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Full Length Research Paper

Comparison of the REMBRANDT system with the Wang and Elhag approach: A practical example of the rank reversal problem

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Analytic hierarchy process (AHP) has been considerably criticized for possible rank reversal phenomenon caused by: i, when new alternatives are added or old ones deleted; and ii, when new criteria are added or old ones deleted with the caveat that the priorities of alternatives would be tied under these criteria and hence argued that the criteria should be irrelevant when ranking the alternatives. While in many cases this is a perfectly valid phenomenon, there are also many cases where rank should be preserved. This paper deals with rank reversal due to the inconsistency of the inputs. The preference intensities on REMBRANDT scale are more compatible than AHP scale. The REMBRANDT system is therefore proposed to avoid rank reversal phenomenon. We have provided a practical example to show that the rank reversal phenomenon did not occur with the REMBRANDT system, but did occur with the Wang and Elhag approach.

Key words: Analytic hierarchy process (AHP), REMBRANDT system, rank reversal, multiple-attribute decision making (MADM), uncertainty.

INTRODUCTION

Analytic hierarchy process (AHP), as a very popular multiple criteria decision making (MCDM) tool, has been considerably criticized for its possible rank reversal phenomenon, which means changes of the relative rankings of the other alternatives after an alternative is added or deleted. Such a phenomenon was first noticed and pointed out by Belton and Gear (1983), which leads to a long-lasting debate about the validity of AHP (Dyer, 1990a, b; Harker and Vargas, 1987; Leung and Cao, 2001; Perez, 1995; Saaty, 1990a; Saaty, 1986, 1990,1994; Saaty et al., 1983; Schoner et al., 1989; Stewart, 1992; Troutt, 1988; Vargas, 1994; Wang and Elhag, 2006; Watson and Freeling, 1982, 1983), especially about the legitimacy of rank reversal (Forman, 1990; Millet and Saaty, 2000; Saaty, 1987a,b; Saaty and Vargas, 1984; Schoner et al.,1992).

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In order to avoid the rank reversal, Belton and Gear (1983) suggested normalizing the eigenvector weights of alternatives using their maximum rather than their sum, which was usually called B–G modified AHP. Saaty and Vargas (1984) provided a counterexample to show that B– G modified AHP was also subject to rank reversal. Belton and Gear (1985) argued that their procedure was misunderstood and insisted that their approach would not result in any rank reversal if criteria weights were changed accordingly.

Schoner and Wedley (1989) presented a referenced AHP to avoid rank reversal phenomenon, which requires the modification of criteria weights when an alternative is added or deleted.

Schoner et al. (1993) also suggested a method of normalization to the minimum and a linking pin AHP [see also (Schoner et al., 1997)], in which one of the alternatives under each criterion is chosen as the link for criteria comparisons and the values in the linking cells are assigned a value of one, with proportional values in the other cells. Barzilai and Golany (1994) showed that no normalization could prevent rank reversal and suggested a multiplicative aggregation rule, which replaces normalized weight vectors with weight–ratio matrices, to avoid rank reversal. Lootsma (1993) and Barzilai and Lootsma (1997) suggested a multiplicative AHP for rank preservation. Vargas (1997) provided a practical counterexample to show the invalidity of the multiplicative AHP. Triantaphyllou (2001) offered two new cases to demonstrate that the rank reversals do not occur with the multiplicative AHP, but do occur with the AHP and some of its additive variants. Leung and Cao (2001) showed that Sinarchy, a particular form of analytic network process (ANP), could prevent rank reversal. As an integrative view, the AHP now supports four modes, called Absolute, Distributive, Ideal and Supermatrix modes, respectively, for scaling weights to rank alternatives (Millet and Saaty, 2000; Saaty, 1986, 1994; Saaty and Vargas, 1993). In absolute mode, alternatives are rated one at a time and there is no rank reversal when new alternatives are added or removed. The distributive mode normalizes alternative weights under each criterion so that they sum up to one, which does not preserve rank. The ideal mode preserves rank by dividing the weight of each alternative only by the weight of the best alternative under each criterion. The supermatrix mode allows one to consider dependencies between different levels of a feedback network. More recently, Ramanathan (2006) suggested a DEAHP, which is claimed to have no rank reversal phenomenon. But in fact, it still suffers from rank reversal. Zahir (2007a, b) provided an explicit derivation of a new aggregation rule that incorporated the unit of measure as defined by the norm of a vector. He believed that because of normalization the relative priorities of the objects have an arbitrary unit of measure. When relative-preferences are fully consistent, aggregate alternative ranks are preserved; however, inconsistent preferences may alter ranks when a new alternative is added or deleted (Zahir and Al-Mahmud, 2009).

Wang and Elhag suggested an approach, in which the local priorities remained unchanged. Hence, the ranking among the alternatives would be preserved. In this paper, we have provided a practical example to show that the Wang and Elhag approach is also subject to rank reversal.

In general, it is known in decision making that if one alters criteria or criteria weights, then the outcome of a decision will change, possibly leading to rank reversal. This is precisely what some authors use to criticize the AHP. There are two situations. The first is called "wash criteria" which involves the deletion of criteria that are assumed irrelevant because the alternatives have equal or nearly equal priorities under them (Finan and Hurley, 2002). The second is called "indifferent criteria" which involves the addition of criteria again assumed irrelevant for the same reason as "wash criteria" (Perez et al., 2006). In the first case, the authors made the error of renormalizing the weights of the remaining criteria that then gave rise to rank reversal because the weights of the criteria were changed (Saaty and Vargas, 2006; Wijnmalen and Wedley, 2009). In the second case, the addition of a new criterion that was irrelevant also led to rank reversal for exactly the same reason of changing the weights of the criteria. It is surprising that anyone would want to add irrelevant criteria and use it to make an important decision. This approach treats the weights of the criteria not as representative of their importance but as scaling constants like in Multi-Attribute Utility Theory (Keeney and Raiffa, 1976).

Our literature review shows that the rank reversal phenomenon has not been perfectly resolved and there still exist debates about the ways of avoiding rank reversals in AHP. So, this paper offers the REMBRANDT system to avoid rank reversal.

