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Economic selection of Dodge-Romig AOQL sampling plan under the quality investment and inspection error

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In this article, the author proposes a modification of Dodge-Romig single rectifying inspection plan with average outgoing quality limit (AOQL) protection. The quality investment and inspection error are considered in the modified model. The optimal parameters of sampling inspection plan and quality investment level are simultaneously determined by minimizing the expected total cost of product under the specified AOQL value. Finally, the comparison of solution between the modified model with/without inspection error will be provided for illustration.

Key words: Dodge-Romig Single Rectifying Inspection Pan, average outgoing quality limit (AOQL), quality investment, inspection error.

INTRODUCTION

Sampling inspection plan is one of the fields for statistical quality control. It can be applied in procurement, production, and shipment of product when the product quality is not stable. The lot-by-lot attribute single sampling plans (SSP) are easy to adopt for evaluating the quality of lot. In 1959, Dodge-Romig provided the rectifying SSP and double sampling plans for attributes with the protection of lot tolerance percent defective (LTPD) or average outgoing quality limit (AOQL). For the attribute rectifying sampling plans with AOQL protection, they are derived to provide assurance that the long-run average lots, will be no more worse than the indexed AOQL value. Klufa (1994, 1997) further presented the modified Dodge-Romig's model based on variable single sampling inspection plan. The classical Dodge-Romig (1959) AOQL SSP are based on the following assumptions:

1. The manufacturing process is normally in binomial control with a process average fraction defective equal to $\bar{p}$.
2. Inspection is rectifying and rejected lots are totally inspected.
3. To make sure that the average quality of his product is satisfactory, the producer chooses an AOQL value and considers only sampling plans satisfying this specification.
4. Among plans having the specified AOQL, the producer chooses the one minimizing average total inspection (ATI) for product of process average fraction defective.
5. Inspection is perfect without error.

One assumption of the classical Dodge-Romig (1959) AOQL SSP is perfect inspection without error. However, the inspection error usually occurs in industrial or medical application.
If we just use the product inspection for providing the quality assurance, then it is a short-term method. For modern industrial statistics, we usually address the preventive method for quality improvement and adopt the

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process control and quality design for improving product quality and satisfying the need of customer. The on-line 100% rectifying inspection can be used as a short-term available method for controlling the product quality shipped to the customers. However, quality investment is an available method for improving the process parameters in the long-term. For example, one can buy a new machine for manufacturing the product and address the continuous education training for personnel. Hong et al. (1993), Ganeshan et al. (2001), and Chen and Tsou (2003) have presented the exponential reduction of process mean and standard deviation as the function of quality investment. Abdul-kader et al. (2010) further adopted Chen and Tsou's (2003) quality investment function for determining the optimum quality investment and corresponding improved process mean and standard deviation. Chen (2011a, 2011b) further extended Chen and Tsou's (2003) method for designing the sampling inspection plan under the economic selection.

In this paper, the author proposes a modified Dodge-Romig (1959) AOQL SSP under the quality investment and inspection error. It is an extension of Chen's (2011a) work. The objective of this research is to integrate the process improvement and sampling inspection for obtaining the minimum expected total cost of product when the inspection error occurs. The motivation behind this work stems from the fact that neglecting the effect of inspection error should underestimate the expected total cost of product. This paper also presents the effect of inspection error on the modified Dodge-Romig (1959) model. The solution procedure for obtaining the optimal parameters of Dodge-Romig AOQL SSP and quality investment level are presented. Finally, the comparison of solution between the modified model with/without inspection error will be provided for illustration.

DODGE-ROMIG AOQL SSP WITH INSPECTION ERROR

Consider the inspection error exists for Dodge-Romig (1959) AOQL SSP. According to Beaing and Case (1981), we have the modified Dodge-Romig (1959) AOQL SSP as follows:

Minimize $$ATI_e = \frac{n+(N-n)(1-P'_{ae})}{1-p_e} \qquad (1)$$

Subject to $$\max_{0\le p\le 1} AOQ_e = p_L \qquad (2)$$

where

$$P'_{ae} = \sum_{x=0}^{c} \frac{e^{-n\overline{p_e}}(n\overline{p_e})^x}{x!} \qquad (3)$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-np_e}(np_e)^x}{x!} \qquad (4)$$

$$\overline{p_e} = \overline{p}(1-e_2)+(1-\overline{p})e_1 \qquad (5)$$

P_{ae} is the acceptance probability of inspection lot with average fraction defective $\overline{p}_e$; $AOQ_e = \frac{npe_2 + p(N-n)(1-p_e)P_{ae} + p(N-n)(1-P_{ae})e_2}{N(1-p_e)}$; p is the true fraction defective; p_e is the apparent fraction defective, $p_e = p(1-e_2)+(1-p)e_1$; e_1 is the probability that a good item classified as a defective; e_2 is the probability that a defective item classified as good; $\overline{p}$ is the average fraction defective; *n* is the sample size; *X* is the number of non-conforming items found in the sample size *n*; *c* is the acceptance number; p_L is the specified AOQL value.

Case et al. (1973) demonstrated that, in general, the AOQ function with the inspection error, AOQ_e, will not be unimodal. Throughout this paper, we still use the generally accepted definition of the $AOQL_e$ as the first mode of the AOQ_e function, even though higher AOQ_e values may be realized as the process fraction defective increases. From Appendix, we can obtain some combinations of parameters (*c*, *n*) that satisfies Equation (2). The unique combination of parameters (c^*, n^*) can minimize the objective function ATI_e is the optimal solution.

MODIFIED DODGE-ROMIG AOQL SSP WITH QUALITY INVESTMENT AND INSPECTION ERROR

Similarly to Chen (2011a), we have the following modified Dodge-Romig (1959) AOQL SSP with quality investment and inspection error as follows:

Minimize $$TC_f = ATI_e \cdot TC_1 + I \qquad (6)$$

Subject to $$\max_{0\le p\le 1} AOQ_e = p_L \qquad (7)$$

where

$$ATI_e = [n + (N-n)(1-P_{ae})]/(1-p_e) \quad (8)$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-np_e}(np_e)^x}{x!} \quad (9)$$

$$p_e = (1-p)e_1 + p(1-e_2) \quad (10)$$

$$p = 1 - \int_{LSL}^{USL} f(y,I)dy = 1 - [\Phi(\frac{USL-\mu_I}{\sigma_I}) - \Phi(\frac{LSL-\mu_I}{\sigma_I})] \quad (11)$$

$$TC_1 = \int_{LSL}^{USL} k(y-y_0)^2 f(y,I)dy + (1-p)\cdot C_r + C_i \quad (12)$$

$$\int_{LSL}^{USL} k(y-y_0)^2 f(y,I)dy = k\left\{[(\mu_I - y_0)^2 + \sigma_I^2]\cdot\left[\Phi\left(\frac{USL-\mu_I}{\sigma_I}\right) - \Phi\left(\frac{LSL-\mu_I}{\sigma_I}\right)\right]\right.$$

$$\left. + \sigma_I\left[(\mu_I - 2y_0 + LSL)\phi\left(\frac{LSL-\mu_I}{\sigma_I}\right) - (\mu_I - 2y_0 + USL)\phi\left(\frac{USL-\mu_I}{\sigma_I}\right)\right]\right\} \quad (13)$$

$$f(y,I) = \frac{1}{\sqrt{2\pi}\sigma_I}\exp[-\frac{1}{2}(\frac{y-\mu_I}{\sigma_I})^2] \quad (14)$$

$$\mu_I^2 = \mu_T^2 + (\mu_0^2 - \mu_T^2)\exp(-\beta I) \quad (15)$$

$$\sigma_I^2 = \sigma_T^2 + (\sigma_0^2 - \sigma_T^2)\exp(-\alpha I) \quad (16)$$

α is the exponential reduction coefficient of process standard deviation for the function of quality investment, $\alpha > 0$; β is the exponential reduction coefficient of process mean for the function of quality investment, $\beta > 0$; $\Phi(\cdot)$ is the cumulative distribution function of standard normal random variable ; $\phi(\cdot)$ is the probability density function of standard normal random variable; *k* is the quality loss coefficient; *Y*, is normally distributed with known process mean μ_0 and process standard deviation σ_0 ; y_0 is the target value of product; μ_T is the target value of process mean; σ_T is the target value of process standard deviation; μ_I is the improved process mean; σ_I is the improved process standard deviation; LSL is the lower specification limit of product; USL is the upper specification limit of product; C_r is the replacement cost per unit for a non-conformance product; C_i is the inspection cost per unit; *I* is the quality improvement.

