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Full Length Research Paper

Incorporating a cost lessening distribution policy into a production system with random scrap rate

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This paper incorporates a cost lessening product distribution policy into a production system with random scrap rate, with the purpose of cutting down producer's inventory holding cost. The present study reconsiders a production lot sizing problem examined by a prior paper and improves its replenishment lot size solution in terms of stock holding cost reduction. An *n*+1 product distribution policy is used here in lieu of the *n* multi-delivery plan adopted in prior study. Under the proposed policy, an initial installment of finished products is delivered to customer for satisfying the product demand during producer's production uptime. Then, fixed quantity *n* installments of finished items are delivered to customer at a fixed interval of time at the end of uptime. Mathematical modeling is used and as a result, the optimal production lot size solution is derived. A numerical example with analysis is provided to show practical usage of research result and demonstrate its significant savings in holding costs.

Key words: Optimization, production system, replenishment lot size, multiple deliveries, scrap.

INTRODUCTION

This study incorporates a cost reduction distribution policy into an economic production quantity (EPQ) model with random scrap rate (Chiu et al., 2009a), with the purpose of reducing manufacturer's stock holding cost. The EPQ model uses mathematical modeling to balance production setup and holding cost, to assist producers in determining the economic production lot size that minimizes total production-inventory costs (Hillier and Lieberman, 2001; Nahmias, 2009). Classic EPQ model implicitly assumes that all items produced are of perfect quality. But in real-life production systems, due to various controllable and/ or uncontrollable factors, generation of defective items is inevitable. Hence, many studies have been carried out to enhance EPQ model by addressing the issues of imperfect quality items. Rosenblatt and Lee (1986) studied an EPQ model that deals with imperfect quality. They assumed that at some random point in time the process might shift from an in-control to an out-of-control state, and a fixed percentage of imperfect quality items are produced. Approximate solutions for obtaining an optimal lot size were developed in their paper. Henig and Gerchak (1990) presented a comprehensive analysis of a general periodic review production/inventory model with random (variable) yield.

Cheung and Hausman (1997) developed an analytical model of preventive maintenance (PM) and safety stock (SS) strategies in a production environment subject to random machine breakdowns. They illustrated the trade-off between investing in the two options (PM and SS) and provided optimality conditions under which either one or both strategies should be implemented to minimize the associated cost function. Both the deterministic and exponential repair time distributions are analyzed in detail in their study. Kim et al. (2001) studied the optimal production run length and inspection schedules in a deteriorating production process. They assumed that a production process is subject to a random deterioration from the in-control state to the out-of-control state and thus produces some proportion of defective items. By minimizing overall production-inventory costs, an optimal production run length and an optimal number of inspections are derived and unique properties of proposed model are discussed (Koçyiğit et al., 2009; Baten and Kamil, 2009; Wazed et al., 2010a).

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The imperfect quality items sometimes can be reworked and repaired, so the overall production-inventory costs can be significantly reduced. For example, production processes in printed circuit board assembly, or plastic injection molding, etc., sometimes employs rework as an acceptable process in terms of level of quality. Hutchings (1976) considered a shop scheduling and control system as a two-edged sword for increasing profits. On one hand, it increases sales by creating credibility with customers through meeting shipping commitments or keeps finished goods in stock so that the sales team can beat competition with earlier delivery. The other edge of this sword cuts manufacturing costs; it reduces set-ups, balances the lines, avoids plant congestion through look-ahead and look-back techniques, reduces scrap and rework through better utilization of skilled labor, and reduces the capital costs of WIP. Where a plant is working to capacity, better shop scheduling and control can increase throughout without added facilities, thus increasing sales and reducing costs concurrently.

Chiu (2003) studied optimal lot size for an imperfect quality finite production rate model with rework and backlogging. Grosfeld-Nir and Gerchak (2002) considered multistage production systems where defective units can be reworked repeatedly at every stage. They showed that a multistage system where only one of the stages requires a set-up can be reduced to a single-stage system. They proved that it is best to make the "bottleneck" the first stage of the system and they also developed recursive algorithms for solving two- and three-stage systems. Chiu et al. (2010) studied the optimization of the finite production rate model with scrap, rework and stochastic machine breakdown. They used mathematical modeling and derived the optimal production run time that minimized such a realistic production model (Barlow and Proschan, 1965; Yum and McDowell, 1987; Chiu and Chiu, 2006; Chiu et al., 2007; Chen et al., 2010; Wazed et al., 2010b).

Continuous product issuing policy for satisfying product demand is another implicitly assumption of classic EPQ model. But, in real-life vendor-buyer integrated production-inventory-delivery system, multiple or periodic deliveries of finished products are commonly used. Goyal (1977) studied integrated inventory model for the single supplier-single customer problem. He proposed a method that is typically applicable to those inventory problems where a product is procured by a single customer from a single supplier. An example was provided to illustrate his proposed method. Many studies have since been carried out to address various aspects of supply chain optimization. Hill (1996) studied a model in which a manufacturing company purchases a raw material, manufactures a product (at a finite rate) and ships a fixed quantity of the product to a single customer at fixed and regular intervals of time, as specified by the customers, while minimizing total cost of purchasing, manufacturing and stockholding.