A group in the Netherlands, led by F.A. Lootsma, has developed a system which uses Ratio Estimation in Magnitudes or deci-Bells to Rate Alternatives which are Non-DominaTed (Lootsma, 1992; Lootsma et al., 1990). This system is intended to adjust for three contended flaws in AHP. First, direct rating is on a logarithmic scale (Lootsma, 1992), which replaces the fundamental 1-9 scale presented by Saaty. Second, the Perron-Frobenius eigenvector method (EM) of calculating weights is replaced by geometric mean, which avoids potential rank reversal (Barzilai et al., 1987). And third, aggregation of scores by arithmetic mean is replaced by the product of alternative relative scores weighted by the power of weights obtained from analysis of hierarchical elements above the alternatives.

This paper then compares the REMBRANDT system with the Wang and Elhag approach. A practical example is examined using the REMBRANDT system to verify its validity and practicability in rank preservation.

RANK REVERSAL IN THE AHP

Belton and Gear (1983) showed that rank reversal might occur in the AHP when an exact replica or copy of an alternative was introduced. They considered an example with three consistent comparison matrices over four alternatives; A, B, C and D with respect to three criteria a, b and c, where D was a copy of B and the three criteria were assumed to be of equal D importance. They first considered alternatives A, B and C and derived a ranking for them, and then considered the four alternatives together and got a new ranking for them, only to find that the ranking between A and B was reversed after the addition of D. Tables 1 and 2 show the comparison matrices, the local and the global weights of the four decision alternatives.

As can be seen from Table 2, the ranking between A and B is $B \succ A$ Before D is introduced, but becomes $A \succ B$ After D is added, where the symbol "$\succ$" means " is

Table 1. Paiwise comparision matrices of alternatives A, B, C and D with respect to three criteria and their local weights.

Criterion	Alternative	A	B	C	D	Local weight
Criterion a	A	1	1/9	1	----	1/11
	B	9	1	9	----	9/11
	C	1	1/9	1	----	1/11
Criterion b	A	1	9	9	----	9/11
	B	1/9	1	1	----	1/11
	C	1/9	1	1	----	1/11
Criterion c	A	1	8/9	8	----	8/18
	B	9/8	1	9	----	9/18
	C	1/8	1/9	1	----	1/18
Criterion a	A	1	1/9	1	1/9	1/20
	B	9	1	9	1	9/20
	C	1	1/9	1	1/9	1/20
	D	9	1	9	1	9/20
Criterion b	A	1	9	9	9	9/12
	B	1/9	1	1	1	1/12
	C	1/9	1	1	1	1/12
	D	1/9	1	1	1	1/12
Criterion c	A	1	8/9	8	8/9	8/27
	B	9/8	1	9	1	9/27
	C	1/8	1/9	1	1/9	1/27
	D	9/8	1	9	1	9/27

Table 2. Global weights of the four alternatives A, B, C and D and their ranks.

Alternative	Local weight			Global weight	Rank
	Criterion a (1/3)	Criterion b (1/3)	Criterion c (1/3)		
A	1/11	9/11	8/18	0.4512	2
B	9/11	1/11	9/18	0.4697	1
C	1/11	1/11	1/18	0.0791	3
A	1/20	9/12	8/27	0.3654	1
B	9/20	1/12	9/27	0.2889	2
C	1/20	1/12	1/27	0.0568	4
D	9/20	1/12	9/27	0.2889	2

superior to". The ranking is reversed after the addition of alternative D. Such a phenomenon is referred to as rank reversal, which may occur not only when a copy of an alternative is added, but also when a new alternative is added as well as when an existing alternative is removed. Dyer (1990a) provided an example, as shown in Tables 3 and 4, to illustrate the rank reversal phenomenon between A_1 and A_3 when a new alternative A_4 was added. Troutt (1988) provided an example, as shown in Table 5, to demonstrate the rank reversal phenomenon between A_1 and A_2 when alternative A_3 was removed. There may be other AHP examples that can lead to rank reversals.

Perez et al. (2006) showed that the addition of different criteria (for which all alternatives performed equally)

Table 3. Decision matrix of four alternatives A_1-A_4 with respect to four decision criteria.

Alternative	Criterion 1	Criterion 2	Criterion 3	Criterion 4
A_1	1	9	1	3
A_2	9	1	9	1
A_3	8	1	4	5
A_4	4	1	8	5

Table 4. AHP weights and the rankings of the four alternatives A_1-A_4.

Alternative	Local weight				Global weight	Rank
	Criterion 1 (1/4)	Criterion 2 (1/4)	Criterion 3 (1/4)	Criterion 4 (1/4)		
A_1	1/18	9/11	1/14	3/9	0.3196	3
A_2	9/18	1/11	9/14	1/9	0.3362	2
A_3	8/18	1/11	4/14	5/9	0.3442	1
A_1	1/22	9/12	1/22	3/14	0.2638	1
A_2	9/22	1/12	9/22	1/14	0.2432	4
A_3	8/22	1/12	4/22	5/14	0.2465	2
A_4	4/22	1/12	8/22	5/14	0.2465	2

Table 5. Decision matrix and AHP weights of three alternatives with respect to two decision criteria.

Alternative	Decision matrix		Criteria and local weight		Global weight	Rank
	Criterion 1	Criterion 2	Criterion 1 (3/10)	Criterion 2 (7/10)		
A_1	57	3	57/100	3/10	0.381	1
A_2	33	4	33/100	4/10	0.379	2
A_3	10	3	10/100	3/10	0.240	3
A_1	57	3	57/90	3/7	0.490	2
A_2	33	4	33/90	4/7	0.510	1

caused a significant alternation of the aggregated priorities of alternatives, with important consequences. Although not in three-level hierarchies, in more complex hierarchies, rank reversal might happen. They provided an example, as shown in Tables 6 and 7, to analyze the rank reversal phenomenon when an indifferent criterion was added.

Finan and Hurley (2002) showed that where the wash criterion was a subcriterion and DM was perfectly consistent, the rank-order might change by leaving it out. They provided an example, as shown in Tables 8 and 9, to illustrate the rank reversal phenomenon between A_1 and A_2 when wash criteria was removed. Note that, with subcriteria 3, A_2 is prefered to A_1, and in the case where subcriteria 3 is left out, A_1 is prefered to A_2.