For Equation (6), TC_1 denotes the expected product cost per unit which includes the expected quality loss within specification limits, the expected replacement cost per unit out of specification limits, and the unit inspection cost. TC_f denotes the expected total cost of product which includes the quality investment cost and the expected product cost per inspection lot.

The solution procedure of Equations (6) to (7) is as follows:

Step 1. From Appendix, find the possible combination of parameters (c, n) satisfying Equation (7).

Step 2. For a given *c* and its corresponding *n* from Step1, one can adopt the direct search method for obtaining the optimal quality investment level satisfying Equation (6).

Step 3. Let *c* = *c*+1. Repeat Step 2 until obtaining the optimal parameters ($c^*, n^*, \mu_I^*, \sigma_I^*, I^*$) with minimum TC_f .

NUMERICAL EXAMPLE AND SENSITIVITY ANALYSIS OF PARAMETERS

Numerical example

Assume that the declining exponential function of the quality investment can be used for describing the improved process mean and standard deviation. By regression analysis of the historical production data, the exponential reduction coefficients are $\alpha = 0.01$ and $\beta = 0.05$. Consider the normal quality characteristic with known process mean $\mu_0 = 10$ and standard deviation $\sigma_0 = 0.5$. The lower specification limit of product is LSL = 9.24, the upper specification limit of product is USL = 10.56, the target value of product $y_0 = 9.9,$ and the quality loss coefficient k = 5. The replacement cost for non-conformance product is $C_r = 2$ and the inspection cost per unit is $C_i = 0.1$. The product lot size N = 2000. The average outgoing quality limit AOQL = 1%. The probability for classifying a good unit as a defective is $e_1 = 0.01$ and the probability for classifying a defective unit as a good one is $e_2 = 0.02$. The long-term improved process mean is $\mu_T = 9.9$ and improved process standard deviation is $\sigma_T = 0$. We would like to adopt Dodge-Romig (1959) AOQL SSP for controlling the output quality of product by integrating the concept of quality investment.

By the aforementioned solution procedure, the solution for modified Dodge-Romig (1959) AOQL SSP with quality investment and inspection error is ($c^*, n^*, \mu_I^*, \sigma_I^*, I^*$) = (4,146, 9.9, 0.212, 171.73) with $TC_f = 238.31$. The solution for Chen (2011a) is ($c^*, n^*, \mu_I^*, \sigma_I^*, I^*$) = (1, 81, 9.9, 0.214, 169.60) with $TC_f = 204.04$.

Sensitivity analysis

Tables 1 to 2 list ±20% change for the parameter values and present the effect on the acceptance number, sample size, improved process mean, improved process standard deviation, quality investment level, and expected total cost of product. If the change percentage of the expected total profit of product is greater than10%, then the parameter has a significant effect on the expected total profit of product. Figures 1 to 11 present the effect of parameters on the quality investment level and the expected total cost of product for modified model with/without inspection error. From Tables 1 to 2 and Figures 1 to 11, we have the following conclusions:

1. As the known process standard deviation, σ_0, increases, the quality investment level and expected total cost of product also increase. The known process standard deviation has a major effect on the quality investment level and expected total cost of product for both modified models.
2. As the exponential reduction coefficient of process standard deviation for quality improvement, α, increases, the quality investment level and expected total cost of product decrease.
The exponential reduction coefficient of process standard deviation for quality improvement has a significant effect on the quality investment level and expected total cost of product for both modified models.
3. The improved process mean, μ_I, is equal to the target value of product for both modified models.
4. The probabilities of inspection error, e_1 and e_2, have the slight effects on the quality investment level and expected total cost of product for modified model with inspection error.
5. As the average outgoing quality limit, AOQL, increases, the quality investment level and expected total cost of product decrease. The average outgoing quality limit has a slight effect on the quality investment level and expected total cost of product for modified model with inspection error.
6. The modified model with quality investment and inspection error needs more acceptance number, sample size, quality investment level, and expected total cost of product than those of one without inspection error.

The aforementioned results show that (1) we need to input more resource in quality improvement for decreasing the effect of known process standard deviation and the effect of exponential reduction coefficient of process standard deviation; (2) we need to have the optimum target value for the long-term quality improvement; (3) the modified model without inspection error maybe underestimate the quality investment level and the expected total cost of product.

CONCLUSIONS

In this paper, the modified Dodge-Romig (1959) AOQL SSP under the quality investment and inspection error has been proposed.

It considers an integrated optimum problem for the process improvement and inspection plan. From the aforementioned numerical results, one has the conclusions that (1) the exponential reduction coefficient of process standard deviation for quality improvement has a major effect on the quality investment level and the expected total cost of product; (2) the modified model with quality investment and inspection error needs more acceptance number, sample size, quality investment level, and expected total cost of product than those of one without inspection error.

The managerial implications of this work are that the joint optimization of product inspection and quality investment can improve the product quality/service quality shipped to the customer and decrease the expected total cost of product for the manufacturer. The extension to the economic design of modified Dodge-Romig (1959) with variable sampling plan, quality investment, and measurement error may be left for further study.

Table 1. The effect of parameters on optimal solution for modified Dodge-Romig (1959) AOQL SSP with inspection error.

N	***c***	***n***	μ_I	σ_I	***I***	TC_f
1600	4	143	9.9	0.217	167.06	231.77
2400	4	147	9.9	0.208	175.08	242.98
k	*c*	*n*	μ_I	σ_I	*I*	TC_f
4	4	146	9.9	0.216	168.28	228.98
6	4	146	9.9	0.208	175.07	247.09
μ_0	*c*	*n*	μ_I	σ_I	*I*	TC_f
9	4	146	9.9	0.212	171.84	238.31
11	4	146	9.9	0.212	171.79	238.31
σ_0	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.4	4	146	9.9	0.212	127.22	193.68
0.6	4	146	9.9	0.212	208.27	274.77
C_r	*c*	*n*	μ_I	σ_I	*I*	TC_f
1.6	4	146	9.9	0.212	171.63	238.16
2.4	4	146	9.9	0.212	171.87	238.46
C_i	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.08	4	146	9.9	0.213	171.05	234.18
0.12	4	146	9.9	0.211	172.51	242.41
α	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.008	4	146	9.9	0.217	208.83	280.64
0.012	4	146	9.9	0.207	146.61	209.35
β	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.04	4	146	9.9	0.212	171.79	238.31
0.06	4	146	9.9	0.212	171.74	238.31
AOQL	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.8%	4	163	9.9	0.205	178.04	251.70
1.2%	4	132	9.9	0.218	166.29	227.33
e_1	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.8%	3	126	9.9	0.210	173.73	235.30
1.2%	5	163	9.9	0.213	170.41	241.54
e_2	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.8%	4	145	9.9	0.212	171.46	237.57
1.2%	4	147	9.9	0.212	172.05	239.05

Table 2. The effect of parameters on optimal solution for modified Dodge-Romig (1959) AOQL SSP without inspection error.

N	***c***	***n***	μ_I	σ_I	***I***	TC_f
1600	1	81	9.9	0.217	166.90	201.67
2400	1	81	9.9	0.212	171.63	205.52
k	***c***	***n***	μ_I	σ_I	***I***	TC_f
4	1	81	9.9	0.216	167.61	199.19
6	1	81	9.9	0.212	171.35	208.62
μ_0	*c*	*n*	μ_I	σ_I	*I*	TC_f
9	1	81	9.9	0.214	169.62	204.04
11	1	81	9.9	0.214	169.62	204.04
σ_0	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.4	1	81	9.9	0.214	125.06	159.42
0.6	1	81	9.9	0.214	206.09	240.51
C_r	*c*	*n*	μ_I	σ_I	*I*	TC_f
1.6	1	81	9.9	0.214	169.58	203.96
2.4	1	81	9.9	0.214	169.73	204.13
C_i	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.08	1	81	9.9	0.215	168.98	201.93
0.12	1	81	9.9	0.213	170.23	206.13
α	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.008	1	81	9.9	0.217	208.46	246.08
0.012	1	81	9.9	0.212	143.32	175.58
β	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.04	1	81	9.9	0.214	169.63	204.04
0.06	1	81	9.9	0.214	169.63	204.04
AOQL	*c*	*n*	μ_I	σ_I	*I*	TC_f
0.8%	1	81	9.9	0.209	174.01	213.11
1.2%	1	81	9.9	0.218	165.88	197.11

