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Full Length Research Paper

Comments on preservation technology investment for deteriorating inventory

Yi-Hsien Huang[1]*, Ching-Cheng Wang[1], Chung-Jen Huang[2] and Chung-Yuan Dye[3]

[1]Institute of Manufacturing Engineering, National Cheng Kung University, Tainan, Taiwan, 701, R. O. C.
[2]Graduate Institute of Business Administration, National Chiayi University, Chiayi , Taiwan, 600, R. O. C.
[3]Department of Business Management, Shu-Te University, Yen Chao, Kaohsiung, Taiwan, 824, R. O. C.

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Recently, in the Preservation technology investment for deteriorating inventory, by the International Journal of Production Economics, Hsu et al. (2010) in a press proposed a deteriorating inventory with time-dependent partial backlogging rate. In addition, the retailer is allowed to invest on the preservation technology to reduce the rate of product deterioration. However, the property of the retailer's unit time profit of Hsu et al. (2010) has remained unexplored. In this comment, we complement the shortcomings of this paper.

Key words: Preservation technology, deteriorating inventory, capital constraint.

INTRODUCTION

In practice, the deterioration rate of products can be controlled and reduced through various efforts such as procedural changes and specialized equipment acquisition. The results of the sensitivity analysis in numerous studies (Taso and Sheen (2008), Yang et. al. (2009), Geetha and Uthayakumar (2010)) also showed that lower deterioration rate is beneficial for an economic viewpoint. In a recent article, Hsu et al. (2010) developed a deterministic inventory model for deteriorating items with time-dependent partial backlogging rate. In addition, the retailer is allowed to invest on the preservation technology to reduce the rate of product deterioration. The main objective in their paper is to find the retailer’s replenishment and preservation technology investment policy which maximizes the retailer’s unit time profit. The graphical analysis approach is used to show the concavity of the objective function. However, the uniqueness of the optimal solution in their model has remained for future research. Furthermore, the property of the retailer’s unit time profit has remained unexplored. In this paper, we complement the shortcomings of Hsu et al. (2010). First, we prove that the optimal replenishment schedule not only exists but is unique, for any given invested capital. Next, we show that the retailer’s unit time profit is a concave function of invested capital when the replenishment schedule is given. We then provide a simple algorithm to find the optimal preservation technology cost and replenishment schedule for the proposed model. Finally, a couple of numerical examples are discussed to illustrate the algorithm.

MODEL FORMULATION

For easy tractability with Hsu et al. (2010), we use the same notations and assumptions as they did except assuming *T* is a continuous variable. The retailer’s unit time profit constructed by Hsu et al. (2010) is reviewed as follows:

$$F(T,v,\xi)=\frac{1}{T}\left\{p\left[dv+\frac{d\lambda\left(T^2-v^2\right)}{2T}\right]-rd\left(\frac{T}{2}+\frac{v^2}{2T}-v\right)-c\left[\frac{d\left(T^2-v^2\right)}{2T}-\frac{d}{k-m(\xi)}\left(1-e^{v[k-m(\xi)]}\right)\right]-\frac{hd}{k-m(\xi)}\left[\frac{e^{v[k-m(\xi)]}}{k-m(\xi)}-v-\frac{1}{k-m(\xi)}\right]-c_0-\xi\right\} \quad (1)$$

Now, the problem is to determine *T*, *v* and *ξ* such that *F* (*T, v, ξ*) is maximized. To maximize the retailer’s unit time profit, it is necessary to solve the following equations simultaneously:

*Corresponding author. E-mail: huang4343@gmail.com.