WANG AND ELHAG APPROACH

Sometimes, it may be argued that rank reversal is a normal phenomenon in some situations where avoiding it does not make sense. As is known, the weights of criteria are usually assumed to be independent of the number of alternatives in most of the real world MCDM problems and MCDM approaches. Although this assumption is also under debate in the AHP (Dyer, 1990a; Schoner and Wedley, 1989), it is not easy to accept the assumption that the weights or the number of criteria should vary with the number of alternatives.

Wang and Elhag introduced an approach to avoid rank reversal phenomenon, which, using the first n weights of alternative, is unchanged. They verified that in order to avoid rank reversal, original local priorities of each alternative under every criterion had to remain unchanged when an alternative was added or removed (Wang and Elhag, 2006). In what follows, they discussed how to keep the original priorities unchanged when an alternative is added.

They have considered $A = (a_{ij})_{n \times n}$ as a comparison

Table 6. Decision matrix and AHP weights of two alternatives with respect to two criteria.

Alternative	Criterion 1 (0.5)		Criterion 2 (0.5)		Global weight	Rank
	Sub criteria 1 (0. 5)	Sub criteria 2 (0. 5)	Sub criteria 1 (0.5)	Sub criteria 2 (0.5)		
X_1	0.2	0.6	0.35	0.8	0.4875	2
X_2	0.8	0.4	0.65	0.2	0.5125	1

Table 7. Decision matrix and AHP weights of two alternatives with respect to two criteria.

Alternative	Criteria 1 (0.5)			Criteria 2 (0.5)		Global weights	Rank
	Sub criteria 1 (0.25)	Sub criteria 2 (0.25)	Sub criteria 3 (0.5)	Sub criteria 1 (0.5)	Sub criteria 2 (0.5)		
X_1	0.2	0.6	0.5	0.35	0.8	0.5125	1
X_2	0.8	0.4	0.5	0.65	0.2	0.4875	2

Table 8. Decision matrix and AHP weights of two alternatives with respect to two criteria.

Alternative	Criteria 1 (0.55)			Criteria 2 (0.45)		Global weights	Rank
	Sub criteria 1 (0.6)	Sub criteria 2 (0.2)	Sub criteria 3 (0.2)	Sub criteria 1 (0.5)	Sub criteria 2 (0.5)		
A_1	0.5	0.8	0.4	0.2	0.6	0.477	2
A_2	0.5	0.2	0.6	0.8	0.4	0.523	1

Table 9. Decision matrix and AHP weights of two alternatives with respect to two criteria.

Alternative	Criteria 1 (0.55)		Criteria 2 (0.45)		Global weights	Rank
	Sub criteria 1 (0.5)	Sub criteria 2 (0.5)	Sub criteria 1 (0.5)	Sub criteria 2 (0.5)		
A_1	0.8	0.4	0.2	0.6	0.51	1
A_2	0.2	0.6	0.8	0.4	0.49	2

matrix with respect to some criterion and $B=(b_{ij})_{(n+1)\times(n+1)}$ as the augmented comparison matrix with the same criterion after the $(n+1)^{th}$ alternative is added. Their eigenvector weights are denoted by $W_A=(w_{1A},...,w_{nA})^T$ and $W_B=(w_1,...,w_{n+1})^T$, respectively. Since W_B is the normalized principal right eigenvector of the comparison matrix B, namely, $BW_B=\lambda W_B$, it follows that $B(KW_B)=\lambda_{\max}(KW_B)$ for any $k\succ 0$, which means KW_B is also a principal right eigenvector of B. The only difference between W_B and $\hat{W}_B=KW_B$ is that $\sum_{i=1}^{n+1} w_{iB}=1$ while $\sum_{i=1}^{n+1}\widehat{w}_{iB}=k\neq 1$. In order to keep the original priorities of the first n alternatives unchanged, the following condition has to be met:

$$\sum_{i=1}^{n} w_{iA}=\sum_{i=1}^{n} kw_i \qquad (1)$$

Since $\sum_{i=1}^{n} w_{iA}=1$, they have gotten from Equation (1):

$$k=\frac{1}{\sum_{i=1}^{n} w_i} \qquad (2)$$

Accordingly,

$$\widehat{W}_B=kW_B=(\frac{w_1}{\sum_{i=1}^{n} w_i},\frac{w_2}{\sum_{i=1}^{n} w_i},...,\frac{w_{n+1}}{\sum_{i=1}^{n} w_i})^T \qquad (3)$$

where $\hat{W}_B$ can be interpreted as the normalization with

Table 10. Preference value.

Verbal description	Saaty ratio w_j / w_k	REMBRANDT δ_{jk}
Very strong preference for object k	1/9	-8
Strong preference for object k	1/7	-6
Definite preference for object k	1/5	-4
Weak preference for object k	1/3	-2
Indifference	1	0
Weak preference for object j	3	2
Definite preference for object j	5	4
Strong preference for object j	7	6
Very strong preference for object j	9	8

respect to the original n alternatives. Therefore, as long as they have used the rescaled eigenvector $\hat{W}_B$ instead of the original eigenvector W_B, the original priorities of the first n alternatives under each criterion will be kept unchanged. Accordingly, the ranking among them will be able to be preserved. If an alternative is going to be removed, then the remaining alternatives have to keep unchanged their original local priorities with respect to each criterion. Accordingly, their composite weights will not change and there will be no rank reversal to happen in this situation.

THE REMBRANDT SYSTEM

The REMBRANDT system has been designed to address three criticized features of AHP. The first issue addressed by Lootsma is the numerical scale for verbal comparative judgment. Saaty presented a verbal scale for the ratio of relative value between two objects where 1 represents roughly equal value, 3 represents the base objects as being moderately more important than the other objects, 5 reflects essential advantage, 7 very strong relative advantage, and 9 the ultimate overwhelming relative advantage. Lootsma feels that relative advantage is more naturally concave, and presents a number of cases where a more nearly logarithmic scale would be appropriate, such as planning horizons, loudness of sounds, and brightness of light. Therefore, Lootsma presents a geometric scale where the gradations of decision maker judgment are reflected by the scale as follows:

1/16: strict preference for object 2 over base object.
1/4: weak preference for object 2 over the base object.
1: indifference.
4: weak preference for the base object over object 2.
16: strict preference for the base object over object 2.