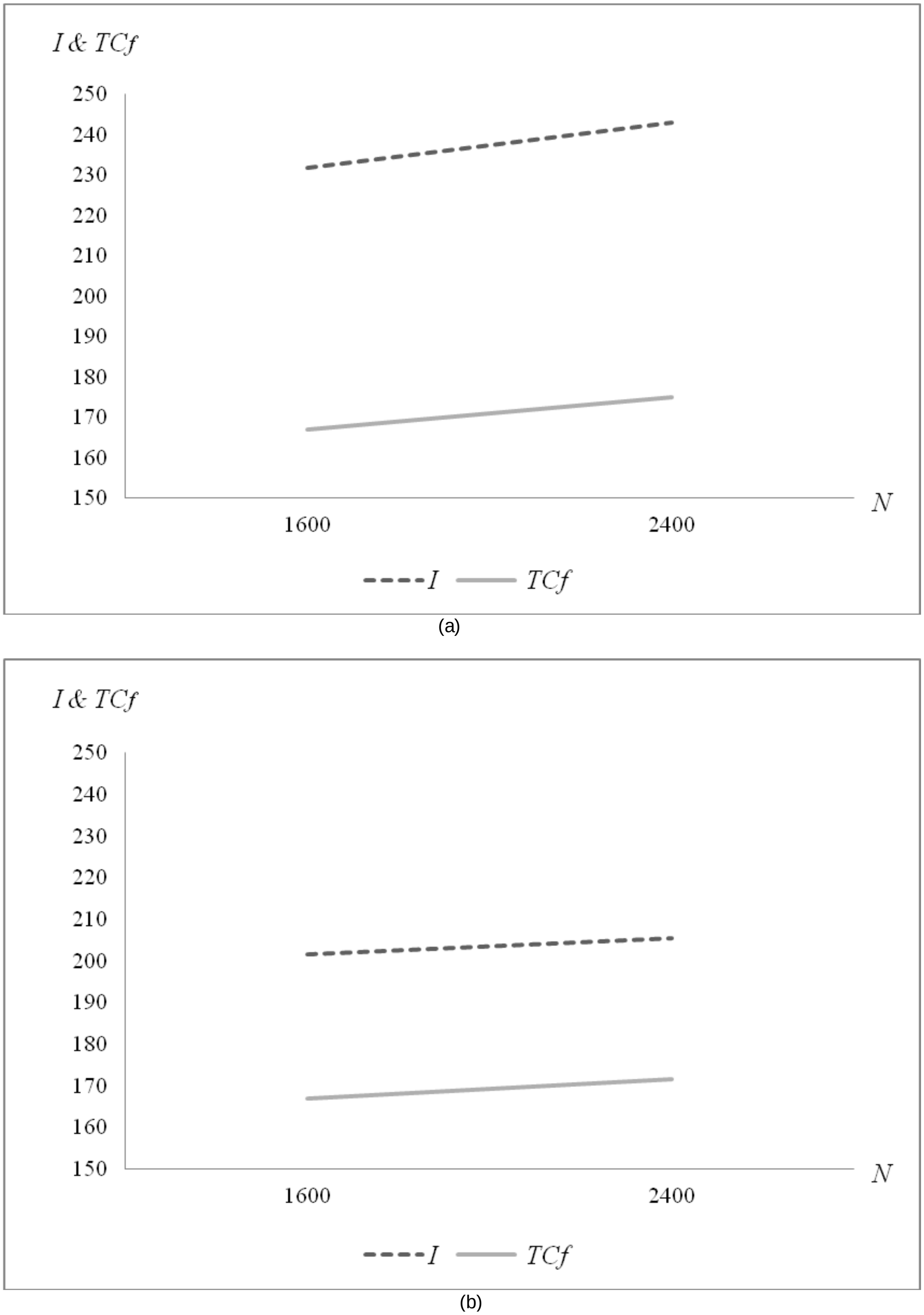
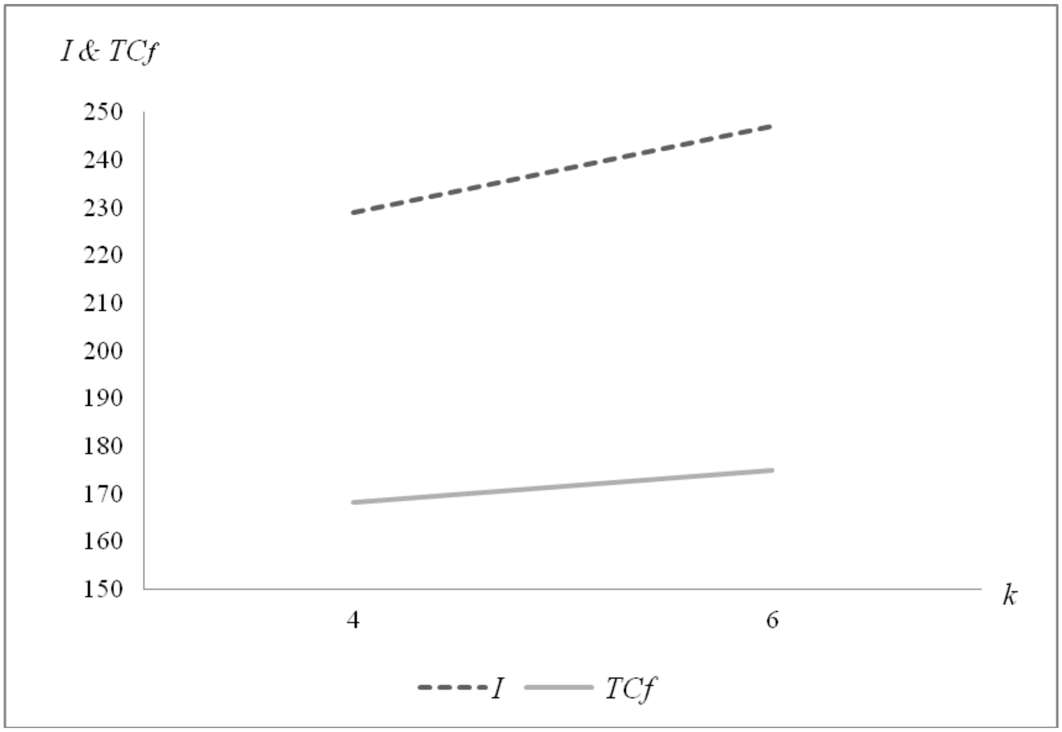
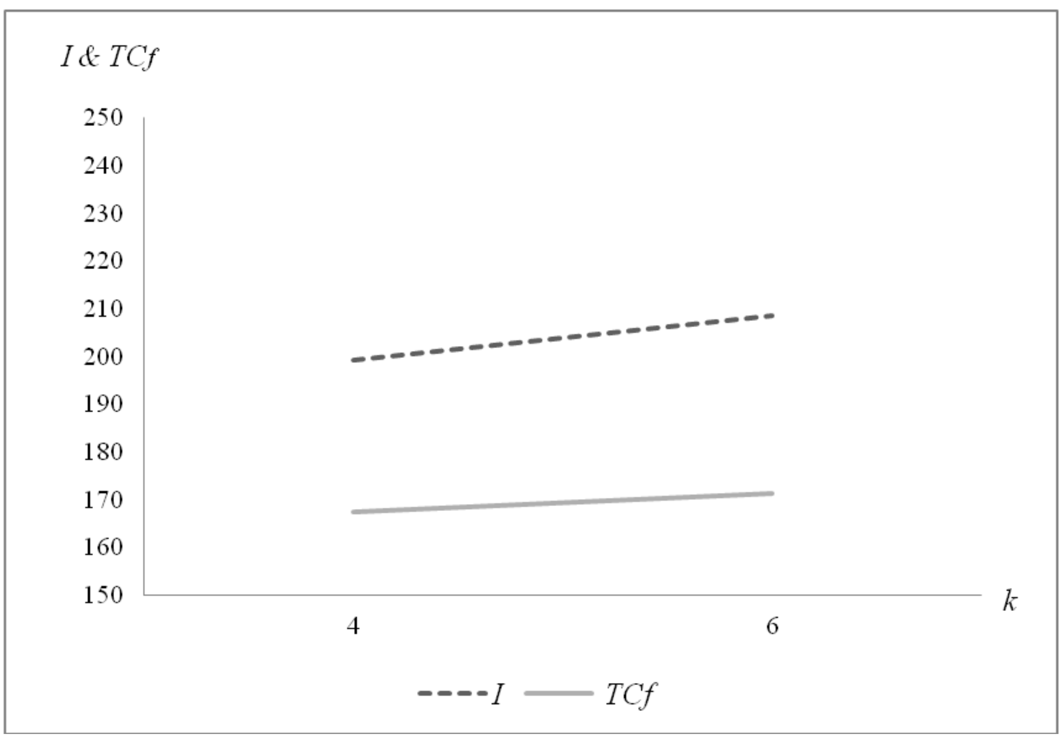


Figure 1. The effect of *N* on model (a) with inspection error; (b) without inspection error.

