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Full Length Research Paper

Comparisons of non-parametric disturbance simulations and Monte Carlo approach

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This paper utilized the proposed historical simulation, where the effect of GARCH (1,1) model on price path were considered, and the Monte Carlo approach were also used to examine the difference in option payoff values between these simulation approaches and the original path. Furthermore, we showed which simulation model would have smaller root mean squared pricing error by examining the difference of root mean squared pricing error between these approaches. We applied these approaches to simulate option payoff values on the Shenzhen composite index series in China during the period 2005 to 2009, and the common back-testing approach was used. The results showed that the estimated option values were significant and differ from the actual Shenzhen composite index option payoff values for the observed period. Finally, we found that the root mean squared pricing error of the adjusted historical simulation is less than the other two simulation approaches.

Key words: Simulation approaches, option payoff values, GARCH, valuation, price paths.

INTRODUCTION

Simulation methods were extensively used in assets pricing, such as covariance matrix based approaches, historical simulation approaches and the scenario based approaches (Deutsche, 2002) were usually mentioned. However, one assumption of the covariance matrix based approaches is portfolio value changes in a linear manner with changes in the risk factor. But Glasserman et al. (2002) showed that the relationship between risk factors and portfolio values should include at least quadratic terms. Rouvinez (1997) and Wilson (1999) described delta-gamma approximations relied on the value of a portfolio changes not only linearly with changes in market risk factors. Britten et al. (1999) explored the relationship between asset values and the risk factor methods whicw were not just only linear but is likely to be adapted better than the assumption of linear method. Therefore, this assumption limited its accuracy.

Using historical simulations requires relatively fewer assumptions, and its main advantage is that it is non-parametric. Beside original path, alternative outcomes are projected by sampling with replacement from the original set, and it is likely to introduce sampling error.

Tompkins and D'Ecclesia (2006) developed a non-parametric approach using unconditional return disturbances and alternative outcomes were projected by sampling without replacement from the original set. The results showed that the simulated options payoff values were insignificantly different from the actual S&P 500 option payoff values for the observed period.

The principal disadvantage of the traditional historical simulation method is that, it assigns an equal weighted of 1/N probability to compute the empirical conditional density function of each return. This assumption is inconsistent with the well-known fact shown by Bollerslev (1986), who indicated that the volatility of asset returns was time varying and displayed clustering. Boudoukh et al. (1998) assumed that returns represented the current risk and from the recent past was better than returns from the distant past. Eberlein et al. (2003) had shown that volatility was stochastic. To overcome the problem arising with the usual assumption of constant volatility, Barone-Adesi, Giannopoulos and Vosper (1999, 2001) allowed the volatilities of historical returns to change, as it involved the calibration of the appropriate GARCH model.

Akgiray (1989) utilized data obtained from the center for research in security prices (CRSP) which contained 6,030 daily returns from January 1963 to December 1986 to examine whether the GARCH model were better than

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other models in forecasting and the result showed that out-of-sample forecasts of return variances of stock indices based on a GARCH model, are superior predictors to others. Chu and Freund (1996) selected the raw data to calculate daily returns for the S&P 500 and the S&P 100 indices which were obtained from the CRSP and the S&P 100 reporter respectively, and the samples contained 1,263 daily returns from March 3, 1981 through February 28, 1986 for each index. The result showed that the use of GARCH models will significantly reduce model mispricing while forecasting option values using only historical returns data.

Also, Hull and White (1998) proposed a procedure for using a GARCH model in conjunction with historical simulation when computing value at risk. Daily data were chosen on five stocks indices including the S&P 500, CAC-40, FT-SE 100, Nikkei 225, and Toronto stock exchange 300 from July 11, 1988 to February 10, 1998. They found out that results were mixed for the investments in stock indices. Tompkins and D'Ecclesia (2006) pointed out that while utilizing GARCH (1,1) model, the simulated options payoff values were insignificant different from actual options payoff values within one year expiration.