The ratio of value r_{jk} on the geometric scale is expressed as an exponential function of the difference between the echelons of value on the geometric scale δ_{jk}, as well as a scale parameter y. Lootsma considers two alternative scales y to express preferences. For calculating the weight of criteria, $y = \ln\sqrt{2} \approx 0.347$ is used. For calculating the weight of alternatives on each criterion, $y = \ln 2 \approx 0.693$ is used. The difference in echelons of value δ_{jk} is graded as in Table 10.

The second suggested improvement is the calculation of impact scores. The arithmetic mean is subject to rank reversal of alternatives. The geometric mean is not subject to rank reversal, nor is logarithmic regression. Note that Saaty (1990a) argues that rank reversal when new reference points are introduced is a positive feature. Barzilai et al. (1987), taking an opposing view, argued that the geometric mean was more appropriate for calculation of relative value (through weights) than the arithmetic mean used by Saaty.

Lootsma proposes logarithmic regression, minimizing $\sum_{j<k}(\ln r_{jk} - \ln v_j + \ln v_k)^2$ where r_{jk} are the ratio comparisons made by the decision maker for base object j and compared object k, and the weight for j (w_j) is represented by $\ln v_j$. The analysis is to calculate these weights. Since $r_{jk} = \frac{w_j}{w_k}$, error is represented minimizing the squared error yields the set of weights w_i which best fit the decision maker expressed preferences. Solving this is complicated by the fact that the resulting data set is singular. However, a series of normal equations can be solved to yield the desired weights.

The ratio matrix in REMBRANDT for criteria is

Table 11. Failure mode and effect analysis.

Operation	Potential mode of the failure	Potential effect of the failure	Causes of the failure
Casting	Breaking of the case	Deterioration of the pump body	Lack of proper mold Sands of bad material Quickly cooling molten
Turning	Conical shape of the edge	Problem in getting Fit	Improper feeding rate Fatigue of the wheels Carelessness of the operator
Milling	Surface scratches	High fatigue	Tolerance problem Fatigue of the help surface Not enough pressure
Assembly	Wobble of the gears	Improper working of the pump	Not firm gears in place

transformed through the operator $e^{0.347\,r(jk)}$ to generate the set of values transformed to the logarithmic scale. Krovac (1987) notes that the geometric means of row elements of such a matrix yields the solution minimizing the sum of squared errors of the form $\sum_{i=1}^{n}\sum_{i=1}^{n}(\ln r_{jk} - w_j + w_k)^2$.

This solution is normalized by product. It is a simple matter to normalize by sum, simply divide each element by the total. The third improvement proposed by Lootsma is aggregation of scores. This lowest level is normalized multiplicatively, so that the product of components equals 1 for each of the k factors over which the alternatives are compared. Therefore, each alternative has an estimated relative performance w_k for each of the k factors. The components of the hierarchical level immediately superior to this lowest level are normalized additively, so that they add to 1, yielding weights O(*i*). The aggregation rule for each alternative j is:

$$w_j = \prod_{i=1}^{k} w_i^{o(i)} \qquad (4)$$

EXAMPLE

The present model in this article was performed in ARA-FAN Company which manufactures hydraulic gear pumps. The company produces hydraulic gear pumps and focuses on manufacturing Ferguson 285 tractor hydraulic pumps as its most important product that the present article is going to analyze the failures and the effects of them in the production of the mentioned. Evaluating the production process, it was determined that there are generally four major and influential operations in the production: 1- casting operation 2- turning operation 3- milling operation 4- assembly.

To increase the reliability of the product, the failure analysis in relation to these four operations was performed (Table 11). As we know, the potential impact of each cause of a failure is as a function of the three criteria for severity, occurrence and detection in FMEA. First, an executive manager considers the FMEA for the potential causes of the failure in the AHP method. He obtains the assessment scores for any potential cause of the failure in each of these three criteria using the ideas of production and maintenance experts as well as preference value (Table 10). After recognizing the hierarchical structure as in Figure 1, first, he obtains the weight of the criteria relative to the goal using pairwise comparison. Table 16 shows the results of pairwise comparison of the criteria relative to the goal. As can be seen, the occurrence criterion is the most important one based on the weight it has gained. Now, he obtains the weight of alternatives relative to criteria using pairwise comparison matrices.

This example demonstrates the rank reversal phenomenon in the Wang and Elhag approach, which involves three comparison matrices over ten alternatives with respect to three criteria occurrence, detection and severity, respectively, when alternative "improper feeding rate" is added. This example has been examined using the REMBRANDT system.

AHP calculations: Wang and Elhag approach

Tables 12, 13 and 14 show the comparison matrices, and Table 15 shows the local and composite weights of the ten decision alternatives. As can been seen from Table 15, the ranking between A,..., J will be $B \succ E \succ H \succ G \succ F \succ C \succ A \succ I \succ J$ before D is introduced, but will be become $B \succ E \succ D \succ G \succ H \succ F \succ C \succ A \succ I \succ J$ after D is

