a great deal of simplifications, the above equations(except(4)) become

$$\phi_i = \begin{cases} \phi_{Q_1} (-1)^{Q_1 - i} \underset{j=Q_1-1}{\overset{i}{\Omega}} (B_{j+1} A_j^{-1}), & i = 0, 1, \ldots, Q_1 - 1, \\ \phi_{Q_1} \sum_{k=s}^{i-Q_1} (-1)^{2Q_1+1-i} \underset{j=Q_1-1}{\overset{k}{\Omega}} (B_{j+1} A_j^{-1})(M_{s-k+1} A_{Q_1+k}^{-1}) \underset{l=Q_1+k-1}{\overset{i}{\Omega}} (B_{l+1} A_l^{-1}), & i = Q_1 + 1, Q_1 + 2, \ldots, S_1. \end{cases}$$

The above expressions yields the values of ϕ_i interms of ϕ_{Q_1}. Then ϕ_{Q_1} can be obtained by solving the equations (2) and (4),

$$\phi_{Q_1+1} B_{Q_1+1} + \phi_{Q_1} A_{Q_1} + \phi_0 M_{s+1} = 0\ and\ \Phi e = 1.$$

that is,

$$\phi_{Q_1}\left[\left(\sum_{k=s}^{1} (-1)^{Q_1} \underset{j=Q_1-1}{\overset{k}{\Omega}} (B_{j+1} A_j^{-1})(M_{s-k+1} A_{Q_1+k}^{-1}) \underset{l=Q_1+k-1}{\overset{Q_1+1}{\Omega}} (B_{l+1} A_l^{-1})\right) B_{Q_1+1} + A_{Q_1} + \left((-1)^{Q_1} \underset{j=Q_1-1}{\overset{0}{\Omega}} (B_{j+1} A_j^{-1})\right) M_{s+1}\right] = 0$$

and

$$\phi_{Q_1}\left[\sum_{i=0}^{Q_1-1}\left\{(-1)^{Q_1-i} \underset{j=Q_1-1}{\overset{i}{\Omega}} (B_{j+1} A_j^{-1})\right\} + I + \sum_{i=Q_1+1}^{S_1}\left\{\sum_{k=s}^{i-Q_1} (-1)^{2Q_1+1-i} \times \underset{j=Q_1-1}{\overset{k}{\Omega}} (B_{j+1} A_j^{-1})(M_{s-k+1} A_{Q_1+k}^{-1}) \underset{l=Q_1+k-1}{\overset{i}{\Omega}} (B_{l+1} A_l^{-1})\right\}\right] e = 1.$$

(1)

(2)

This gives the complete limiting probability distribution of the joint inventory levels of two commodities in stock $(\phi_{(i,k)})_{(i,k) \in E}$.

System performance measures

(3)

In this section, some performance measures of the system in the steady state are derived using the limiting probability distribution $(\phi_{(i,k)})$ obtained above.

(4)

(5)

Mean inventory level

(6)

Let β_1 denote the expected inventory level of the first commodity in the steady state which is given by

$$\beta_1 = \sum_{i=1}^{S_1} i\left(\sum_{j=0}^{S_2}\phi_{(i,j)}\right) \quad (7)$$

Let β_2 denote the expected inventory level of the second commodity in the steady state which is given by

$$\beta_2 = \sum_{j=1}^{S_2} j\left(\sum_{i=0}^{S_1}\phi_{(i,j)}\right). \quad (8)$$

Mean reorder rate

Let β_3 denote the mean joint reorder rate for both the commodity in the steady state which is given by

$$\beta_3 = \sum_{k=0}^{s}(\lambda_1 + k\gamma_1)\phi_{(s+1-k,k)} + \sum_{k=0}^{s}(\lambda_2 + k\gamma_2)\phi_{(k,s+1-k)}. \quad (9)$$

Mean shortage rate

Let β_4 denote the mean shortage rate of the first commodity in the steady state then

$$\beta_4 = \lambda_1\sum_{k=0}^{S_2}\phi_{(0,k)}. \quad (10)$$

Let β_5 denote the mean shoratge rate of the second commodity in the steady state then

$$\beta_5 = \lambda_2\sum_{k=0}^{S_1}\phi_{(k,0)}. \quad (11)$$

Mean perishable rate

Let β_6 denote the mean perishable rate of the first commodity in the steady state then we have

$$\beta_6 = \sum_{i=1}^{S_1}\sum_{j=0}^{S_2} i\gamma_1\phi_{(i,j)}. \quad (12)$$

Let β_7 denote the mean perishable rate of the second commodity in the steady state then we have

$$\beta_7 = \sum_{i=0}^{S_1}\sum_{j=1}^{S_2} j\gamma_2\phi_{(i,j)}. \quad (13)$$

Total expected cost rate

To compute the total expected cost rate(total expected cost per unit time), the following costs are considered:

h_1: The inventory carrying cost per item per unit time of first commodity.
h_2 : The inventory carrying cost per item per unit time of second commodity.
c_s : Setup cost per order.
c_{p1}: Perishable cost per unit item per unit time of first commodity.
c_{p2}: Perishable cost per unit item per unit time of second commodity.
c_{sh1} : Shortage cost per unit item of the first commodity.
c_{sh2} : Shortage cost per unit item of the second commodity.