Viswanathan (1998) reexamined the integrated vendor -buyer inventory models with two different strategies that had been proposed in the literature for the problem: one where each replenishing quantity delivered to the buyer is identical and the other strategy where at each delivery all the inventory available with the vendor is supplied to the buyer. He showed that there is no one strategy that obtains the best solution for all possible problem parameters. Abdul-Jalbar et al. (2005) examined a multi-stage distribution/inventory system with a central warehouse and *N* retailers. Customer demand arrives at each retailer at a constant rate. The retailers replenish their inventories from the warehouse, which in turn orders from an outside supplier. It is assumed that shortages are not allowed and lead times are negligible. The goal is to determine policies which minimize the overall cost in the system, that is, the sum of the costs at each facility consisting of a fixed charge per order and a holding unit cost. We propose a heuristic procedure to compute near-optimal policies. Computational results on several randomly generated problems are reported.

Sarmah et al. (2006) reviewed literature dealing with buyer vendor coordination models that have used quantity discount as coordination mechanism under deterministic environment and classified the various models. An effort was also made to identify critical issues and scope of future research. Sarker and Diponegoro (2009) considered an optimal policy for production and procurement in a supply-chain system with multiple non-competing suppliers, a manufacturer and multiple non-identical buyers. They assumes that the manufacturer procures raw materials from suppliers, converts them to finished products and ships the products to each buyer at a fixed-interval of time over a finite planning horizon. The demand of finished product is given by buyers and the shipment size to each buyer is fixed. Their objective was to determine the production start time, the initial and ending inventory, the cycle beginning and ending time, the number of orders of raw materials in each cycle, and the number of cycles for a finite planning horizon so as to minimize the system cost. A surrogate network representation of the problem developed to obtain an efficient, optimal solution to determine the production cycle and cycle costs with predetermined shipment schedules in the planning horizon. They prescribed optimal policies for a multi-stage production and procurements for all shipments scheduled over the planning horizon.

Chiu et al. (2009a) derived the production lot size with random scrap rate and fixed quantity multiple deliveries. They assumed that fixed quantity multiple installments of the finished batch can only be delivered to customers at the end of the production. A closed-form optimal lot size solution to the problem was obtained (Schwarz, 1973; Schwarz et al., 1985; Sarker and Parija, 1994; Goyal and Nebebe, 2000; Sarker and Khan, 2001; Diponegoro and Sarker, 2006; Buscher and Lindner, 2005; Chiu et al., 2009b; Abolhasanpour et al., 2009; Golmohammadi et

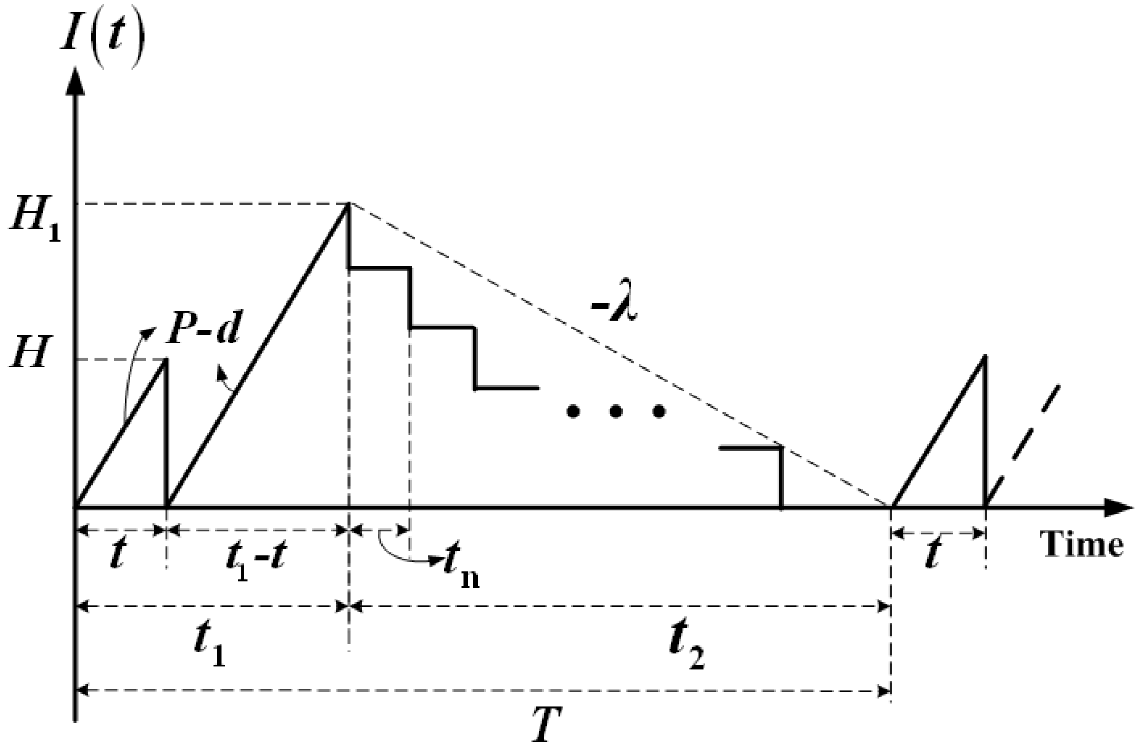

Figure 1. On-hand inventory of perfect quality items in EPQ model with random scrap rate and (n+1) delivery policy.

al., 2010).