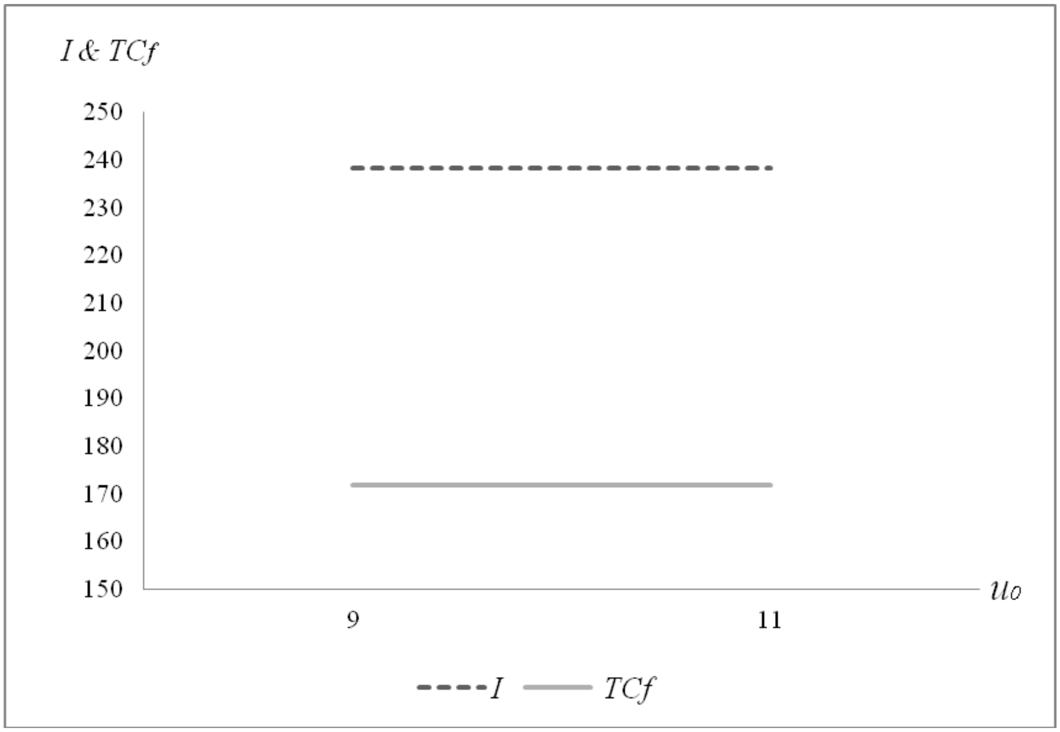


(a)

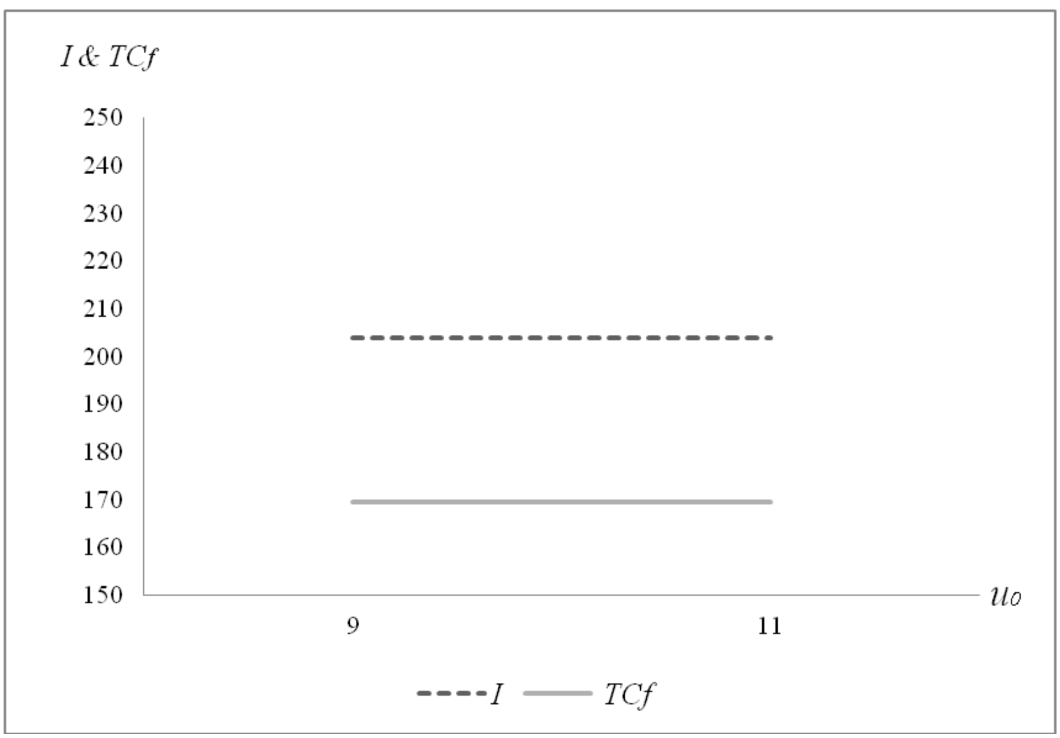


(b)

Figure 2. The effect of *k* on model (a) with inspection error; (b) without inspection error.

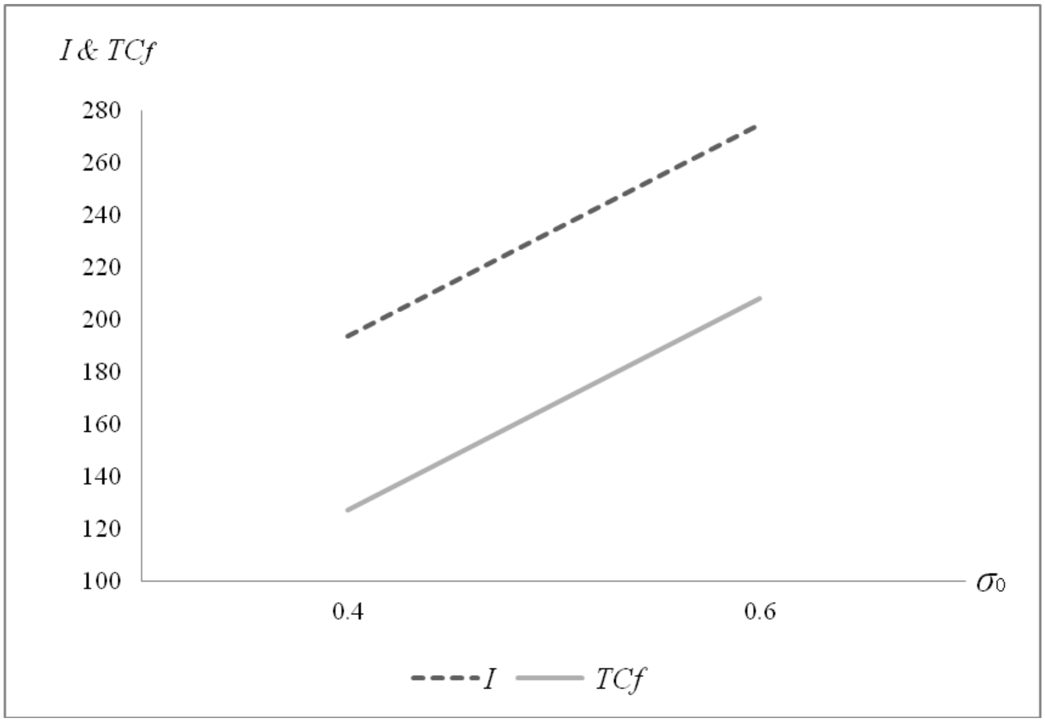


(a)