$$\frac{\partial F(T,v,\xi)}{\partial T}=\frac{1}{T}\left\{p\left[d\lambda+\frac{d\lambda(T^2-v^2)}{2T^2}\right]-dr\left(\frac{1}{2}+\frac{v^2}{2T^2}\right)-c\left[d-\frac{d(T^2-v^2)}{2T^2}\right]\right\}-\frac{F(T,v,\xi)}{T}=0 \quad (2)$$

$$\frac{\partial F(T,v,\xi)}{\partial v}=\frac{d}{T}\left\{r\left(1-\frac{v}{T}\right)-c\left(e^{v[k-m(\xi)]}-\frac{v}{T}\right)+p\left(1-\frac{v\lambda}{T}\right)-h\frac{e^{v[k-m(\xi)]}-1}{k-m(\xi)}\right\}=0 \quad (3)$$

and

$$\frac{\partial F(T,v,\xi)}{\partial \xi}=\frac{1}{T}\left\{-1+hd\frac{2+v[k-m(\xi)]-2e^{v[k-m(\xi)]}v[k-m(\xi)]}{[k-m(\xi)]^3}m'(\xi)+cd\frac{1-e^{v[k-m(\xi)]}+v[k-m(\xi)]e^{v[k-m(\xi)]}}{[k-m(\xi)]^2}m'(\xi)\right\}=0 \quad (4)$$

After some algebraic manipulation, equations (2) and (3) reduce to:

$$(p+r-c)-\frac{v}{T}(p\lambda+r-c)-\frac{h+c[k-m(\xi)]}{[k-m(\xi)]}\left[e^{v[k-m(\xi)]}-1\right]=0 \quad (5)$$

and

$$c_0+\xi+d(p\lambda+r-c)\frac{v^2}{T}-dv(p+r-c)+\frac{d\{h+c[k-m(\xi)]\}}{[k-m(\xi)]^2}\times\left\{e^{v[k-m(\xi)]}-1-v[k-m(\xi)]\right\}=0 \quad (6)$$

Equation (5) gives, after simplification,

$$T=\frac{v(p\lambda+r-c)}{p+r-c-\frac{h+c[k-m(\xi)]}{k-m(\xi)}\left[e^{v[k-m(\xi)]}-1\right]} \quad (7)$$

Because, by assumption,

$$0\le\frac{v}{T}=\frac{p+r-c-\frac{h+c[k-m(\xi)]}{k-m(\xi)}\left[e^{v[k-m(\xi)]}-1\right]}{p\lambda+r-c}\le 1$$

The region of v^* can be represented as follows:

$$p(1-\lambda)\le\frac{h+c[k-m(\xi)]}{[k-m(\xi)]}\left[e^{v[k-m(\xi)]}-1\right]\le p+r-c$$

or

$$\hat{v}_{min}\le v^*\le\hat{v}_{max}$$

where

$$\hat{v}_{min}=\frac{1}{k-m(\xi)}\ln\left[\frac{p(1-\lambda)[k-m(\xi)]}{h+c[k-m(\xi)]}+1\right] \quad (8)$$

and

$$\hat{v}_{max}=\frac{1}{k-m(\xi)}\ln\left[\frac{(p+r-c)[k-m(\xi)]}{h+c[k-m(\xi)]}+1\right] \quad (9)$$

Hence, $\hat{v}_{min}$ and $\hat{v}_{max}$ are the lower and upper bounds for the optimal v^*, respectively.

Next, in order to prove the uniqueness of the solution for problem, by taking implicit differentiation on equation (5) with respect to *v*, we obtain:

$$(p\lambda+r-c)\left(1-\frac{v}{T}\frac{dT}{dv}\right)=-T\{h+c[k-m(\xi)]\}e^{v[k-m(\xi)]}$$

Obviously, the previous equation holds if and only if,

$$\left(1-\frac{v}{T}\frac{dT}{dv}\right)\le 0$$

Furthermore, from equation (6), we let:

$$G(v)=d(p\lambda+r-c)\frac{v^2}{T}-dv(p+r-c)+d\frac{h+c[k-m(\xi)]}{[k-m(\xi)]^2}\left(e^{v[k-m(\xi)]}-1-v[k-m(\xi)]\right)+c_0+\xi \quad (10)$$