This paper improves the lot size solution derived by Chiu et al. (2009a) by introducing a cost lessening delivery policy to their model, with the purpose of lowering producer's stock holding cost. We propose an n+1 delivery policy is proposed here in lieu of their n multi-delivery plan for this specific EPQ model with random scrap rate. The joint effects of the n+1 multi-delivery policy and the random scrap rate on the optimal replenishment batch size for this integrated EPQ model are studied.

METHODS

Modeling and formulations

This research re-examines the specific EPQ model studied by Chiu et al. (2009a), as stated earlier. The description of the proposed EPQ model is as follows. Consider a production system may produce x portion of random defective items at a production rate d. All nonconforming items are assumed to be scrap items. Under the regular operating schedule, the constant production rate P is larger than the sum of demand rate λ and production rate of defective items d. That is: $(P\text{-}d\text{-}\lambda)>0$; where d can be expressed as $d=Px$. The cost related parameters considered in the proposed model include the unit production cost C, setup cost K per production run, disposal cost per scrap item C_S, unit holding cost h, fixed delivery cost K_1 per shipment, and delivery cost C_T per item shipped to customers. Refer to Figure 1 for the on-hand inventory of perfect quality items of the proposed model. Additional notation is listed as follows:

Q = production lot size to be determined for each cycle,
t = the production time needed for producing enough perfect items for satisfying product demand during the production uptime t_1,
t_1 = the production uptime for the proposed EPQ model,
t_2 = time required for delivering the remaining perfect quality finished products,
T = cycle length,
H = the level of on-hand inventory in units for satisfying product demand during manufacturer's regular production time t_1,
H_1 = maximum level of on-hand inventory in units when regular production ends,
n = number of fixed quantity installments of the rest of finished batch to be delivered to customer during t_2,
t_n = a fixed interval of time between each installment of products delivered during t_2,
$I(t)$ = on-hand inventory of perfect quality items at time t,
$I_d(t)$ = on-hand inventory of scrap items at time t,
$TC(Q)$ = total production-inventory-delivery costs per cycle for the proposed model,
$TC_1(Q)$ = total production-inventory-delivery costs per cycle for the special case model,
$E[TCU(Q)]$ = the long-run average costs per unit time for the proposed model,
$E[TCU_1(Q)]$ = the long-run average costs per unit time for the special case.

Under the proposed n+1 delivery policy, an initial installment of finished (perfect quality) products is delivered to customer for satisfying the demand during production uptime t_1. At the end of production when the rest of lot is completion, fixed quantity n installments of the finished items are delivered to customer at a fixed interval of time.