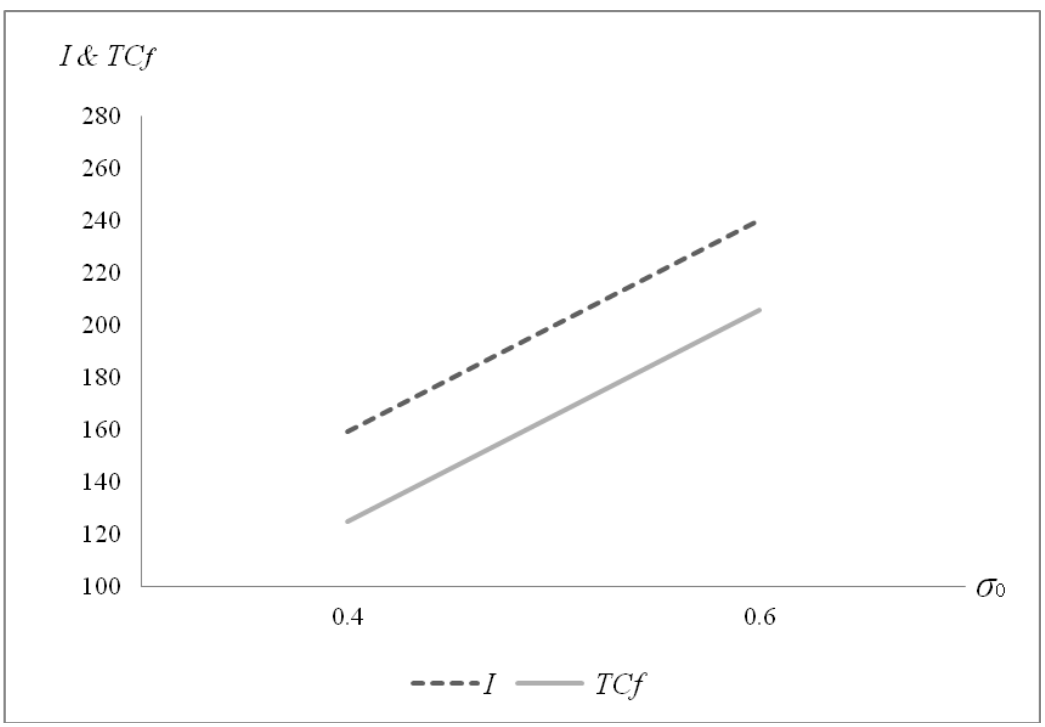


(b)

Figure 3. The effect of μ_0 on model (a) with inspection error; (b) without inspection error.

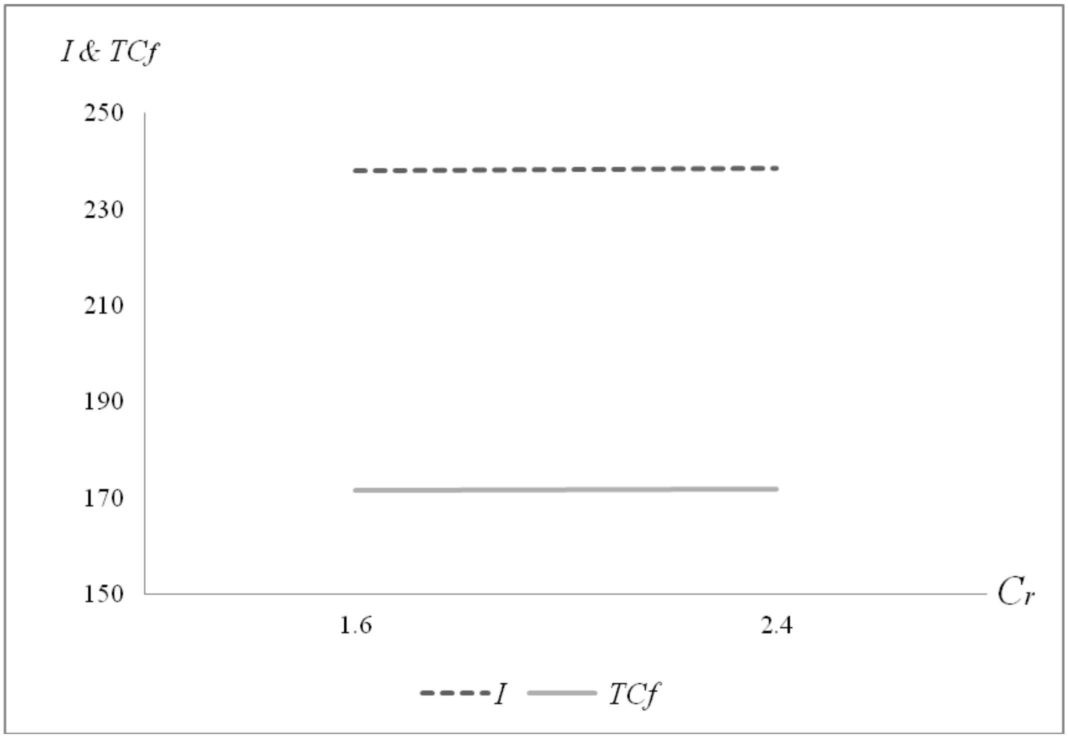


(a)



(b)

Figure 4. The effect of σ_0 on model (a) with inspection error; (b) without inspection error.



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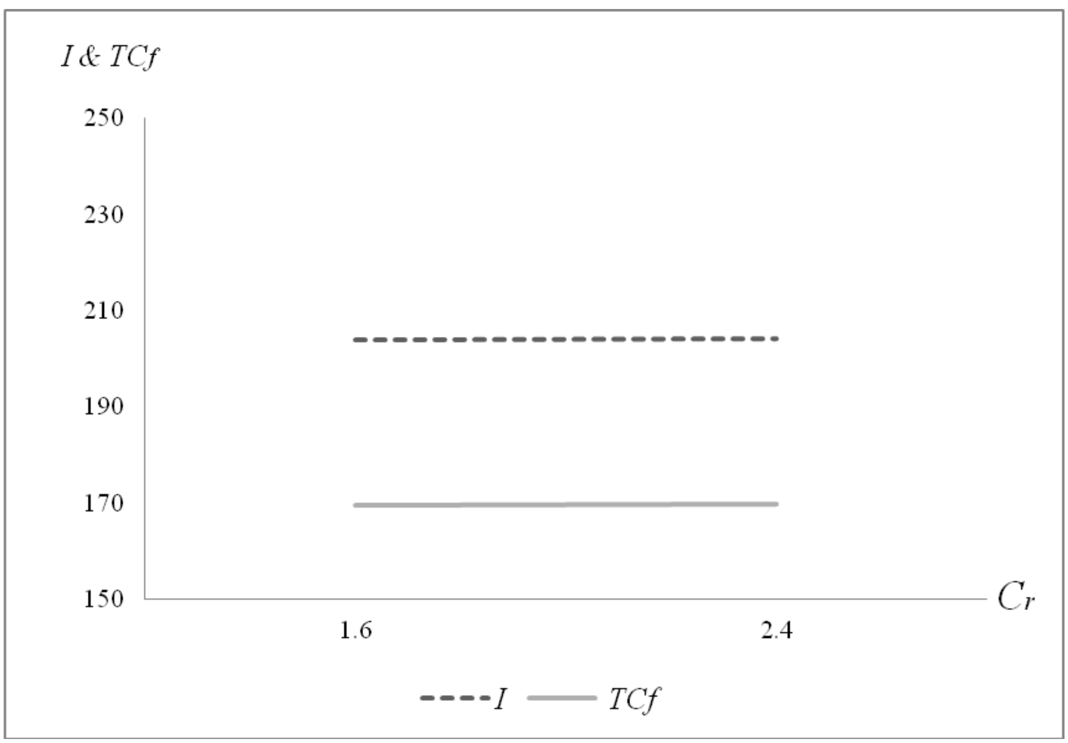
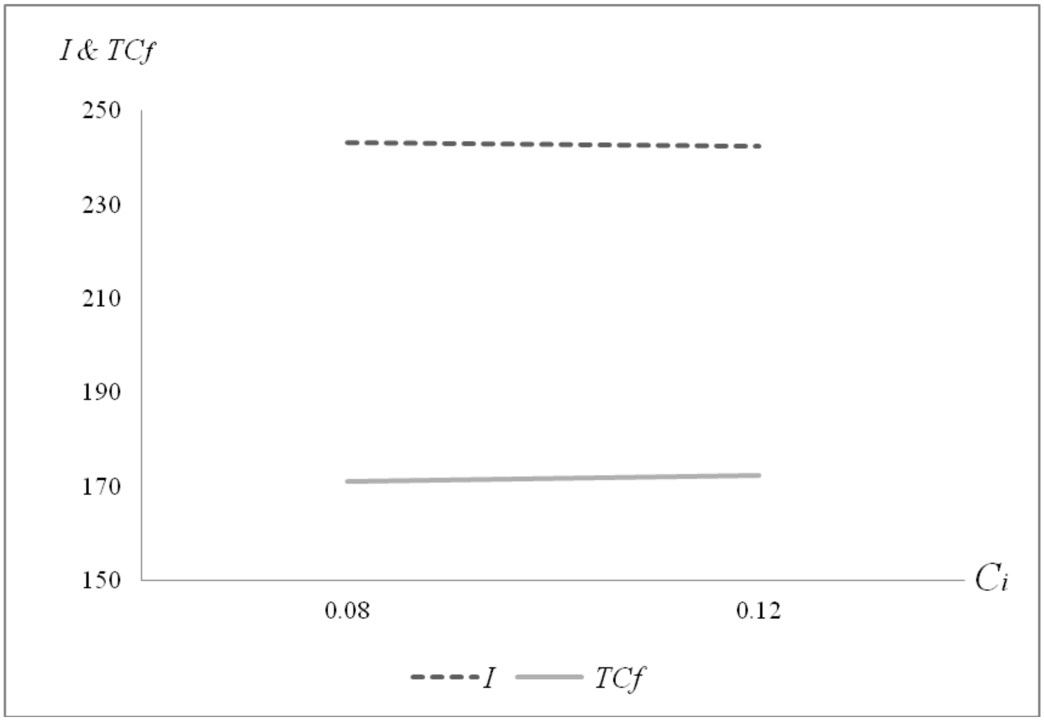


(b)

Figure 5. The effect of C_r on model (a) with inspection error; (b) without inspection error.