Due to the relations shown in equation (5) and using the fact that $1-\frac{v}{T}\frac{dT}{dv}<0$, so that,

$$\frac{dG(v)}{dv}=d(p\lambda+r-c)\left[\frac{2vT-v^2\frac{dT}{dv}}{T^2}\right]-d(p+r-c)+d\frac{h+c[k-m(\xi)]}{[k-m(\xi)]^2}\left(e^{v[k-m(\xi)]}-1\right)$$
$$=d(p\lambda+r-c)\frac{v}{T}-d(p\lambda+r-c)\frac{v^2}{T^2}\frac{dT}{dv}+d\left[(p\lambda+r-c)\frac{v}{T}-(p+r-c)+\frac{h+c[k-m(\xi)]}{k-m(\xi)}\left(e^{v[k-m(\xi)]}-1\right)\right]$$
$$= d(p\lambda+r-c)\frac{v}{T}\left(1-\frac{v}{T}\frac{dT}{dv}\right)<0 \quad (11)$$

As a result, $G(v)$ is a strictly decreasing function of *v*. From the above arguments and the structural induction, we have the following results:

Proposition 1

For any given ξ, we have:
(a) If $G(\hat{v}_{min})>0$ and $G(\hat{v}_{min})<0$, then the optimal value $(v^*,\ T^*)$ can be found by solving (5) and (6) simultaneously, and it not only exists but is unique.
(b) If $G(\hat{v}_{min})<0$, then the model reduces to the model without shortages.
(c) If $G(\hat{v}_{max})>0$, then the optimal value of T is $T\to\infty$.

Proof

(a) Because $G(v)$ is a strictly decreasing function of v, the intermediate value theorem implies that there exists a unique value v^* such that $G(v^*)=0$ if $G(\hat{v}_{min})>0$ and $G(\hat{v}_{max})<0$. Consequently, the point (v^*,T^*) satisfying (2) and (3) simultaneously not only exists but is unique. Let (v^*,T^*) be the solution of equations (2) and (3). To establish sufficiency, substituting this result into Hessian matrix yields:

$$\left.\frac{\partial^2F(T,v,\xi)}{\partial v^2}\right|_{(v^*,T^*)}=-\frac{\left\{p\lambda+r-c+e^{v^*[k-m(\xi)]}T^*\{h+c[k-m(\xi)]\}\right\}}{{T^*}^2}<0$$

$$\left.\frac{\partial^2F(T,v,\xi)}{\partial T^2}\right|_{(v^*,T^*)}=-\frac{d{v^*}^2(p\lambda+r-c)}{{T^*}^4}<0$$

and

$$\left.\frac{\partial^2F(T,v,\xi)}{\partial v\,\partial T}\right|_{(v^*,T^*)}=\frac{dv^*(p\lambda+r-c)}{{T^*}^3}$$

After some algebraic manipulation, the determinant of the Hessian matrix at the stationary point $(v^*,\ T^*)$ becomes:

$$|H|=\left.\frac{\partial^2F(T,v,\xi)}{\partial v^2}\right|_{(v^*,T^*)}\times\left.\frac{\partial^2F(T,v,\xi)}{\partial T^2}\right|_{(v^*,T^*)}-\left\{\left.\frac{\partial^2F(T,v,\xi)}{\partial v\,\partial T}\right|_{(v^*,T^*)}\right\}^2=\frac{d^2e^{v^*[k-m(\xi)]}{v^*}^2(p\lambda+r-c)\{h+c[k-m(\xi)]\}}{{T^*}^5}>0$$

As a result, for any given ξ, $F(T^*,v^*,\xi^*)$must be concave and the resulting stationary point yields a global maximum.
(b) If $G(\hat{v}_{min})<0$, then we have $\partial F(v,T,\xi)/\partial T=G(v)/T^2<(\hat{v}_{min})/T^2<0$, which implies the maximum value of $F(v,T,\xi)$ occurs at $T=v$. Thus the model reduces to the model without shortages.
(c) If $G(\hat{v}_{max})>0$, then we have $\partial F(v,T,\xi)/\partial T=G(v)/T^2>(\hat{v}_{max})/T^2$, which implies that a larger value of T causes a higher value of $F(v,T,\xi)$. Hence, the maximum value of $F(v,T,\xi)$ occurs at the point $T\to\infty$.