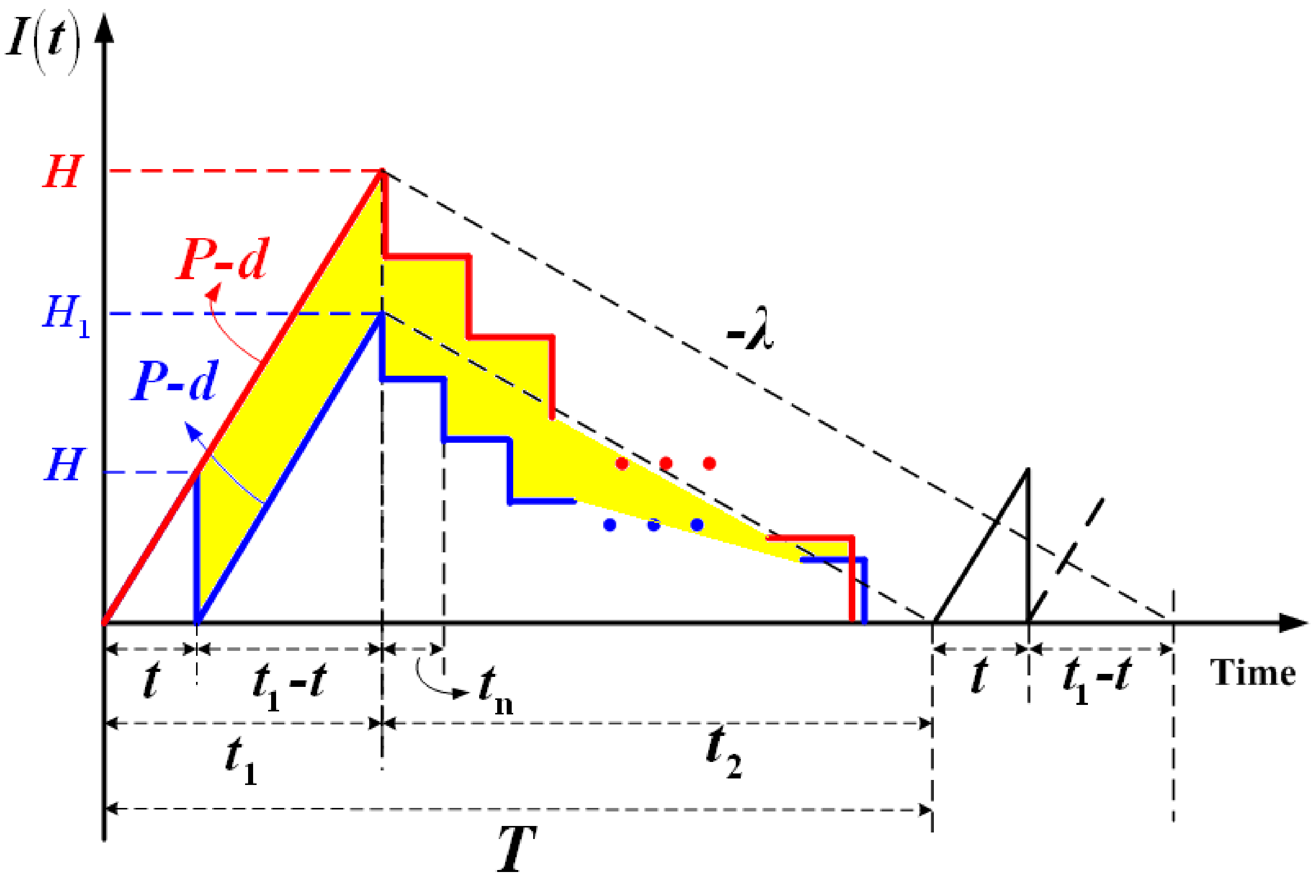

Figure 2. Expected reduction in stock holding costs (in yellow/shade) of the proposed model in comparison with Chiu et al.'s model (2009a).

Such an n+1 delivery policy is intended to reduce the supplier's stock holding cost. Figure 2 depicts the expected reduction in producer's stock holding costs (in yellow/shade) for the proposed model (in blue) in comparison with Chiu et al.'s model (2009a) (in red)From Figure 1 and the assumption of the proposed model, the following expressions can be derived accordingly (as in Chiu et al. (2009a)):

$$t_1 = \frac{Q}{P} \quad (1)$$

$$t = \frac{\lambda t_1}{P-d} \quad (2)$$

$$H = (P-d)t = \lambda t_1 \quad (3)$$

$$H_1 = Q(1-x) - \lambda t_1 \quad (4)$$

$$t_2 = nt_n = T - t_1 \quad (5)$$

$$T = t_1 + t_2 = \frac{Q}{\lambda} \quad (6)$$

The on-hand inventory of random scrap items during production uptime t_1 is displayed in Figure 3. It is noted that the maximum level of scrap items is dt_1. Total production-inventory-delivery costs per cycle $TC(Q)$ consists of the variable production cost, the setup cost, variable disposal cost, (n+1) fixed distribution costs and variable delivery cost, holding cost for perfect quality items during production uptime t_1, holding cost for scrap items during t_1, and holding cost for finished goods during the delivery time t_2 where n fixed-quantity installments of the finished batch are delivered to customers at a fixed interval of time (for computation of the last term refer to Appendix-2 of Chiu et al. (2009b)).

$$TC(Q) = CQ + K + C_S[xQ] + (n+1)K_1 + C_T Q(1-x) + h\left[\frac{H}{2}(t) + \frac{H_1}{2}(t_1 - t) + \frac{dt_1}{2}(t_1)\right] + h\left[\left(\frac{n-1}{2n}\right)H_1 t_2\right] \quad (7)$$

Substituting all related parameters from Equations (1) to (6) in $TC(Q)$, one has

$$TC(Q) = CQ + K + (n+1)K_1 + C_S(xQ) + C_T[Q(1-x)] + \frac{hQ^2}{2}\left\{\frac{2\lambda^2}{P^3(1-x)} - \frac{\lambda}{P^2} + \left(1-\frac{1}{n}\right)\left[\frac{(1-x)^2}{\lambda} - \frac{(1-2x)}{P}\right] - \left(\frac{1}{n}\right)\left[\frac{1}{P}\left(\frac{\lambda}{P} - 1\right)\right]\right\} \quad (8)$$

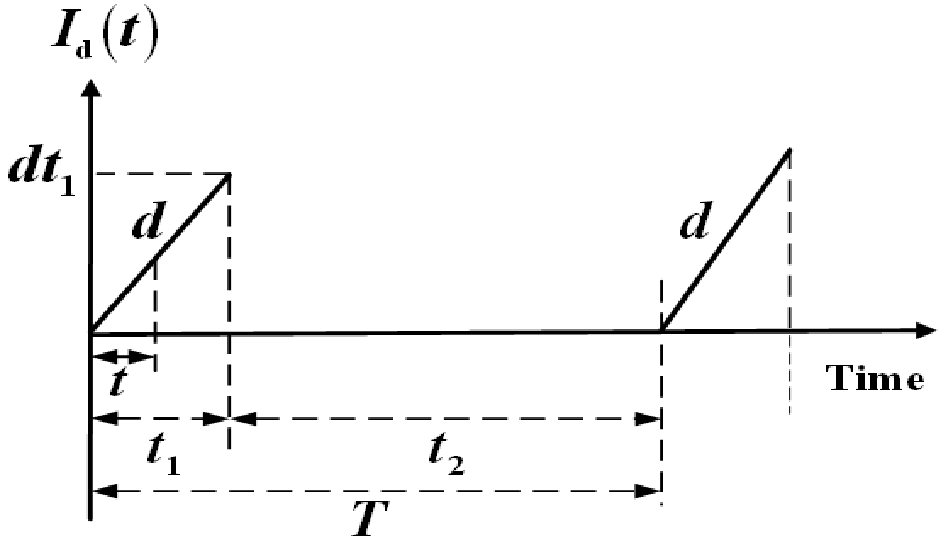

Figure 3. On-hand inventory of scrap items in the proposed EPQ model with (n+1) delivery policy.