In addition to the aforementioned simulation methods, alternative parametric scenario approach have been proposed, which preserves the statistical artifacts that exist in financial data. To generate scenarios, Monte Carlo methods are commonly used, which are only a tool for determining the distributional properties of values associated with a given model. Boyle (1977) introduced Monte Carlo simulation as a numerical method and assumed that the price process followed the usual geometric Brownian motion. Papageorgiou and Traub (1996) referred to a deterministic method and compared this method with Monte Carlo on the valuation of underlying assets in their study. They found that the Monte Carlo method was sensitive to the initial seed and the random point samples were wasted due to clustering. Okten and Eastman (2004) presented a survey of comparison of simulation methods in pricing certain securities such as European call options, the result showed that Monte Carlo methods offered losses in error reduction. But Tompkins and D'Ecclesia (2006) showed that option payoffs estimated with Monte Carlo method were not significantly different from the actual payoffs values.

Therefore, according to these studies, it is still an issue that is worth discussing, whether the historical simulations or the adjusted one with time varying asset returns volatility and Monte Carlo method are appropriate to use in simulations. The study proposes a simple and general simulation approach which minimizes model assumptions and parameter input. Specifically, this study differs from previous historical simulations in that it selects the disturbances for the procedure, and fixes the mean and standard deviation while varying the disturbances. We compared the simulation methods with the actual pay off values in our study, and finally used the root mean squared pricing error (RMSE) as suggested by Figlewski (2002) to show which simulation model would have smaller RMSE by examining the differences of the RMSE between these approaches.

DATA AND SIMULATIONS DESIGN

This research is conducted using the basis of Shenzhen composite index series in China, which is collected from the Taiwan economic journal (TEJ) database. Data sample period is from January 4, 2005 to December 31, 2009, for a total of 1215 daily settlement price, S_{tj} are obtained.

The pseudo random disturbances approach: (PRD model)

Given a historical daily price series S_{tj}, returns are computed using continuously compound interest rate:

$$r_{tj} = \ln\left(\frac{S_{tj}}{S_{tj-1}}\right) \quad for\ j = 1, 2, ..., N$$

The unconditional mean, μ, and standard deviation, σ, are estimated during the time period. Then normalizing the sequence of returns by these two moments yielded:

$$dz_{tj} = \frac{r_{tj} - \mu}{\sigma} \tag{1}$$

Where $\{dz_{tj}\}$ is the series of standardized "disturbances" from t_1 to T, and by design, the resulting disturbances have a mean 0 and standard deviation of 1. The simulated prices $\tilde{S}_{tj+\delta t}$ for each time t_j >0 are obtained according to the following formulation by using the standardized disturbances, dz_{tj}:

$$\tilde{S}_{tj+\delta t} - \tilde{S}_{tj} = \mu \tilde{S}_{tj} \delta t + \sigma \tilde{S}_{tj} dz_{tj} \tag{2}$$

Introducing GARCH (1,1) volatility: The adjusted PRD model

We assume the volatility term as the form of a GARCH (1,1) model in this case according to the Bollerslev (1986) formulation:

$$\tilde{\sigma}_{tj} = \sqrt{\tilde{\sigma}_{tj}^2} = \sqrt{\omega + \alpha u_{tj-1}^2 + \beta \sigma_{tj-1}^2} \tag{3}$$

Following the Tompkins and D'Ecclesia (2006), to present the effect of GARCH (1,1) model, the standardized

Table 1. Unconditional moments of the Shenzhen composite index returns, 2005-2009.

Henzhen composite index	Daily unconditional returns	Daily standardized disturbances	Adjusted daily standardized disturbances
μ	0.00111	0.000067	0.01036
σ	0.02184	1.00014	1.06753
Skewness	-0.61275	-0.61275	-0.58363
Kurtosis	4.97229	4.97229	5.27835