For any given positive ξ, Proposition 1(a) provides that if $G(\hat{v}_{min})>0$ and $G(\hat{v}_{max})<0$, then we can find a unique point (v^*,T^*), where $v^*\in(\hat{v}_{min},\hat{v}_{max})$, such that the retailer's unit time profit $F(v,T,\xi)$ is maximum. On the other hand, Proposition 1(b) reveals that if $G(\hat{v}_{min})<0$, then the model reduces to the model without shortages, that is T = v. Under this situation, equation (1) can be rewritten as:

$$F(v,\xi)=d(p-c)-\frac{c_0+\xi}{v}-\frac{e^{v[k-m(\xi)]}-v[k-m(\xi)]-1}{v}\times\frac{d\{h+c[k-m(\xi)]\}}{[k-m(\xi)]^2} \quad (12)$$

After taking the second derivative of $F(v, \xi)$ with respective *v*, we obtain:

$$\frac{\partial^2 F(v,\xi)}{\partial v^2} = -\frac{c_0+\xi}{v^3} - \frac{d\{h+c[k-m(\xi)]\}}{[k-m(\xi)]^2} \times \frac{f_1\{v[k-m(\xi)]\}}{v^3},$$

where

$$f_1(x) = -2 + 2e^x - 2xe^x + x^2e^x.$$

Because $x = v[k-m(\xi)] > 0$ by assumption, it follows that:

$$\begin{aligned} f_1(x) &= -2 + 2e^x - 2xe^x + x^2e^x \\ &= -2 + 2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) \\ &\quad - 2x\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) \\ &\quad + x^2\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) \\ &= \sum_{n=2}^{\infty}\left[\frac{2}{n!} - \frac{2}{(n-1)!} + \frac{1}{(n-2)!}\right]x^n \\ &= \sum_{n=2}^{\infty}\left[\frac{(n-1)(n-2)}{n!}\right]x^n > 0 \end{aligned}$$

Therefore, $F(v, \xi)$ is a concave function of *v* and there exists a unique v^* which maximizes $F(v, \xi)$. Once this case occurs, the retailer should raise the preservation technology cost to improve the unit time profit.

Finally, Proposition 1(c) reveals that, if $G(\hat{v}_{max}) > 0$, then $T^* \to \infty$. Under this situation, from equation (1), the retailer's unit time profit becomes:

$$\lim_{T\to\infty} F(T, v, \xi) = \frac{d(p\lambda + r - c)}{2} \tag{13}$$

Since *r* represents the penalty cost per unit of a lost sale inclusive of profit, it implies that $r > p - c$. And thus,

$$\lim_{T\to\infty} F(T, v, \xi) = \frac{d(p\lambda + r - c)}{2} < 0$$

That is, at this given preservation technology cost ξ, the inventory system should not be operated. Once this case occurs, the retailer should decrease the preservation technology cost to improve the unit time profit.

From the analysis carried out so far, we have obtained that, for any given ξ, the point (v^*, T^*) which maximizes the retailer's unit time profit not only exists but is unique. Continuing, we study the conditions under which the optimal preservation technology cost also exists and is unique.

For any given feasible *v* and *T*, taking the second partial derivative of equation (1) with respect to ξ yields"

$$\begin{aligned} \frac{\partial^2 F(T,v,\xi)}{\partial \xi^2} = \frac{d}{T}\Big\{ &-\frac{c}{[k-m(\xi)]^3} f_1(v[k-m(\xi)])\left[m'(\xi)\right]^2 \\ &- \frac{c}{[k-m(\xi)]^2} f_2(v[k-m(\xi)])\left[m''(\xi)\right] \\ &+ \frac{h}{[k-m(\xi)]^4} f_3(v[k-m(\xi)])\left[m'(\xi)\right]^2 \\ &- \frac{h}{[k-m(\xi)]^3} f_4(v[k-m(\xi)])\left[m''(\xi)\right]\Big\}, \end{aligned} \tag{14}$$

where

$f_1(x) = -2 + 2e^x - 2xe^x + x^2e^x$, $f_2(x) = -1 + e^x - xe^x$, $f_3(x) = 6 + 2x - -6e^x + 4xe^x - x^2e^x$ and $f_4(x) = 2 + x - 2e^x + xe^x$. From (14), we have the following proposition.