Or

$$TC(Q) = CQ + K + (n+1)K_1 + C_S(xQ) + C_T\left[Q(1-x)\right] + \frac{hQ^2}{2}\left\{\begin{array}{l} \dfrac{2\lambda^2}{P^3(1-x)} - \dfrac{\lambda}{P^2} + \dfrac{(1-x)^2}{\lambda} - \dfrac{(1-2x)}{P} \\ -\left(\dfrac{1}{n}\right)\left[\dfrac{(1-x)^2}{\lambda} - \dfrac{2(1-x)}{P} + \dfrac{\lambda}{P^2}\right] \end{array}\right\} \quad (9)$$

Taking the randomness of defective rate x into account (where x is assumed to be a random variable with a known probability density function), one uses the expected values of x in the related cost analysis. After derivations, the expected production-inventory-delivery cost E[$TCU(Q)$] becomes

$$\frac{E\left[TC(Q)\right]}{E[T]} = \frac{C\lambda}{1-E(x)} + \frac{\left[(n+1)K_1 + K\right]\lambda}{Q\left[1-E(x)\right]} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda + \frac{hQ\lambda}{2\left[1-E(x)\right]}\left\{\begin{array}{l} \dfrac{2\lambda^2}{P^3}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda}{P^2} \\ + \dfrac{\left[1-E(x)\right]^2}{\lambda} - \dfrac{\left[1-2E(x)\right]}{P} \\ -\left(\dfrac{1}{n}\right)\left[\dfrac{\left[1-E(x)\right]^2}{\lambda} - \dfrac{2\left[1-E(x)\right]}{P} + \dfrac{\lambda}{P^2}\right] \end{array}\right\} \quad (10)$$

With further derivations, one has the expected optimal E[$TCU(Q)$] as follows:

$$E\left[TCU(Q)\right] = + \frac{C\lambda}{1-E(x)} + \frac{\left[(n+1)K_1 + K\right]\lambda}{Q\left[1-E(x)\right]} + \frac{C_S E(x)\lambda}{1-E(x)} + C_T\lambda + \frac{hQ}{2}\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2\left[1-E(x)\right]} \\ + \left[1-E(x)\right] - \dfrac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]} \\ -\left(\dfrac{1}{n}\right)\left[\left[1-E(x)\right] - \dfrac{2\lambda}{P} + \dfrac{\lambda^2}{P^2\left[1-E(x)\right]}\right] \end{array}\right\} \quad (11)$$

The optimal replenishment policy

The optimal replenishment lot size can be obtained by minimizing the expected cost function *E[TCU(Q)]*. Differentiating *E[TCU(Q)]* with respect to Q, the first and the second derivatives of *E[TCU(Q)]* are shown in Equations (12) and (13).

$$\frac{dE\left[TCU(Q)\right]}{dQ} = \frac{-\left[(n+1)K_1 + K\right]\lambda}{Q^2\left[1-E(x)\right]} + \frac{h}{2}\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2\left[1-E(x)\right]} \\ + \left[1-E(x)\right] - \dfrac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]} \\ -\left(\dfrac{1}{n}\right)\left[\left[1-E(x)\right] - \dfrac{2\lambda}{P} + \dfrac{\lambda^2}{P^2\left[1-E(x)\right]}\right] \end{array}\right\} \quad (12)$$

$$\frac{d^2E\left[TCU(Q)\right]}{dQ^2} = \frac{2\left[(n+1)K_1 + K\right]\lambda}{Q^3\left[1-E(x)\right]} \quad (13)$$

It is noted that Equation (13) is resulting positive because K, n, K_1, λ, (1-$E[x]$), and Q are all positive. The second derivative of *E[TCU(Q)]* with respect to Q is greater than zero, and hence *E[TCU(Q)]* is a convex function for all Q different from zero. The optimal production lot size Q^* can be obtained by setting the first derivative of *E[TCU(Q)]* equal to zero.

$$\text{Let } \frac{dE\left[TCU(Q)\right]}{dQ} = \frac{-\left[(n+1)K_1 + K\right]\lambda}{Q^2\left[1-E(x)\right]} + \frac{h}{2}\left\{\begin{array}{l} \dfrac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\dfrac{1}{1-x}\right) - \dfrac{\lambda^2}{P^2\left[1-E(x)\right]} \\ + \left[1-E(x)\right] - \dfrac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]} \\ -\left(\dfrac{1}{n}\right)\left[\left[1-E(x)\right] - \dfrac{2\lambda}{P} + \dfrac{\lambda^2}{P^2\left[1-E(x)\right]}\right] \end{array}\right\} = 0 \quad (14)$$