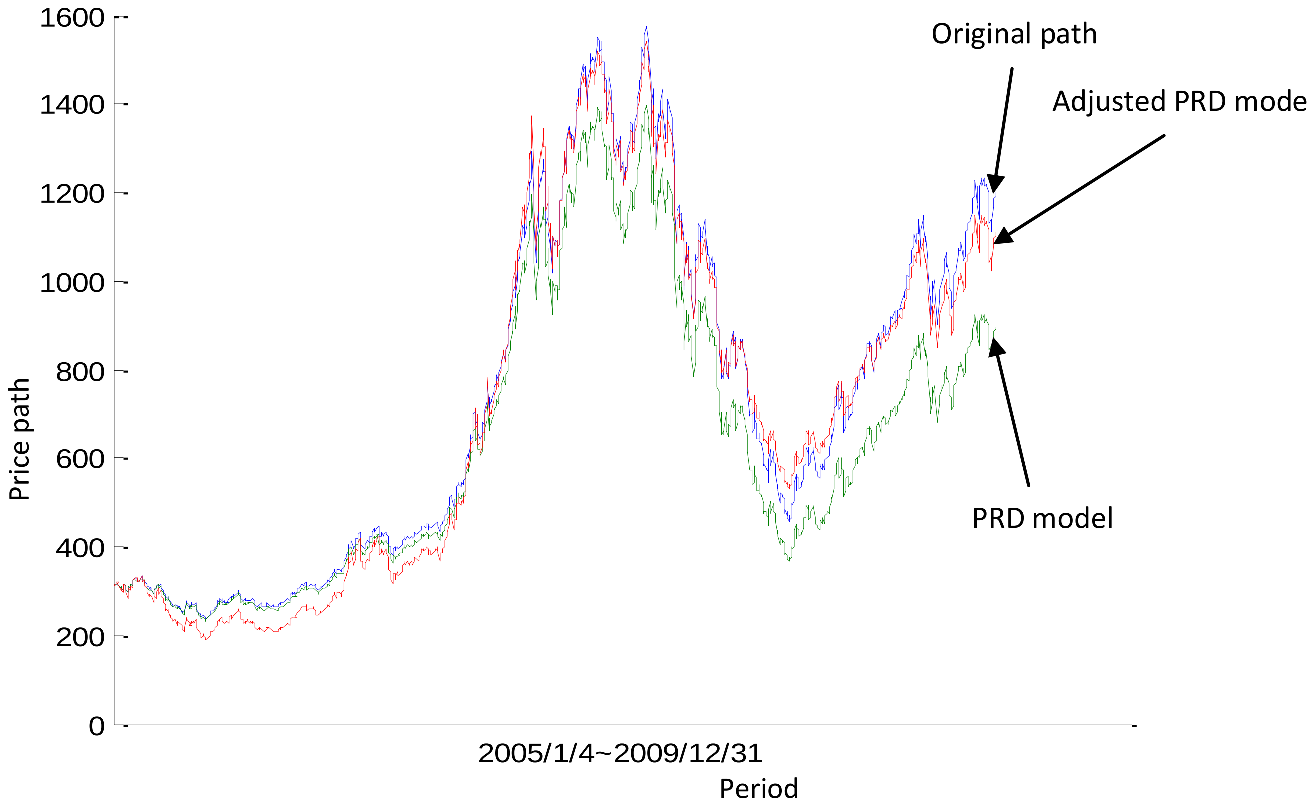

Figure 1. Price paths of original, PRD model and adjusted PRD model.

disturbances, $\{d\tilde{z}_{tj}\}$, should be obtained using the volatility model of equation (3) with the following expression to generate alternative price paths.

$$d\tilde{z}_{tj} = (r_{tj} - \mu) / \tilde{\sigma}_{tj} \quad (4)$$

Where $\tilde{\sigma}_{tj}$ is the estimated volatility path of GARCH (1,1) model for the entire period using equation (3). There are 1215 daily settlement prices and 1214 unconditional returns during this period. For the original path of daily returns and disturbances, the statistical properties are shown in Table 1. Figure 1 presents the price paths of original, PRD model and adjusted PRD model.

Mixing the pseudo random disturbances (the MPRD-model)

A non-parametric simulation approach which provides a tool to generate financial assets price paths was proposed. We use this approach to "re-write" historic financial assets prices paths, and it required neither any parameter estimation nor distributional assumption.