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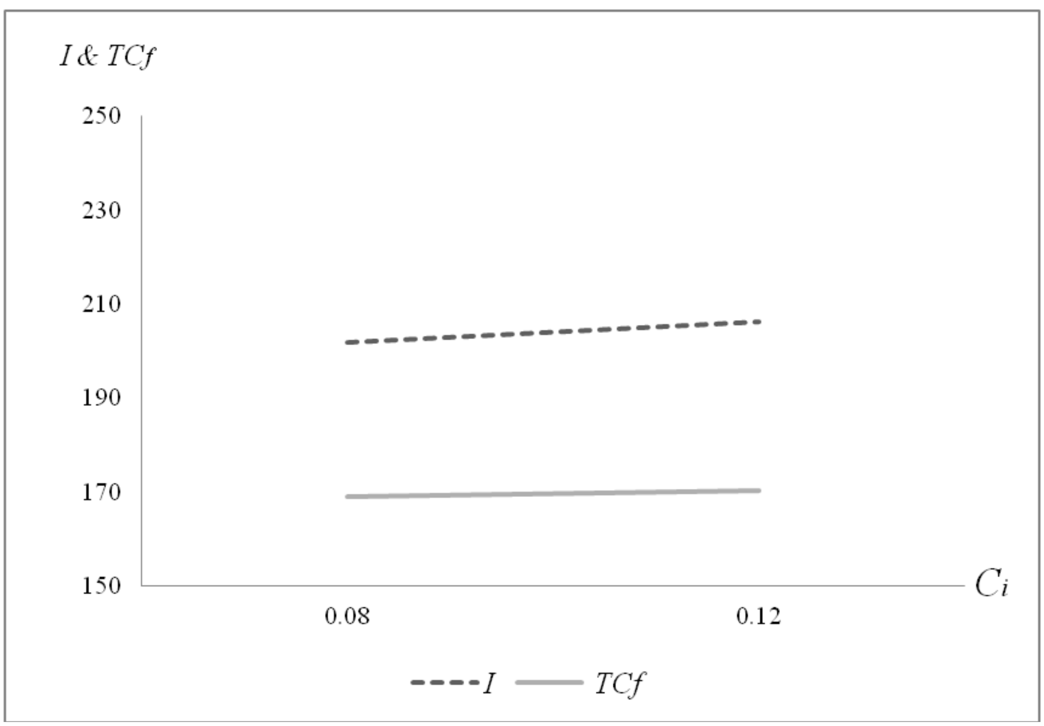
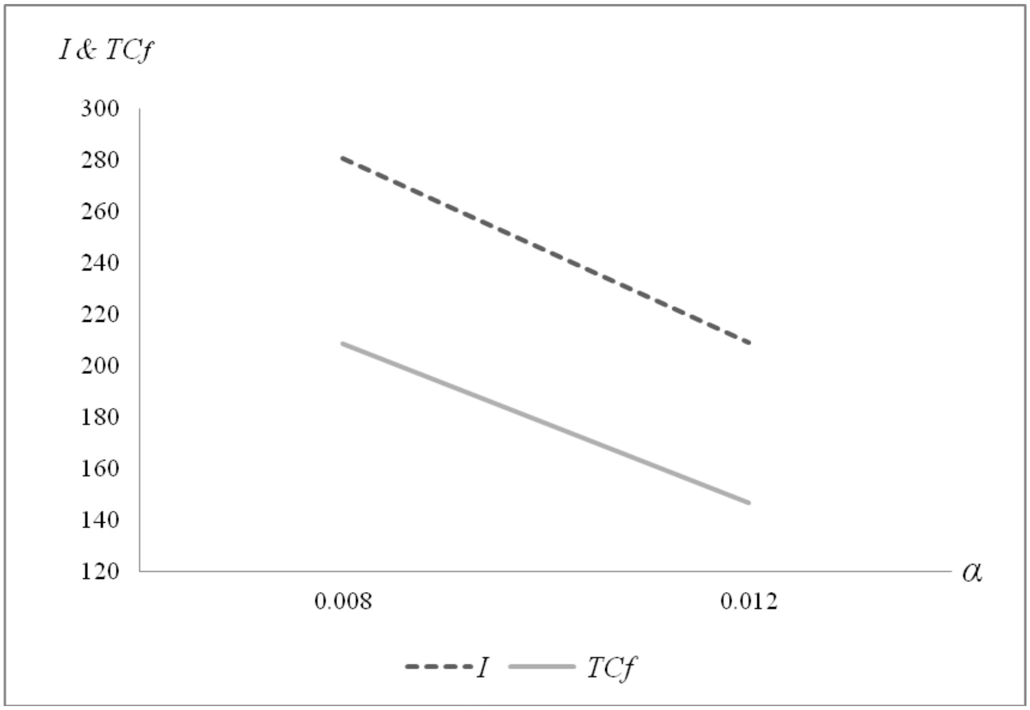


(b)

Figure 6. The effect of C_i on model (a) with inspection error; (b) without inspection error.