Proposition 2

For any given feasible (*v, T),* if the productivity of invested capital, *m(ξ),* is a strictly concave function of *ξ(*that is, $m''(\xi) < 0$ or diminishing marginal productivity of capital), then there exists a unique optimal preservation technology cost ξ^* that maximizes $F(v, T, \xi)$.

Proof

By employing the Taylor series expansion, $f_4(x)$ can be rewritten as:

$$f_4(x) = 2 + x - 2e^x + xe^x = \sum_{n=1}^{\infty}\left[\frac{(n-1)}{(n+1)!}\right]x^{n+1}$$

Because $x = v[k - m(\xi)] > 0$ by assumption, it follows that $f_4(x) > 0$. Moreover, since,

$$\begin{aligned} 2f_1(x) - f_4(x) &= \sum_{n=1}^{\infty}\left[\frac{2(n-1)}{(n+1)!}\right]x^{n+1} \\ &\quad - \sum_{n=2}^{\infty}\left[\frac{(n-1)(n-2)}{n!}\right]x^n \\ &= -\sum_{n=2}^{\infty}\left[\frac{(n-3)(n-2)}{n!}\right] < 0 \end{aligned}$$

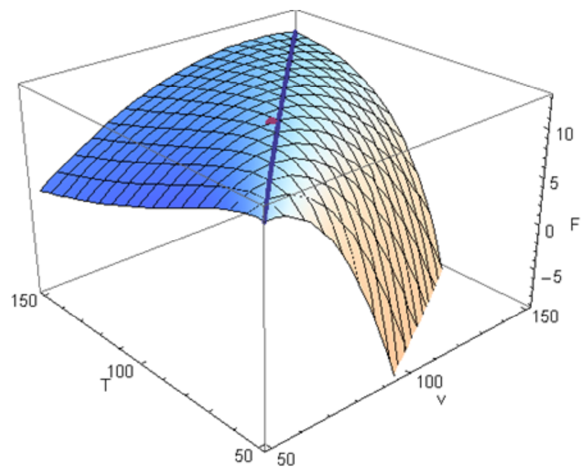

Figure 1. The retailer's unit time profit, $F(T, v, 263.3434)$.

it is clear that,

$$f_3(x) = 6 + 2x - -6e^x + 4xe^x - x^2e^x = 2f_1(x) - f_4(x) < 0$$

and

$$\begin{aligned} f_2(x) &= -1 + e^x - xe^x = 1 + x - e^x - f_4(x) \\ &= 1 + x - \left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) \\ &- f_4(x) < 0 \end{aligned}$$

Using these results, we have $\partial^2 F(T, v, \xi)/\partial\xi^2 < 0$ for any feasible (v, T).

Consequently, $F(T, v, \xi)$ is a strictly concave function of ξ and there exists a unique optimal preservation technology cost ξ^* that maximizes $F(T, v, \xi)$. The optimal ξ^* should be selected to satisfy;

$$\frac{\partial F(T, v, \xi)}{\partial \xi} = 0$$

otherwise,

$$\xi^* = \begin{cases} 0, \text{if } \left.\dfrac{\partial F(T, v, \xi)}{\partial \xi}\right|_{\xi<0} < 0 \\ w, \text{if } \left.\dfrac{\partial F(T, v, \xi)}{\partial \xi}\right|_{\xi=w} > 0 \end{cases}$$

Summarize above results, we can now establish the following algorithm to obtain the optimal solution of the problem.

Algorithm

Step 1: Start with $j = 0$ and the initial trial value of ξ_j, where $m(\xi) = 0.5k$.
Step 2: Find the optimal replenishment schedule, v^* and T^*, for a given preservation technology cost ξ_j.
Step 3: Use the result in Step 2, and then determine the optimal ξ_{j+1}.
Step 4: If the difference between ξ_j and ξ_{j+1} is sufficiently small, set $\xi^* = \xi_{j+1}$, then $(T^*; v^*, \xi^*)$ is the optimal solution and stop. Otherwise, set $j = j+1$ and go back to step 2.

NUMERICAL EXAMPLE

To illustrate the results, let us apply the proposed algorithm to solve the following numerical examples.