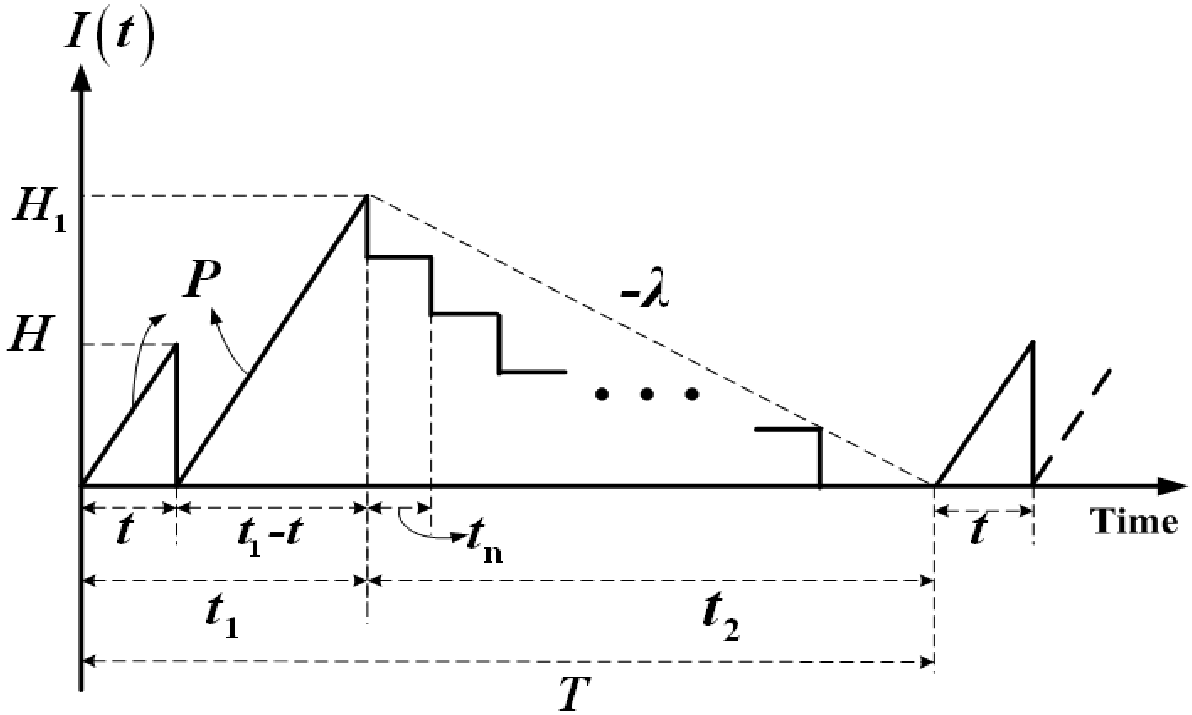

Figure 4. On-hand inventory of finished items in EPQ model with (n+1) delivery policy – the special case (x=0) model.

With rearrangement, one obtains the following:

$$\frac{\left[(n+1)K_1+K\right]\lambda}{Q^2\left[1-E(x)\right]}= +\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3\left[1-E(x)\right]}E\left(\frac{1}{1-x}\right)-\frac{\lambda^2}{P^2\left[1-E(x)\right]}\\+\left[1-E(x)\right]-\frac{\lambda\left[1-2E(x)\right]}{P\left[1-E(x)\right]}\\-\left(\frac{1}{n}\right)\left[\left[1-E(x)\right]-\frac{2\lambda}{P}+\frac{\lambda^2}{P^2\left[1-E(x)\right]}\right]\end{array}\right\} \qquad (15)$$

Or

$$Q^2=\frac{\left[(n+1)K_1+K\right]\lambda}{\frac{h}{2}\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)-\frac{\lambda^2}{P^2}\\+\left[1-E(x)\right]^2-\frac{\lambda\left[1-2E(x)\right]}{P}\\-\left(\frac{1}{n}\right)\left[\left[1-E(x)\right]^2-\frac{2\lambda\left[1-E(x)\right]}{P}+\frac{\lambda^2}{P^2}\right]\end{array}\right\}} \qquad (16)$$

With further rearrangements, one obtains the following optimal replenishment lot size:

$$Q^*=\sqrt{\frac{2\lambda\left[(n+1)K_1+K\right]}{h\left\{\begin{array}{l}\frac{2\lambda^3}{P^3}E\left(\frac{1}{1-x}\right)-\frac{\lambda^2}{P^2}\left(1+\frac{1}{n}\right)-\frac{\lambda\left[1-2E(x)\right]}{P}\\+\left[1-E(x)\right]^2\left(1-\frac{1}{n}\right)+\left(\frac{1}{n}\right)\left[\frac{2\lambda\left[1-E(x)\right]}{P}\right]\end{array}\right\}}} \qquad (17)$$

Special case

Suppose all items produced are of perfect quality (i.e. x=0). Figure 4 depicts the on-hand inventory of finished items for this special case model. Let $TC_1(Q)$ denote total production-inventory-delivery cost per cycle:

$$TC_1(Q)=K+CQ+(n+1)K_1+C_TQ +h\left[\frac{H}{2}(t)+\frac{H_1}{2}(t_1-t)\right]+h\left[\left(\frac{n-1}{2n}\right)H_1t_2\right] \qquad (18)$$