Table 12. Comparison matrices relative to severity criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	1	1/4	3	----	2	1	1/5	2	3	2
Sands of bad material (B)	4	1	7	----	6	3	1	6	7	7
Quickly cooling molten (C)	1/3	1/7	1	----	1	1/4	1/8	1	1	1
Fatigue of the wheels (E)	1/2	1/6	1	----	1	1/3	1/7	1	1	1
Carelessness of the operator (F)	1	1/3	4	----	3	1	1/4	3	4	4
Tolerance problem (G)	5	1	8	----	7	4	1	7	8	8
Fatigue of the help surface (H)	1/2	1/6	1	----	1	1/3	1/7	1	1	1
Not enough pressure (I)	1/3	1/7	1	----	1	1/4	1/8	1	1	1
Not firm gears in place (J)	1/2	1/7	1	----	1	1/4	1/8	1	1	1
Lack of proper mold (A)	1	1/4	3	1/4	2	1	1/5	2	3	2
Sands of bad material (B)	4	1	7	1	6	3	1	6	7	7
Quickly cooling molten (C)	1/3	1/7	1	1/7	1	1/4	1/8	1	1	1
Improper feeding rate (D)	4	1	7	1	6	3	1	6	7	7
Fatigue of the wheels (E)	1/2	1/6	1	1/6	1	1/3	1/7	1	1	1
Carelessness of the operator (F)	1	1/3	4	1/3	3	1	1/4	3	4	4
Tolerance problem (G)	5	1	8	1	7	4	1	7	8	8
Fatigue of the help surface (H)	1/2	1/6	1	1/6	1	1/3	1/7	1	1	1
Not enough pressure (I)	1/3	1/7	1	1/7	1	1/4	1/8	1	1	1
Not firm gears in place (J)	1/2	1/7	1	1/7	1	1/4	1/8	1	1	1

Table 13. Comparison matrices relative to detection criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	1	1	1/6	----	1/3	1/4	1	1/3	1	1
Sands of bad material (B)	1	1	1/6	----	1/3	1/4	1	1/3	1	1
Quickly cooling molten (C)	6	6	1	----	3	2	6	3	6	6
Fatigue of the wheels (E)	3	3	1/3	----	1	1	3	1	3	3
Carelessness of the operator (F)	4	4	1/2	----	1	1	4	1	4	4
Tolerance problem (G)	1	1	1/6	----	1/3	1/4	1	1/3	1	1
Fatigue of the help surface (H)	3	3	1/3	----	1	1	3	1	3	3
Not enough pressure (I)	1	1	1/6	----	1/3	1/4	1	1/3	1	1
Not firm gears in place (J)	1	1	1/6	----	1/3	1/4	1	1/3	1	1
Lack of proper mold (A)	1	1	1/6	1/4	1/3	1/4	1	1/3	1	1
Sands of bad material (B)	1	1	1/6	1/4	1/3	1/4	1	1/3	1	1
Quickly cooling molten (C)	6	6	1	2	3	2	6	3	6	6
Improper feeding rate (D)	4	4	1/2	1	1	1	4	1	4	4
Fatigue of the wheels (E)	3	3	1/3	1	1	1	3	1	3	3
Carelessness of the operator (F)	4	4	1/2	1	1	1	4	1	4	4
Tolerance problem (G)	1	1	1/6	1/4	1/3	1/4	1	1/3	1	1
Fatigue of the help surface (H)	3	3	1/3	1	1	1	3	1	3	3
Not enough pressure (I)	1	1	1/6	1/4	1/3	1/4	1	1/3	1	1
Not firm gears in place (J)	1	1	1/6	1/4	1/3	1/4	1	1/3	1	1

added. Note that the composite weights of the alternatives obtained with the Wang and Elhag approach are different from the AHP outcome given in the Table 15. The ranking order between G and H is changed. Such a phenomenon is referred to as rank reversal, which many occur not only when an alternative is added,

Table 14. Comparison matrices relative to occurrence criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	1	1/4	1	----	1/4	1	1	1/3	1	1
Sands of bad material (B)	4	1	5	----	4	4	4	1	3	4
Quickly cooling molten (C)	1	1/5	1	----	1/5	1	1	1/4	1/2	1
Fatigue of the wheels (E)	4	1	5	----	1	4	4	1	3	4
Carelessness of the operator (F)	1	1/4	1	----	1/4	1	1	1/3	1	1
Tolerance problem (G)	1	1/4	1	----	1/4	1	1	1/3	1	1
Fatigue of the help surface (H)	3	1	4	----	1	3	3	1	2	3
Not enough pressure (I)	1	1/3	2	----	1/3	1	1	1/2	1	1
Not firm gears in place (J)	1	1/4	1	----	1/4	1	1	1/3	1	1
Lack of proper mold (A)	1	1/4	1	1	1/4	1	1	1/3	1	1
Sands of bad material (B)	4	1	5	3	4	4	4	1	3	4
Quickly cooling molten (C)	1	1/5	1	1/2	1/5	1	1	1/4	1/2	1
Improper feeding rate (D)	1	1/3	2	1	1/3	1	1	1/2	1	1
Fatigue of the wheels (E)	4	1	5	3	1	4	4	1	3	4
Carelessness of the operator (F)	1	1/4	1	1	1/4	1	1	1/3	1	1
Tolerance problem (G)	1	1/4	1	1	1/4	1	1	1/3	1	1
Fatigue of the help surface (H)	3	1	4	2	1	3	3	1	2	3
Not enough pressure (I)	1	1/3	2	1	1/3	1	1	1/2	1	1
Not firm gears in place (J)	1	1/4	1	1	1/4	1	1	1/3	1	1

but also when an alternative is removed. It is observed from Table 15 that the Wang and Elhag approach for ranking the reversal of any changes that occurred in local priorities has failed to keep the priorities unchanged. However, the cause of the rank reversal is the inconsistency of the inputs.

REMBRANDT calculations

Now, the analysis of the failure mode and effect using REMBRANDT system can be done. The matrices in Tables 17, 18 and 19 show the $\delta(jk)$ matrices equivalent to Saaty's scale used previously, as well as the transformed matrices $e^{0.693\delta(jk)}$. The pairwise comparisons of criteria on the goal use an exponential multiplier of $\ln\sqrt{2}$ (Table 21). These are then aggregated to obtain weighted scores for each of the alternatives. For example:

$$A: 1^{0.316} \times 0.540^{0.423} \times 0.397^{0.261} = 0.606$$

The impact and final scores are shown in Table 20, from which it can be seen very clearly that the REMBRANDT system preserves the ranking between G and H in this example when D is added. Comparative results shown in Table 20 indicate that results obtained by REMBRANDT were different from those obtained using the Wang and Elhag approach.

Rescaled eigenvector ranking indicated alternative B has 4.10 times the value of alternative J. The REMBRANDT scores can be interpreted as indicating that overall, alternative B is 7.76 times as valuable as alternative J. However, REMBRANDT uses a longer scale than the Wang and Elhag approach.