Example 1

We first redo the same example of Hsu (2010) to see the optimal replenishment policy: $d = 1$, $\lambda = 0.8$, $k = 0.02$, $h = 0.05$, $p = 30$, $c = 10$, $c_o = 120$, $r = 3$ and $m(\xi) = k(1-e^{-0.01\xi})$. Then, applying the algorithm, the optimal values of ξ, v and T are $\xi^* = 263.3434$, $v^* = 103.8139$ and $T^* = 106.4338$, respectively. The retailer's unit time profit obtained here is $F^* = 12.8087$, respectively. The three-dimensional retailer's unit time profit graph and contour plot as $\xi^* = 263.3434$ are respectively shown in Figures 1 and 2. Note that we run the numerical results with distinct starting values of $\xi = 150, 160, 170, \ldots, 350$. The numerical results indicate that F is strictly concave in ξ, as shown in Figure 3.

Consequently, we are sure that the local maximum obtained here is indeed the global maximum solution.

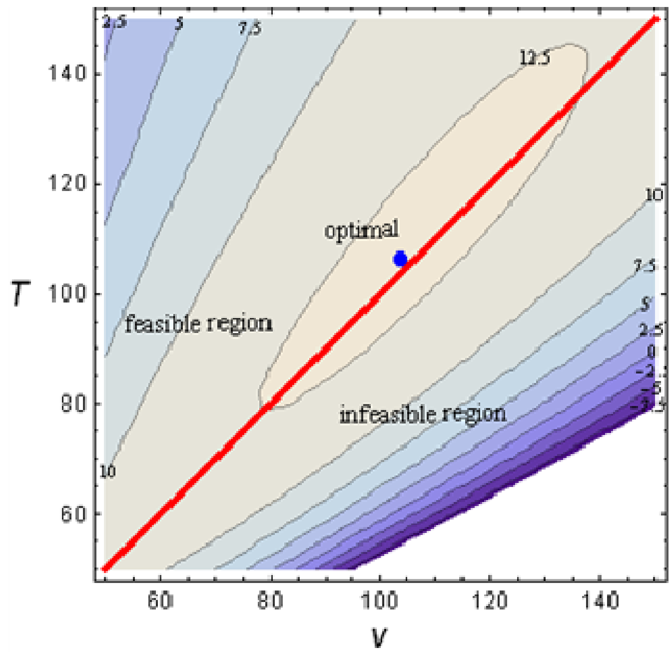

Figure 2. Contour of $F(T, v, 263.3434)$.

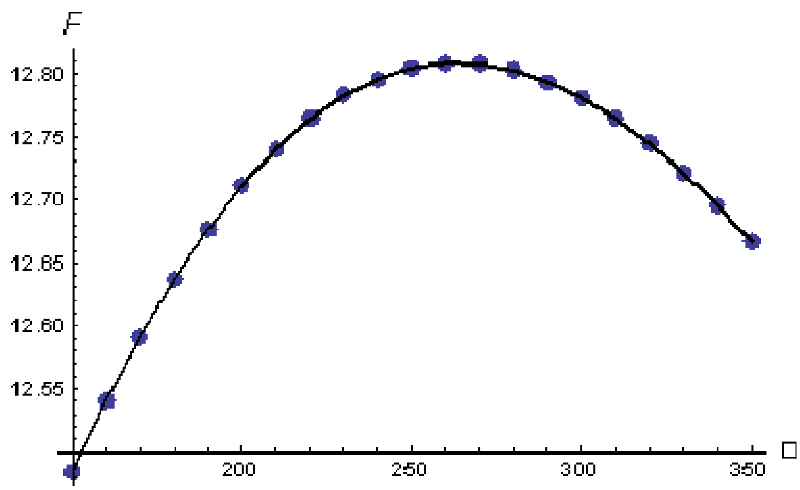

Figure 3. Graphical representation of $F(T^*, v^*, \xi^*)$ in example 1.

Example 2

In this example, the same data in example 1 are used except putting limited capital w = 200. From example 1, we know that $F(T, v, \xi)$ reaches its maximum at ξ= 263.3434. Because $F(T, v, \xi)$ is a strictly concave function of ξ, it follows that $\xi^*= w =$ 200. Then, by proposition 1, we get v^* = 84.3875 and T^*= 86.69858. The retailer's unit time profit obtained here is F^*=12.7112. The three-dimensional retailer's unit time profit graph and contour plot as ξ^*= 200 are respectively shown in Figures 4 and 5.