The long run total expected cost rate is given by

$$TC(S_1, S_2, s) = h_1\beta_1 + h_2\beta_2 + c_s\beta_3 + c_{sh1}\beta_4 + c_{sh2}\beta_5 + c_{p1}\beta_6 + c_{p2}\beta_7. \quad (14)$$

Numerical Illustration

We have computed the limiting probability distribution of the inventory level for specific values of the parameters. For Table 1, we have assumed

$$S_1 = 8; S_2 = 9; s = 2; \lambda_1 = 2.4; \lambda_2 = 3.5; \mu = 1.5; \gamma_1 = 1.1; \gamma_2 = 2.5$$

$$h_1 = 1.2; h_2 = 1.4; c_s = 300; c_{sh1} = 2; c_{sh2} = 1.2; c_{p1} = 0.3; c_{p2} = 0.2; \quad (11)$$

Expected inventory level for first commodity	= 2.48
Expected inventory level for second commodity	= 1.49
Expected reorder rate	= 0.50
Expected shortage rate for first commodity	= 0.69
Expected shortage rate for second commodity	= 1.93
Expected perishable rate for first commodity	= 2.73
Expected perishable rate for second commodity (12)	= 3.73
Total expected cost rate	= 161.64

As a second example we have considered the following values for the parameters and costs (Table 2):

$$S_1 = 10; S_2 = 7; s = 4; \lambda_1 = 5.6; \lambda_2 = 4.5; \mu = 4.5; \gamma_1 = 2.4; \gamma 2 = 4.5;$$

$$h_1 = 5.2; h_2 = 6.4; c_s = 50; c_{sh1} = 3; c_{sh2} = 5.2; c_{p1} = 1.3; c_{p2} = 1.2;$$

Table 1. Limiting Probabilities for both commodities.

Commodity 1	Commodity 2				
	0	**1**	**2**	**3**	**4**
0	0.2665	0.0129	0.0050	0.0020	0.0005
1	0.0920	0.0154	0.0082	0.0040	0.0016
2	0.0799	0.0216	0.0146	0.0079	0.0039
3	0.0628	0.0241	0.0182	0.0125	0.0077
4	0.0314	0.0187	0.0179	0.0154	0.0120
5	0.0128	0.0110	0.0132	0.0143	0.0141
6	0.0039	0.0045	0.0067	0.0088	0.0110
7	0.0008	0.0011	0.0019	0.0028	0.0038
8	0.0001	0.0002	0.0004	0.0007	0.0012
	5	**6**	**7**	**8**	**9**
0	0.0002	3.64e-005	6.25e-006	7.39e-007	8.02e-008
1	0.0005	0.0002	3.25e-005	4.36e-006	5.96e-007
2	0.0017	0.0006	0.0002	2.23e-005	3.82e-006
3	0.0042	0.0018	0.0006	9.23e-005	2.05e-005
4	0.0082	0.0048	0.0020	0.0003	9.57e-005
5	0.0126	0.0099	0.0060	0.0009	0.0004
6	0.0131	0.0151	0.0170	0.0020	0.0015
7	0.0048	0.0057	0.0063	0.0007	0.0000
8	0.0018	0.0026	0.0037	0.0000	0.0000

Table 2. Limiting Probabilities for both commodities.

Commodity 1	Commodity 2			
	0	**1**	**2**	**3**
0	0.1442	0.0123	0.0038	0.0010
1	0.0672	0.0145	0.0062	0.0021
2	0.0682	0.0219	0.0118	0.0049
3	0.0640	0.0285	0.0192	0.0085
4	0.0560	0.0322	0.0232	0.0130
5	0.0319	0.0264	0.0246	0.0177
6	0.0162	0.0186	0.0222	0.0213
7	0.0070	0.0106	0.0161	0.0209
8	0.0025	0.0047	0.0082	0.0117
9	0.0007	0.0017	0.0039	0.0069
10	0.0001	0.0004	0.0011	0.0026
	4	**5**	**6**	**7**
0	0.0003	4.52e-005	7.62e-006	7.74e-007
1	0.0006	0.0001	2.65e-005	3.48e-006
2	0.0014	0.0003	8.87e-005	1.47e-005
3	0.0029	0.0008	0.0002	5.34e-005
4	0.0053	0.0015	0.0006	0.0002
5	0.0092	0.0024	0.0012	0.0005
6	0.0151	0.0034	0.0021	0.0013
7	0.0246	0.0042	0.0028	0.0033
8	0.0138	0.0025	0.0005	0.0000
9	0.0105	0.0018	0.0000	0.0000
10	0.0055	0.0000	0.0000	0.0000

Expected inventory level for first commodity = 3.68
Expected inventory level for second commodity = 1.31
Expected reorder rate = 1.38
Expected shortage rate for first commodity = 0.90
Expected shortage rate for second commodity = 2.06
Expected perishable rate for first commodity = 8.84
Expected perishable rate for second commodity = 5.89
Total expected cost rate = 128.69.

Conclusion

In this paper we have discussed a two commodity continuous review perishable inventory system with joint reorder policy. This model is most suitable for two different items which are perishable or obsolence in technology. The joint probability distribution of the inventory levels in the steady state is obtained to derive operational characteristics and expression for long run total expected cost rate. In this model for analytical tractable solution purpose we assumed Poisson type demand for both commodities. This result can be extended to renewal type demands for both items. The total cost function $TC(S_1, S_2, s)$ is obtained in terms of system performance measures. This expected value criterion may be used to take managerial decisions. The results are numerically illustrated.

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