By using the similar derivations, one obtains $E[TCU_1(Q)]$ as follows.

$$E\left[TCU_1(Q)\right]=C\lambda+\frac{\left[(n+1)K_1+K\right]\lambda}{Q}+C_T\lambda +\frac{hQ}{2}\left[\frac{\lambda^2}{P^2}\left(\frac{2\lambda}{P}-1\right)+\left(1-\frac{\lambda}{P}\right)\left[1-\left(\frac{1}{n}\right)\left(1-\frac{\lambda}{P}\right)\right]\right] \qquad (19)$$

The second derivative of $E[TCU_1(Q)]$ is shown in Eq. (20). One verifies that $E[TCU_1(Q)]$ is a convex function.

$$\frac{d^2E\left[TCU_1(Q)\right]}{dQ^2}=\frac{2\left[(n+1)K_1+K\right]\lambda}{Q^3} \qquad (20)$$

By setting the first derivative of $E[TCU(Q)]$ equal to zero, the following optimal production lot size Q^* can be obtained.

$$Q^*=\sqrt{\frac{2\left[(n+1)K_1+K\right]\lambda}{h\left[\frac{\lambda^2}{P^2}\left(\frac{2\lambda}{P}-1\right)+\left(1-\frac{\lambda}{P}\right)\left[1-\left(\frac{1}{n}\right)\left(1-\frac{\lambda}{P}\right)\right]\right]}} \qquad (21)$$

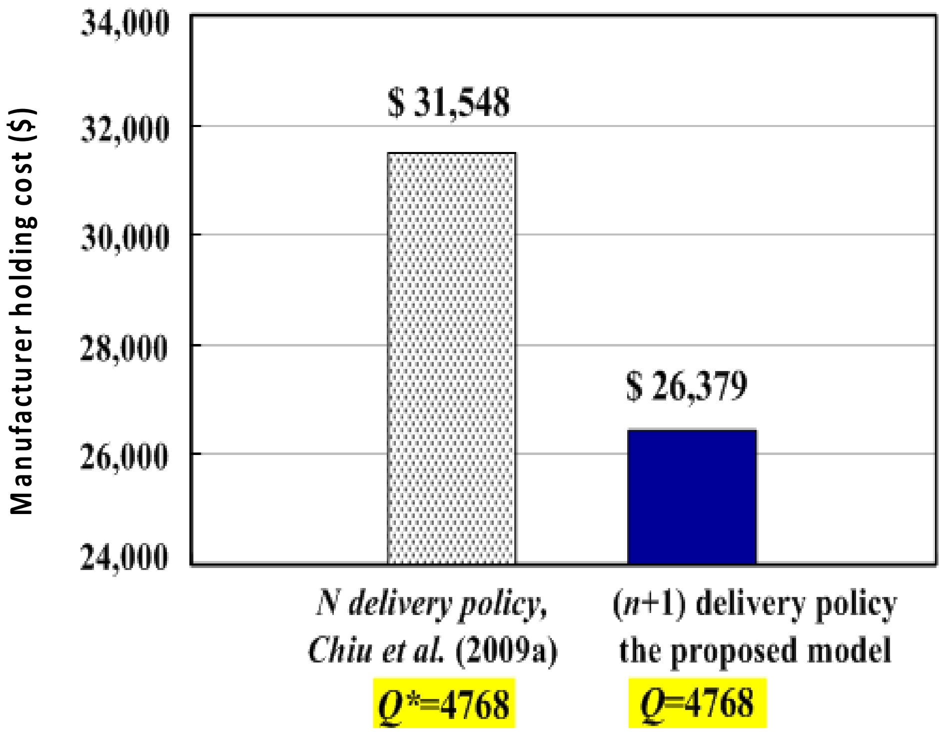

Figure 5. Comparison of the producer's stock holding cost for the proposed (n+1) delivery policy to that of Chiu et al. (2009a).

NUMERICAL EXAMPLE AND DISCUSSION

This section adopts the same numerical example as in (Chiu et al., 2009a) for the purpose of comparison. Consider that a product can be manufactured at a rate of 60,000 units per year and this item has experienced a flat demand rate of 3,400 units per year. During production process a random defective rate is assumed to be uniformly distributed over the interval [0, 0.3] and all defective items are considered to be scrap. Other parameters include C=$100 per item; K= $20,000 per production run; C_S=$20 per scrap item; h=$20 per item per year; n=4 installments of the finished batch are delivered per cycle; K_1=$4,400 per shipment; and C_T=$0.1 per item delivered. In order to show practical usages of our research results, the following two different scenarios are demonstrated, respectively.

Scenario 1: Let total number of deliveries remain 4 (that is, n=4 as was used in Chiu et al. (2009a)).