Conclusion

In this paper, we provide some properties of the retailer's unit time profit that appears in Hsu et al. (2010) and prove that the optimal solution not only exists but is unique. The proposed model can be extended in several ways. Firstly, we can easily extend the backlogging rate of unsatisfied demand to any decreasing function $\beta(x)$, where x is the waiting time up to the next replenishment, and $0 \leq \beta(x) \leq 1$ with $\beta(0)$ = 1. Secondly, we can also incorporate the quantity discount, and variable deterioration

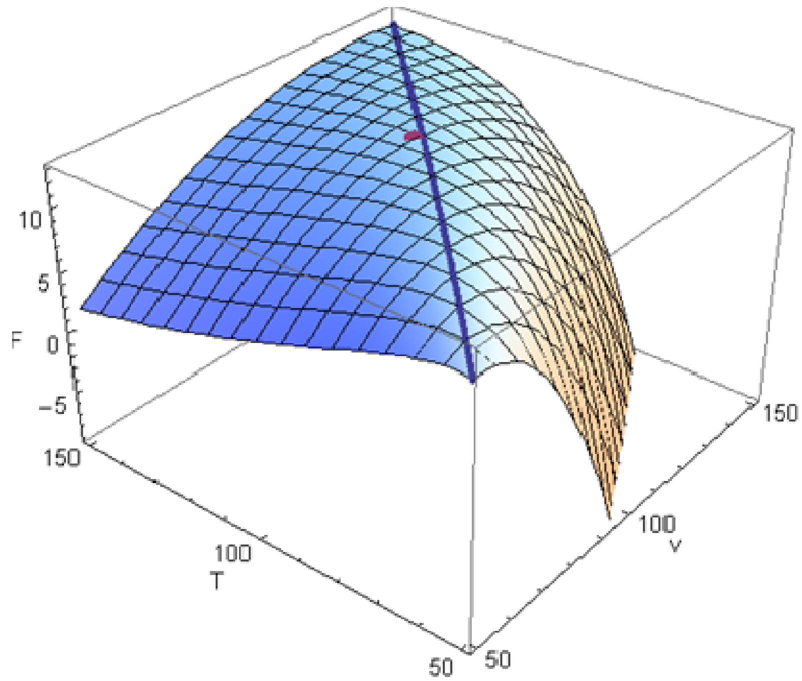

Figure 4. The retailer's unit time profit, *F*(*T, v,* 200).

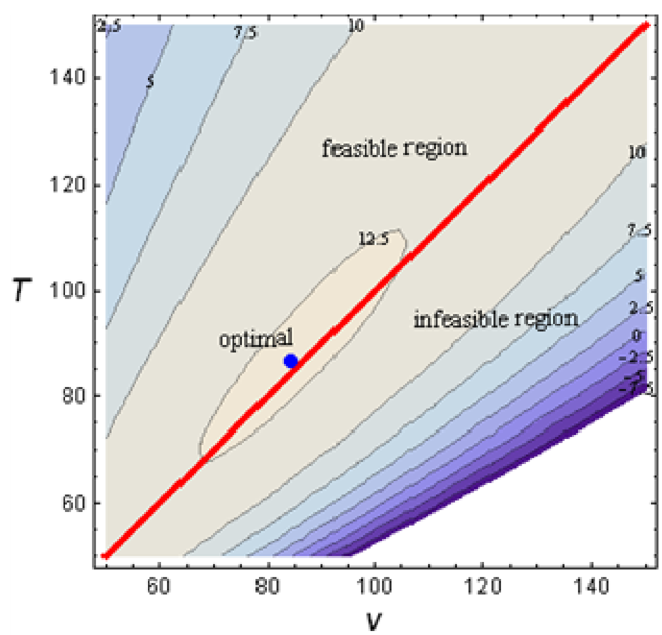

Figure 5. Contour of *F*(*T, v,* 200).

rates (for example, Weibull deterioration rate) into the model.

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