DECISION MAKING UNDER UNCERTAINTY

The decision maker provides a subjective cardinal judgment about the intensity of his/her preference for each alternative over each other alternative under each of a number of criteria or attributes. Often, a DM might be uncertain about his/her preference intensity when comparing two alternatives. When the preference judgments contain elements of uncertainty, both the rescaled composite weights of alternatives and the final impact scores of alternatives will also be uncertain.

Sources of uncertainty in modeling a decision problem

In the decision-making context, uncertainty may be categorized into one of three distinct classes.

Imprecision or vagueness

The linguistic or semantic qualifications describing each category of preference intensity are rather vague or imprecise; so that the DM has difficulty deciding which

Table 15. Composite weights of the ten alternatives and their ranks.

Alternative	Eigenvector weights relative to severity	Eigenvector weights relative to occurrence	Eigenvector weights relative to detection	Composite weights	Rescaled composite weights	Priority
Lack of proper mold (A)	0.087	0.059	0.047	0.065	----	7
Sands of bad material (B)	0.28	0.227	0.047	0.213	----	1
Quickly cooling molten (C)	0.037	0.051	0.318	0.09	----	6
Fatigue of the wheels (E)	0.041	0.227	0.138	0.157	----	2
Carelessness of the operator (F)	0.118	0.059	0.171	0.095	----	5
Tolerance problem (G)	0.321	0.059	0.047	0.135	----	4
Fatigue of the help surface (H)	0.041	0.187	0.138	0.136	----	3
Not enough pressure (I)	0.037	0.072	0.047	0.058	----	8
Not firm gears in place (J)	0.038	0.059	0.047	0.051	----	9
Lack of proper mold (A)	0.086	0.061	0.046	----	0.066	8
Sands of bad material (B)	0.282	0.226	0.046	----	0.213	1
Quickly cooling molten (C)	0.037	0.049	0.319	----	0.09	7
Improper feeding rate (D)	0.282	0.072	0.17	----	0.15	3
Fatigue of the wheels (E)	0.041	0.226	0.141	----	0.157	2
Carelessness of the operator (F)	0.117	0.061	0.17	----	0.095	6
Tolerance problem (G)	0.319	0.061	0.046	----	0.135	4
Fatigue of the Help Surface (H)	0.041	0.183	0.141	----	0.134	5
Not enough pressure (I)	0.037	0.072	0.046	----	0.058	9
Not firm Gears in place (J)	0.039	0.061	0.046	----	0.052	10

category best describes his/her feelings about a comparison or rating. In his/her mind, several of the linguistic qualifications or categories might be more or less appropriate, some more so than others. For example, the DM might know that he/she prefers alternative A over alternative B, but the categories `definite preference' and `strong preference' both seem more or less acceptable descriptions to him/her. An example of this type of uncertainty might arise when a decision is to be taken by a project co-ordinator, based on written inputs from experts in a number of distinct fields (for example, an environmental impact assessment). The co-ordinator has to interpret the experts' written responses and convert these into subjective judgments.

Inconsistency

The categories of preference intensity offered to the DM are distinct (that is, with crisp, non-overlapping endpoints) and are well understood by the DM. However, if the DM is asked to provide a number of replications of a specific pairwise comparison under different environmental conditions, his/her responses are not unique; the range of conditions and his/her recent experiences create an inability for him/her to classify his/her preference intensity into the same single category each time. Psychological evidence exists to support this: experimentation has shown that preferences may vary even when the underlying decision context appears to be the same on all identifiable counts.

Stochastic judgment

The level of preference intensity depends on some event whose outcome is not known with certainty at the time of the decision and is not

Table 16. Comparison matrix of criterion relative to the goal.

Criteria	Occurrence	Detection	Severity	Priority
Occurrence	*	3	2	0.54
Detection	1/3	*	1/2	0.163
Severity	1/2	2	*	0.297

Inconsistency rate = 0.01.

Table 17. Comparison matrices relative to severity criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	0	-3	2	----	1	0	-4	1	2	1
Sands of bad material (B)	3	0	6	----	5	2	0	5	6	6
Quickly cooling molten (C)	-2	-6	0	----	0	-3	-7	0	0	0
Fatigue of the wheels (E)	-1	-5	0	----	0	-2	-6	0	0	0
Carelessness of the operator (F)	0	-2	3	----	2	0	-3	2	3	3
Tolerance problem (G)	4	0	7	----	6	3	0	6	7	7
Fatigue of the help surface (H)	-1	-5	0	----	0	-2	-6	0	0	0
Not enough pressure (I)	-2	-6	0	----	0	-3	-7	0	0	0
Not firm gears in place (J)	-1	-6	0	----	0	-3	-7	0	0	0
Lack of proper mold (A)	0	-3	2	-3	1	0	-4	1	2	1
Sands of bad material (B)	3	0	6	0	5	2	0	5	6	6
Quickly cooling molten (C)	-2	-6	0	-6	0	-3	-7	0	0	0
Improper feeding rate (D)	3	0	6	0	5	2	0	5	6	6
Fatigue of the wheels (E)	-1	-5	0	-5	0	-2	-6	0	0	0
Carelessness of the operator (F)	0	-2	3	-2	2	0	-3	2	3	3
Tolerance problem (G)	4	0	7	0	6	3	0	6	7	7
Fatigue of the help surface (H)	-1	-5	0	-5	0	-2	-6	0	0	0
Not enough pressure (I)	-2	-6	0	-6	0	-3	-7	0	0	0
Not firm gears in place (J)	-1	-6	0	-6	0	-3	-7	0	0	0

under the control of the DM. The stochastic nature thus reflects either subjective probabilities that a particular alternative better achieves a given goal or objective probabilities that reflect uncertain consequences of selecting a particular alternative. For example, a DM is asked to choose one of two investment alternatives that require an identical one-time investment at the beginning of the planning period. The choice is likely to depend on the interest rate ruling over the entire investment period, which may be unknown at the time of the investment decision. Although each of these classes signifies a different form of indecision, we will loosely label the above classes under the general heading of `uncertainty'. In each case, however, the DM is likely to be reluctant to supply a single value or category to represent the intensity of his/her preference or rating.