Given the pseudo random disturbances, dz_{tj}, which was estimated using equation (1), the unconditional mean, μ, and stand deviation, σ, of the original price series, we randomly mix the dz_{tj} and drew the sample without replacement until all the disturbances, $dz_{tj}(\varepsilon_i)$, j=1,…,N are selected, where the ε_i denotes a random permutation

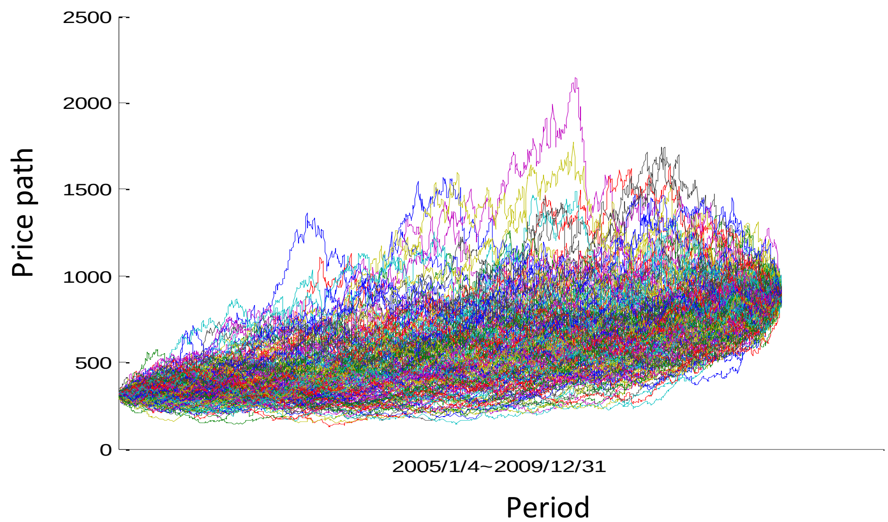

Figure 2. Price paths of mixing pseudo random disturbances model (200 paths).

of the original sequence of the disturbances for i=1,…,200 times. New price paths with these new sequences of disturbances can be estimated, extending from equation (2).

MPRD1: non-adjusted disturbances

The entire time series disturbances, dz_{tj}, are randomly mixed 200 times, and generating different sequences of $dz_{tj}(\varepsilon_i)$, new price paths are determined according to the following expression:

$$\tilde{S}_{tj+\delta t} - \tilde{S}_{tj} = \mu \tilde{S}_{tj}\delta t + \sigma \tilde{S}_{tj} dz_{tj}(\varepsilon_i) \quad (5)$$

According to the definition, the disturbance moments of the entire time series would not change in the mixing process. However, the sequence of the disturbances has changed, therefore, the price paths changed. Figure 2 shows the price paths of MPRD model (200 paths).

MPRD2: adjusted disturbances by GARCH(1,1) volatility

Assuming the process of the volatility is measured as a GARCH (1,1) process, and we mix the set of $\{d\tilde{z}_{tj}\}$ of the GARCH (1,1) to devolatilize the disturbances using the following expression:

$$\tilde{S}_{tj+\delta t} - \tilde{S}_{tj} = \mu \tilde{S}_{tj}\delta t + \sigma \tilde{S}_{tj} d\tilde{z}_{tj}(\varepsilon_i) \quad (6)$$

We get a dispersion of final terminal prices as it is shown in Figure 3, and the average moments of the 200 simulated paths of the MPRD model and the adjusted MPRD model are reported in Table 2.

Monte Carlo simulation method (MC model)

Monte Carlo method is a tool for determining the values of distributional properties associated with a given model. It is commonly argued that this method should be supported because alternative distributions can also be used. In the usual Monte Carlo simulation, specific assumptions on the distribution of the variables have to be made and correlations are often assumed to be zero, this implies that the processes are all independent. Boyle (1977) pointed out that, in general, such simulations assumed that the price process followed the usual geometric Brownian motion, and the process of asset returns followed a standard normal distribution (dB). In this study, we utilize implied volatility for each moneyness/maturity combinations as different parameters input and to generate the price paths using the following form:

$$\tilde{S}_{tj+\delta t} - \tilde{S}_{tj} = \mu \tilde{S}_{tj}\delta t + \hat{\sigma}_{IV} \tilde{S}_{tj} dB_{tj}(\varepsilon_i) \quad (7)$$