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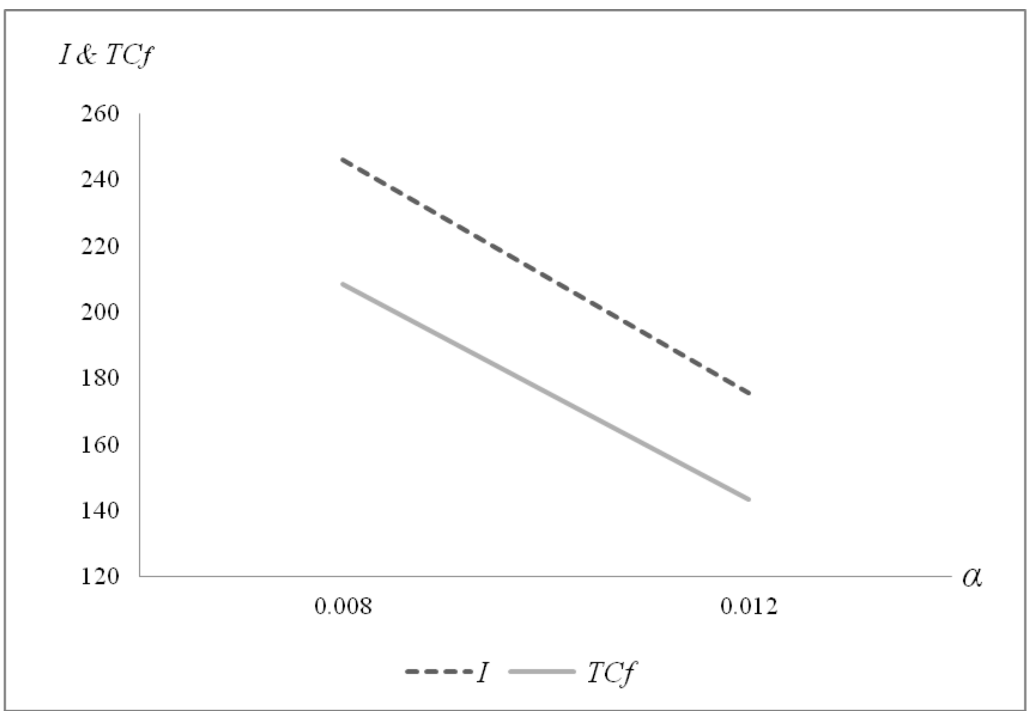
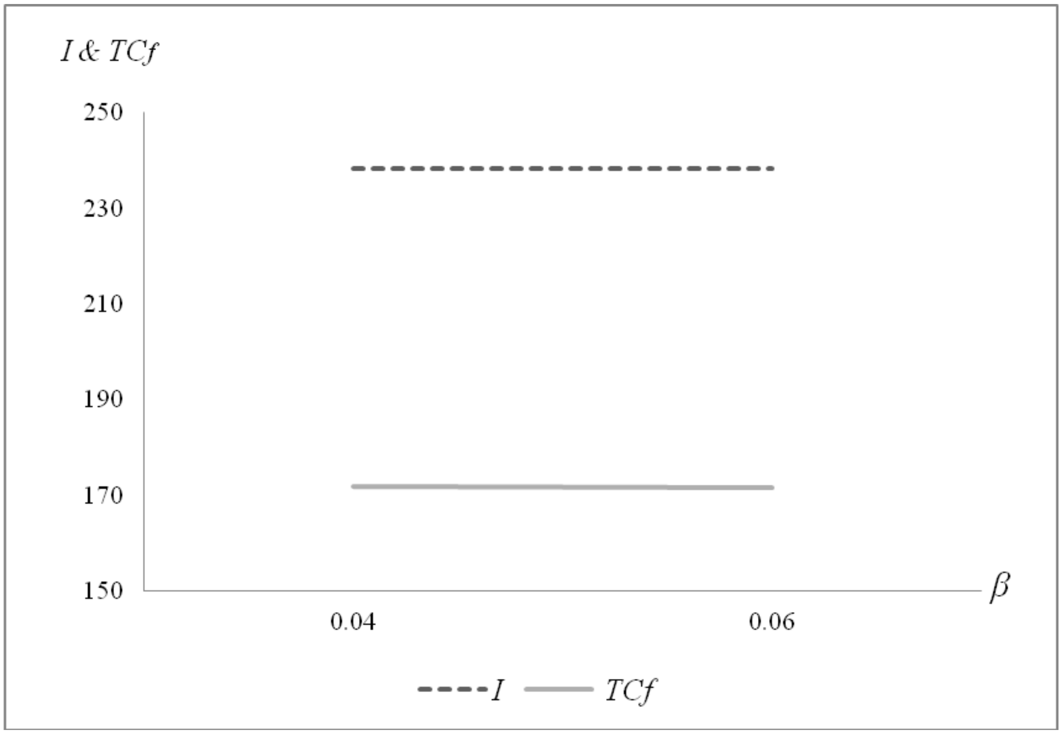


(b)

Figure 7. The effect of α on model (a) with inspection error; (b) without inspection error.



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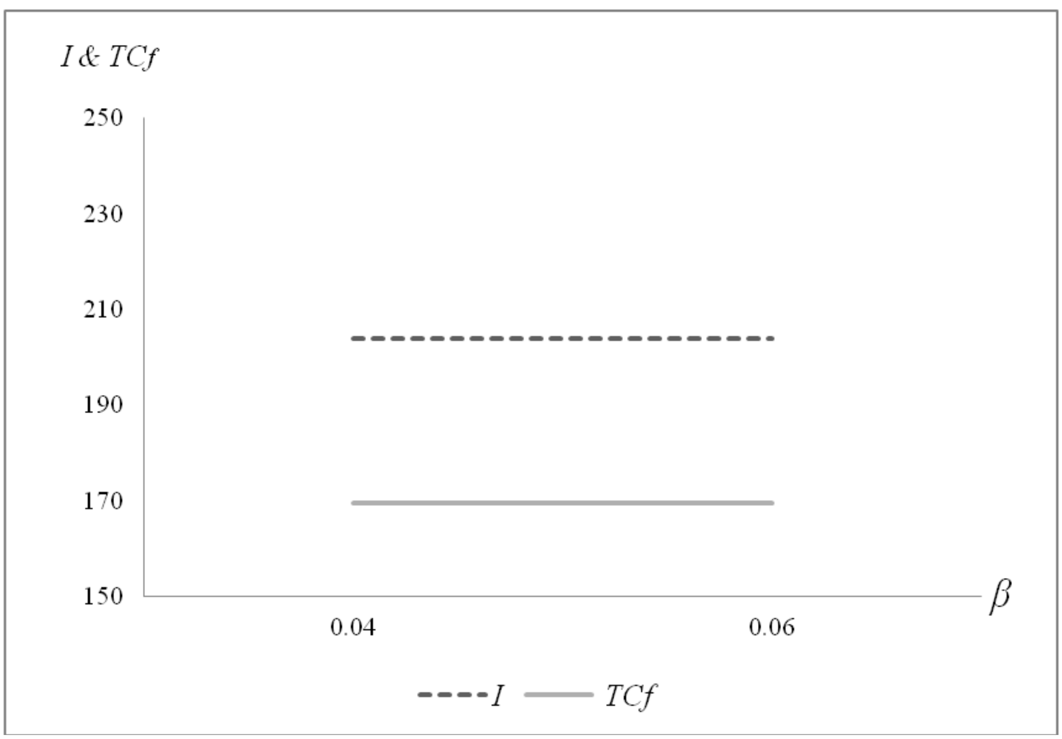
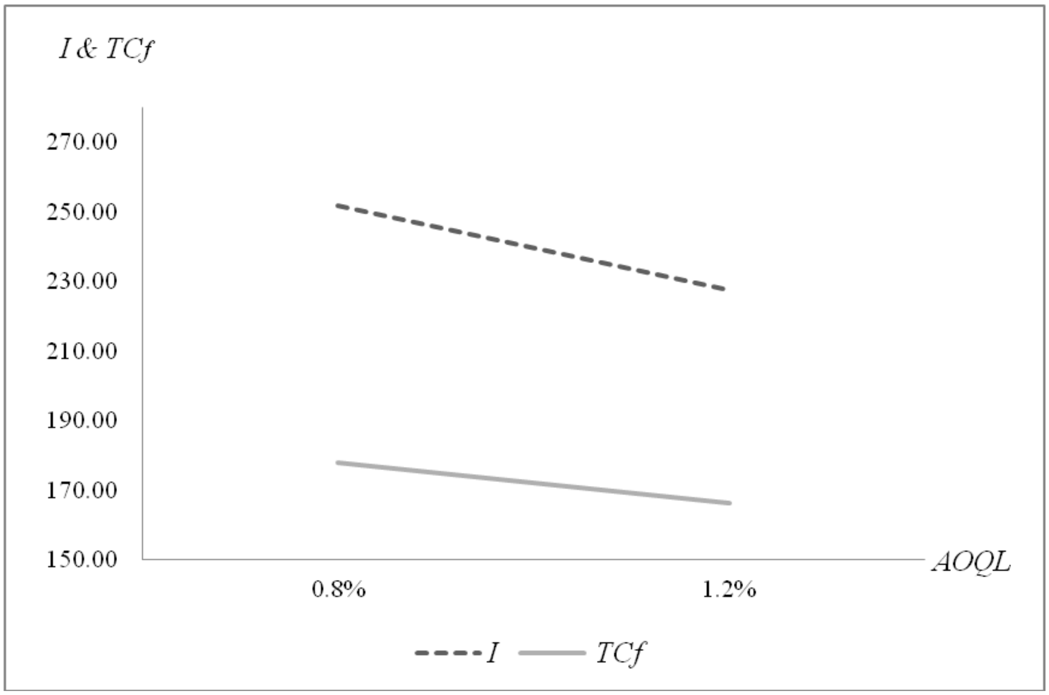
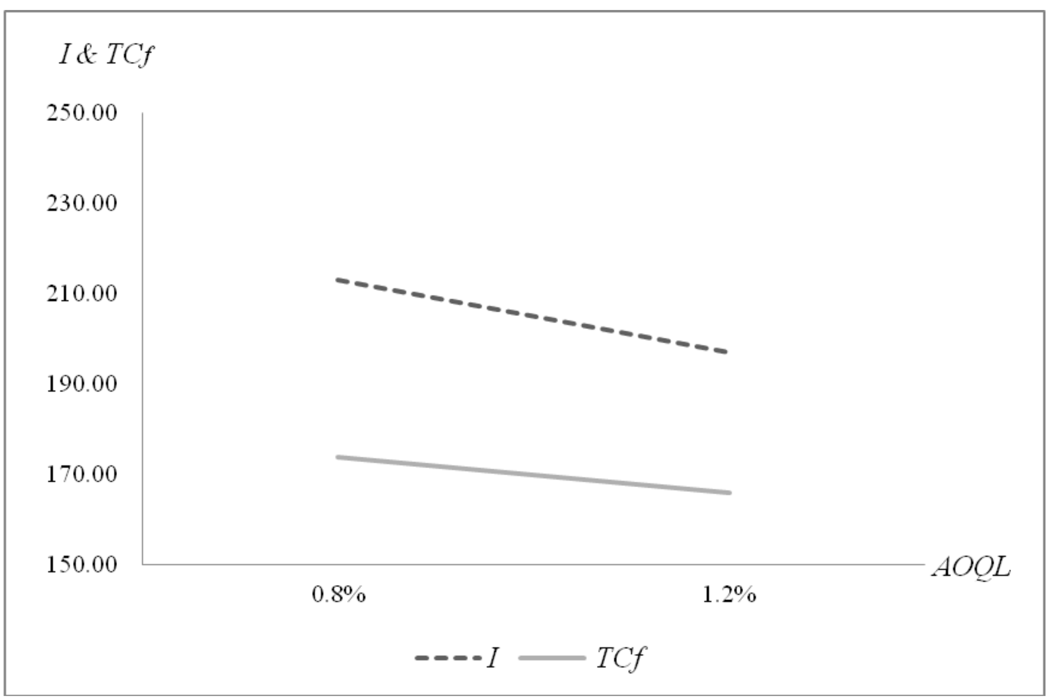