For the proposed model, it is (n+1) =4. An initial installment of finished products is distributed to customer during t_1 for satisfying the product demand during producer's production uptime. Then, at the end of production, fixed quantity 3 other installments of finished items are delivered to customer at a fixed interval of time. Also, for the purpose of comparison, we use the lot-size solution Q=4,768 (from Chiu et al. 2009a) in calculating expected production-inventory- delivery cost (Equation (11) of the proposed model) and obtain E[TCU(4,768)]=$470,263. It is noted that that there is a reduction in manufacturer holding costs amounts to $5,169 (Figure 5) or 3.97% of total other related costs (i.e. E[TCU(Q)]-(λC): total cost excludes the variable production cost).

Scenario 2: Let total number of deliveries remain 4 again (that is (n+1)=4 in our model). By applying Equations (17) and (11), one obtains the optimal replenishment lot size Q^*=5,214 and the expected total costs E[TCU(Q^*)]=$470,032, respectively. One notes that the overall reduction in production- inventory-delivery costs is $5,401, or 4.15% of total other related costs. Figure 6 depicts the variation of replenishment lot size effects on E[TCU(Q)] and on different components of E[TCU(Q)].

Conclusions

This paper incorporates a cost lessening product distribution policy into an imperfect EPQ model with random scrap rate (Chiu et al., 2009a), for the purpose of lowering producer's stock holding cost. Chiu et al. (2009a) derived the production lot size for an EPQ model with the random scrap rate and fixed quantity multiple deliveries. They assumed that fixed quantity multiple

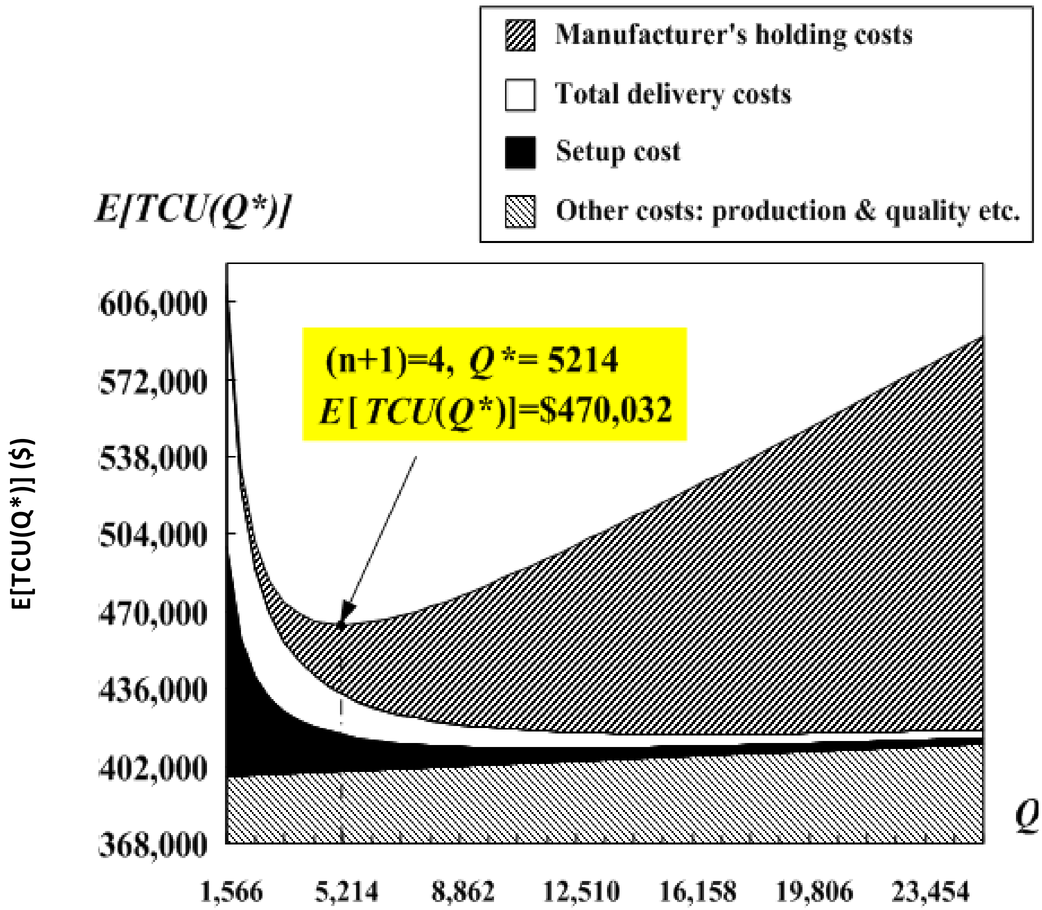

Figure 6. Variation of the replenishment lot size effects on E[*TCU*(*Q*)] and on different components of E[*TCU*(*Q*)].

installments of the finished batch can only be delivered to customers if the production of the whole lot is completed. With the purpose of reducing supplier's stock holding cost, this paper extends Chiu et al.'s model (2009a) and proposes an *n*+1 delivery policy in lieu of their *n* multi-delivery plan.

Mathematical modeling and analysis is used and expected integrated production-inventory-delivery cost per unit time is derived and proved to be convex function. The closed-form optimal lot size solution to the problem is derived. A numerical example is provided to show practical usage of our research result and demonstrate its significant savings in producer's stock holding cost. For future research, one of the interesting issues is to investigate effect of multiple customers on the lot size decision for the same model.

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