The back-tested option payoff values of Shenzhen composite index

The focus of this research is to examine the estimation ability of simulation approaches relative to the benchmark results instead of developing a new option pricing theory. Therefore, option payoff values are estimated using the original spot price path. We determine the option payoffs by setting a variety of fixed striking prices and terms to expiration, and the simple average of the option payoffs for the entire period are computed to provide benchmarks of the option payoff values on the Shenzhen composite index. The main advantage of this, is the fact that we need fewer assumptions, and we do not have to estimate

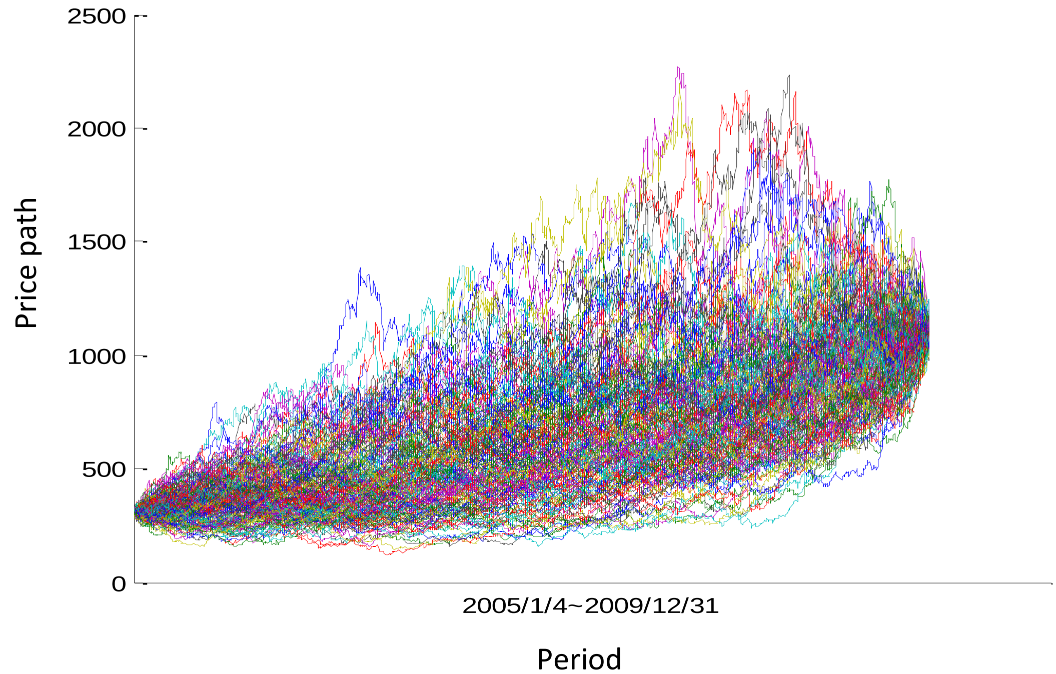

Figure 3. Price paths of adjusted mix pseudo random disturbances model (200 paths.

Table 2. Statistical feature of the original price returns and the simulated ones.

Variable	Average	Standard deviation	Skewness	Kurtosis
Original price series	0.00111	0.02184	-0.61275	4.97229
MPRD price series (average on 200 paths)	0.00086	0.02198	-0.72921	5.11595
Adjusted MPRD price series (average on 200 paths)	0.00103	0.02347	-0.72144	5.53542

the discounted value of the terminal payoffs. A similar approach has been proposed previously by Stutzer (1996), he transforms his estimated objective density to a "risk neutral" equivalent density. Using this approach he can price derivative products by taking the conditional expectations relative to discount the expectation into present value using the risk-free rate. Tompkins and D'Ecclesia (2006) further simplified this approach, by solely defining option values as, their expected terminal payoffs. Therefore, we do not need to introduce a discounting factor.