(b)

Figure 8. The effect of β on model (a) with inspection error; (b) without inspection error.



(a)



(b)

Figure 9. The effect of *AOQL* on model (a) with inspection error; (b) without inspection error.

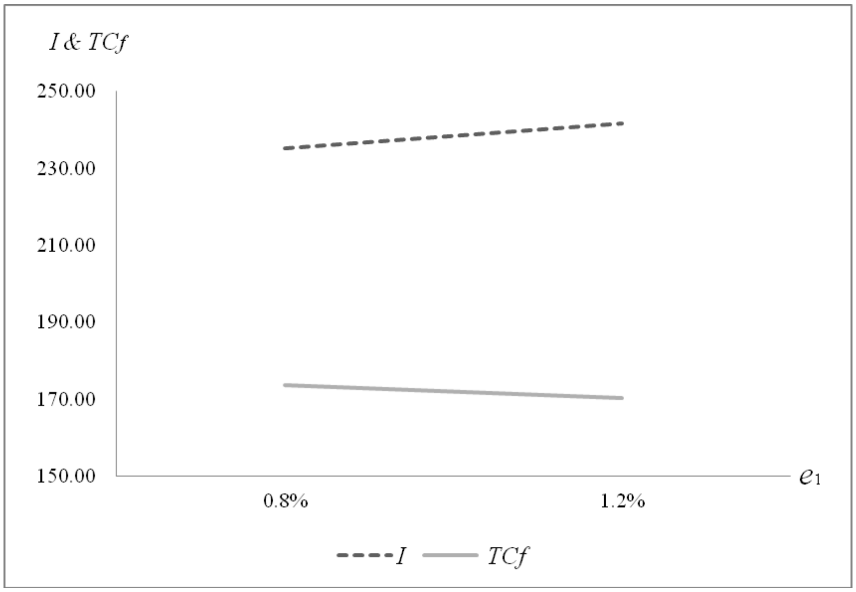


Figure 10. The effect of e_1 on model with inspection error.

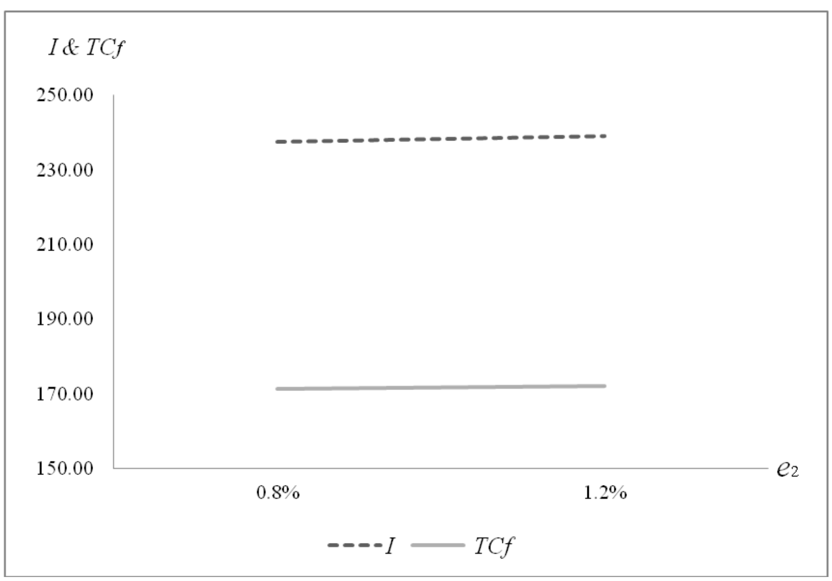


Figure 11. The effect of e_2 on model with inspection error.

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APPENDIX

The determination of the parameters (*c*, *n*) with maximum AOQ_e function

From Beaing and Case (1981), we have

$$AOQ_e = \frac{npe_2 + p(N-n)(1-p_e)P_{ae}}{N(1-p_e)} + \frac{p(N-n)(1-P_{ae})e_2}{N(1-p_e)} \tag{A1}$$

$$P_{ae} = \sum_{x=0}^{c} \frac{e^{-np_e}(np_e)^x}{x!} \tag{A2}$$

Let $A = npe_2 + p(N-n)(1-p_e)P_{ae} + p(N-n)(1-P_{ae})e_2$. Differentiating Equation (A1) with respect to *p* and equating the result to zero (that is, let $d\,AOQ_e/dp = 0$), we obtain

$$\{N(1-p_e)[ne_2 + (N-n)(1-p_e)P_{ae} - p(N-n)P_{ae}(1-e_1-e_2) + p(N-n)(1-p_e)\frac{dP_{ae}}{dp} + (N-n)(1-P_{ae})e_2 - p(N-n)e_2\frac{dP_{ae}}{dp}] + AN\frac{dp_e}{dp}\}/[N^2(1-p_e)^2] = 0 \tag{A3}$$

where $\frac{dp_e}{dp} = 1-e_1-e_2$ and $\frac{dP_{ae}}{dp} = -\frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2).$

Equation (A3) can be rewritten as

$$N(1-p_e)[ne_2 + (N-n)(1-p_e)P_{ae} - p(N-n)P_{ae}(1-e_1-e_2) - p(N-n)(1-p_e)\frac{e^{-np_e}(np_e)^c}{c!}\cdot n(1-e_1-e_2) + (N-n)(1-P_{ae})e_2 + p(N-n)e_2\frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2)] + AN(1-e_1-e_2) = 0 \tag{A4}$$

Let $p = p_1$ is the incoming fraction defective when AOQ_e reaches a maximum value p_L. That is $\max_{0\le p\le 1} AOQ_e = \frac{A}{N(1-p_e)}$ = p_L. Hence, we have

$$A = p_L N(1-p_e) \tag{A5}$$

Substituting Equation (A5) into Equation (A4), we obtain

$$ne_2 + (N-n)(1-p_e)P_{ae} - p(N-n)\cdot P_{ae}(1-e_1-e_2) - p(N-n)(1-p_e)\cdot \frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2) + (N-n)(1-P_{ae})e_2 + p(N-n)e_2\frac{e^{-np_e}(np_e)^c}{c!}n(1-e_1-e_2)] + Np_L(1-e_1-e_2) = 0 \tag{A6}$$

Equation (A6) can be rewritten as

$$
\begin{aligned}
&AOQ_e^{'} \\
&= ne_2 + (N-n)(1-p_e)P_{ae} - (N-n)P_{ae}(p_e - e_1) + (N-n)(1-e_1-e_2)\cdot \\
&\quad (e_2 - 1 + p_e)\frac{e^{-np_e}(np_e)^{c+1}}{c!} + (N-n)(1-P_{ae})e_2 + Np_L(1-e_1-e_2) = 0
\end{aligned}
\tag{A7}
$$

Assume that the maximum AOQ_e function occurs when $p = p_1 = \frac{x}{n}$, where *n* = $\frac{yN}{p_L N + y}$. Hence, we have $p_1 = \frac{x(p_L N + y)}{yN}$. For the given *c*, we can obtain the corresponding values of *x* and *y* from Tables 2 to 3 of Dodge-Romig (1959). Substituting $p = p_1$ into Equation (A7), the corresponding *n* that satisfies minimum | $AOQ_e^{'} - 0$ | is the solution for the given *c*.