ANALYSIS OF DIFFERENCES

The rank reversal occurred in the case of using Wang and Elhag approach because of the presence of inconsistency matrices. The reason why this rank reversal did not appear in the REMBRANDT is because it uses different scales. The REMBRANDT scale is longer than Saaty's fundamental scale with the respective values $..., \frac{1}{5}, \frac{1}{3}, 1, 3, 5, ...$. It is easy to see now why the last-named scale is controversial.

Saaty (1988, 1990b) invokes Fechner's law to explain the choice of the echelons, 3, 5, 7,…, although Stevens' power law is now generally accepted in psychophysics, and he chooses the echelons $\frac{1}{3}, \frac{1}{5}, \frac{1}{7}, ...$ in order to maintain reciprocity. Such a scale, neither geometric nor arithmetic, may introduce inconsistencies which are not necessarily present in the mind of decision maker. Consider three stimuli S_1, S_2 and S_3, for instance. Weak preference for S_1 over S_2, estimated by $r_{12}=3$ on the Saaty scale, and weak preference for S_2 over S_3, estimated by $r_{23}=3$, would consistently lead to $r_{13}=9$, a

Table 18. Comparison matrices relative to detection criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	0	0	-5	----	-2	-3	0	-2	0	0
Sands of bad material (B)	0	0	-5	----	-2	-3	0	-2	0	0
Quickly cooling molten (C)	5	5	0	----	2	1	5	2	5	5
Fatigue of the wheels (E)	2	2	-2	----	0	0	2	0	2	2
Carelessness of the operator (F)	3	3	-1	----	0	0	3	0	3	3
Tolerance problem (G)	0	0	-5	----	-2	-3	0	-2	0	0
Fatigue of the help surface (H)	2	2	-2	----	0	0	2	0	2	2
Not enough pressure (I)	0	0	-5	----	-2	-3	0	-2	0	0
Not firm gears in place (J)	0	0	-5	----	-2	-3	0	-2	0	0
Lack of proper mold (A)	0	0	-5	-3	-2	-3	0	-2	0	0
Sands of bad material (B)	0	0	-5	-3	-2	-3	0	-2	0	0
Quickly cooling molten (C)	5	5	0	1	2	1	5	2	5	5
Improper feeding rate (D)	3	3	-1	0	0	0	3	0	3	3
Fatigue of the wheels (E)	2	2	-2	0	0	0	2	0	2	2
Carelessness of the operator (F)	3	3	-1	0	0	0	3	0	3	3
Tolerance Problem (G)	0	0	-5	-3	-2	-3	0	-2	0	0
Fatigue of the Help Surface (H)	2	2	-2	0	0	0	2	0	2	2
Not Enough Pressure (I)	0	0	-5	-3	-2	-3	0	-2	0	0
Not Firm Gears in Place (J)	0	0	-5	-3	-2	-3	0	-2	0	0

Table 19. Comparison matrices relative to occurrence criterion.

Alternative	A	B	C	D	E	F	G	H	I	J
Lack of proper mold (A)	0	-3	0	----	-3	0	0	-2	0	0
Sands of bad material (B)	3	0	4	----	0	3	3	0	2	3
Quickly cooling molten (C)	0	-4	0	----	-4	0	0	-3	-1	0
Fatigue of the wheels (E)	3	0	4	----	0	3	3	0	2	3
Carelessness of the operator (F)	0	-3	0	----	-3	0	0	-2	0	0
Tolerance problem (G)	0	-3	0	----	-3	0	0	-2	0	0
Fatigue of the help surface (H)	2	0	3	----	0	2	2	0	1	2
Not enough pressure (I)	0	-2	1	----	-2	0	0	-1	0	0
Not firm gears in place (J)	0	-3	0	----	-3	0	0	-2	0	0
Lack of proper mold (A)	0	-3	0	0	-3	0	0	-2	0	0
Sands of bad material (B)	3	0	4	2	0	3	3	0	2	3
Quickly cooling molten (C)	0	-4	0	-1	-4	0	0	-3	-1	0
Improper feeding rate (D)	0	-2	1	0	-2	0	0	-1	0	0
Fatigue of the wheels (E)	3	0	4	2	0	3	3	0	2	3
Carelessness of the operator (F)	0	-3	0	0	-3	0	0	-2	0	0
Tolerance problem (G)	0	-3	0	0	-3	0	0	-2	0	0
Fatigue of the help surface (H)	2	0	3	1	0	2	2	0	1	2
Not enough pressure (I)	0	-2	1	0	-2	0	0	-1	0	0
Not firm gears in place (J)	0	-3	0	0	-3	0	0	-2	0	0

value which represents absolute preference for S_1 over S_3 on the Saaty scale. These preference intensities (weak, weak and absolute) are however not likely to be compatible. A geometric scale yields a more plausible relationship between preference intensities (weak, weak and strict).

Table 20. Final scores of the ten alternatives and their ranks.

Alternative	Impact score relative to severity	Impact score relative to occurrence	Impact score relative to detection	Final scores	Final scores	Priority
Lack of proper mold (A)	1	0.54	0.397	0.606	----	7
Sands of bad material (B)	12.692	3.999	0.397	3.154	----	1
Quickly cooling molten (C)	0.25	0.397	10.074	0.798	----	6
Fatigue of the wheels (E)	0.34	0.54	0.397	1.5	----	3
Carelessness of the operator (F)	1.852	3.999	1.852	1.241	----	4
Tolerance problem (G)	21.758	0.54	2.939	1.605	----	2
Fatigue of the help surface (H)	0.34	0.54	0.397	1.234	----	5
Not enough pressure (I)	0.25	2.519	1.852	0.445	----	8
Not firm gears in place (J)	0.27	0.735	0.397	0.4	----	9
Lack of proper mold (A)	0.812	0.574	0.354	----	0.565	8
sands of bad material (B)	9.844	3.999	0.354	----	2.824	1
Quickly cooling molten (C)	0.19	0.406	8.57	----	0.707	7
Improper feeding rate (D)	9.844	0.758	2.638	----	2.363	2
Fatigue of the wheels (E)	0.268	3.999	1.741	----	1.368	4
Carelessness of the operator (F)	1.516	0.574	2.638	----	1.162	5
Tolerance problem (G)	15.991	0.574	0.354	----	1.45	3
Fatigue of the help surface (H)	0.268	2.462	1.741	----	1.115	6
Not enough pressure (I)	0.19	0.758	0.354	----	0.401	9
Not firm gears in place (J)	0.203	0.574	0.354	----	0.364	10

THE EFFECT OF DECISION MAKER(S) ON DECISION MAKING

The decision making has always been influenced by decision maker because of some points that are almost entirely true to decision makers: first of all, the decision situation is essential to the organization, which is probably different from their prior experiences and that is why they would be spending the money. In addition, as a second point of view, there are various stakeholders involved with numerous different viewpoints, including competing demands for limited resources and differing areas of expertise associated with the decision situation they cope with. Thirdly, the organization goals, the expertise, and the information used to inform their decisions, and the possible courses of action are almost poorly organized, ambiguously defined, conflicting and uncertain.

Concerning the complexity of the modern environment and technology, it is extremely unlikely that a person or a homogeneous team will possess the depth and breadth of knowledge and experience to develop a balanced understanding of a complicated decision situation. The realism of the modern business environment is that a single "decision-maker" could be rarely found, particularly for the big decisions. All different parts of an organization will have varying responsibilities which often result in competing goals and governmental tension.

Furthermore, refining and enunciating goals and objectives for a complex decision are often so difficult. A hard and complex decision usually has at least one or more of the characteristics of complexity, uncertainty, conflicting competing objectives, and multiple stakeholders. All mentioned points force us to accept this quote "hard

Table 21. Comparison matrix of criterion relative to the goal.

Criteria	Occurrence	Detection	Severity	Priority before normalization	Priority after normalization
Occurrence	*	3	2	2.001	0.423
Detection	1/3	*	1/2	1.236	0.261
Severity	1/2	2	*	1.499	0.316

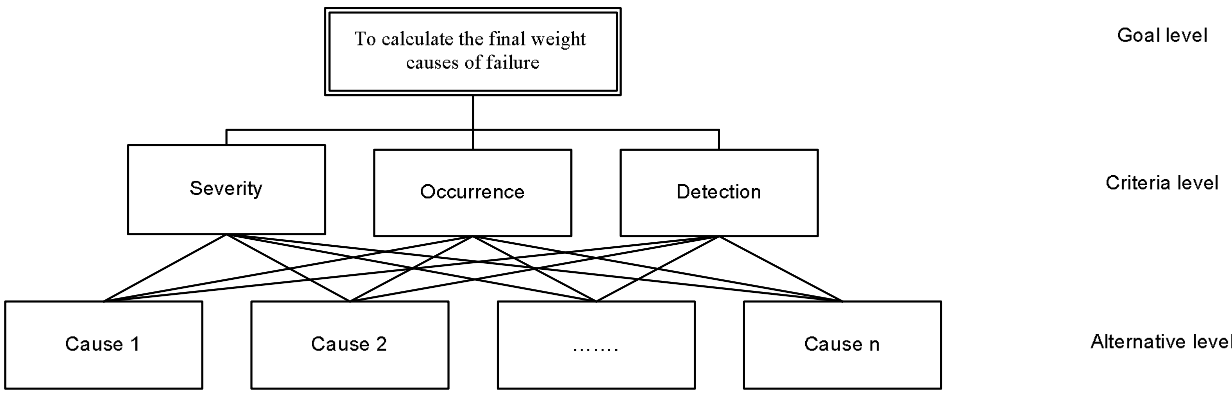

Figure 1. Hierarchical structure of failure mode and effect analysis.

decisions will keep us in business".

If we just take a look around ourselves, we will find plenty of hard decisions that surround us. Just two simple examples, a family's decision to buy a used car has multiple objectives, uncertainty, and likely multiple stakeholders. A high school senior's college decision has competing objectives with uncertain outcomes. All of these examples are a proof that the environment surrounding us is full of messy decision circumstances with significant long term consequences. Decision maker(s) has a key role to make trades among a complicated set of competing objectives over the most hard decision situations. They require a number of multi criteria decision techniques that are in hand to decision maker(s). These techniques have strengths, weaknesses, and limitations which deserve some research before they are applied in practice. They should seriously understand that these techniques have different underlying axioms and different logics about how decision models should formulated. Otherwise considering only importance increases the risk that the model will produce unreliable results.

Conclusion

Ranking or ordering things according to preference is a purely human activity. On the other hand, ranking according to importance or likelihood is a more a scientific or objective activity in which one attempts to project what can happen in the natural word. Nature has no predetermined rank for preference of alternatives on specially chosen criteria of its own. It is people who establish the criteria and make their ranking on these criteria. It frequently happens that the people find it difficult to choose a gradation for their comparative judgment. Since, the inconsistency cannot be ignored in real-life, in order to reduce notorious inconsistency of human judgment, they should consider as many pairs as possible.

In this paper, we have pointed out that rank reversal is due to the inconsistency of the inputs. We have compared the REMBRANDT system with the Wang and Elhag approach for avoiding rank reversal. The examination of the example has shown the validity and practicability of the REMBRANDT system in rank preservation. The performance of Wang and Elhag approach when used with matrices with different levels of inconsistencies was practically studied. The example also shows that the Wang and Elhag approach still suffer from rank reversal. For Wang and Elhag approach there is no rank reversal when the matrix is consistent.

Finally, we point out the REMBRANDT system is in no conflict with Barzilai and Golany's (1994) claim that no normalization can prevent rank reversal. What they proved is that the composite weights computed respectively in terms of normalized local weights and non-normalized local weights may lead to two different rankings. But they did not prove which ranking was true. If non-normalization gives a correct ranking, then normalization may give a wrong ranking; if normalization gives a correct ranking, then non-normalization may be wrong. As is well known, normalization is necessary for most

most MCDM problems and approaches in order to eliminate the dimensions of different decision attributes (Wang and Lue, 2009).

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