To establish our benchmark on the Shenzhen composite index option values, we consider the real payout that would have occurred from buying a European call option by using the given historical price series with a strike price, X_{tj}, and compare this to the price of the underlying asset, S_T, on the expiration date T, and the moneyness, K_m, is expressed in standard deviation terms:

$$K_m = \frac{\ln(X_{tj} / S_{tj})}{\sigma\sqrt{\tau_h / 252}} \qquad m = 1,\ldots,15 \tag{8}$$

Where X_{tj} is the original strike price; S_{tj} is the underlying spot price; σ is the unconditional stand deviation observed during the entire period; $\sqrt{\tau_h / 252}$ reflects the square root of percentage in a year for the time to expiration of the option. This expression in standard deviation terms allows us observe the relative moneyness for different maturity options directly. In this study, we assume that the relevant time is trading days and express time to expiration as a percentage of the total trading time in a year (is assumed to be 252 days). For each fixed K_m, it is possible to determine the correspondent X_{tj}, and it could be expressed as a percentage of the underlying spot price such that: $X_{tj}=\beta_{mh}S_{tj}$. In this application $-3.5 \le K_m \le 3.5$, the expiration dates, τ_h, are set equal to 5, 10, 15, 20, 25, 30, 35, 40, 45, 50, 55, 60, 80, 100, 120, 140, 160, 190, 220 and 250 days, where h=1,…,20. However, the average payoffs would simply be the percentage return of option prices relative to the initial spot prices:

$$average\ payoffs = \frac{1}{T-\tau_h}\sum_{j=0}^{N-\tau_h}[MAX(S_{tj}-\beta_{mh}S_{tj-\tau_h},0)]/S_{tj-\tau_h} \quad (9)$$

for each chosen moneyness K_m and maturity τ_h. (9)

For example, consider a call option with a strike price which corresponds to a percentage β_{mh}=0.84 of the spot price and τ_1=5 trading days to expiration. It is a simple to estimate the actual payoffs from the observed underlying historical price series. In the example, we would examine price from S_{t1} to S_{t6} and from S_{t2} to S_{t7} and so on until S_{t1210} to S_{1215}. The terminal call price would simply be $C = MAX(S_{tj}-\beta_{mh}S_{tj-\tau_h},0)$, for t=0,…,T-$\tau_h$. Therefore, in this case of τ_1=5, we would have T-τ_1=1210 instances of call option prices. This models a situation where an investor had purchased option with a strike price equal to β_{mh}=0.84 and one week to expiration of the current spot price every single day.

On the other hand, we will estimate option prices using the parametric Monte Carlo approach to compare the results with the MPRDs models, and the model has been set up and parameters to be estimated. We assume that the price process follows the usual geometric Brownian motion as mentioned in Boyle (1977) and simulate the prices. We choose a random draws from a normal distribution and project the terminal prices of the underlying asset where the same strike prices and terms to expiration described in previously are used, with the only remaining parameter will be the volatility have to input.

By means of choosing the moneyness, K_m, m=1,…,15, and terms to expiration, τ_h, h=1,…,20, it leads to a 15 × 20 matrix including 300 option payoff values, and these option payoff values will transfer to call prices and express them as implied volatilities using the Black-Scholes model. To yield the implied volatilities, the input variables were $F_{tj} = S_{tj}e^{\mu(tj+\tau_h)}$ as the underlying price, $\beta_{mh}S_{tj}$ as the strike price, the zero interest rate, the time to expiration of τ_h, and the usual Newton Raphson iterative approach was used in the following expression:

$$\sigma_1 = \sigma_0 - [F_{tj}\int_{-\infty}^{d_1}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx - \beta_{mh}S_{tj}e^{-rT}\int_{-\infty}^{d_2}\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}dx]/F_{tj}\sqrt{T}\,\frac{1}{\sqrt{2\pi}}e^{-\frac{d_1^2}{2}} \quad (10)$$

Where

$$d_1 = \frac{\ln(F_{tj}/\beta_{mh}S_{tj}) + (r+\sigma_0^2/2)T}{\sigma_0\sqrt{T}}$$

$$d_2 = \frac{\ln(F_{tj}/\beta_{mh}S_{tj}) + (r-\sigma_0^2/2)T}{\sigma_0\sqrt{T}}$$

σ_0 was the initial value we give, σ_1 was computed by σ_0 and substituted as σ_0 until the absolute value of $\sigma_1 - \sigma_0$ converge toward to zero, and $\sigma_{\beta_{mh}S_{tj}}$ were defined as $\hat{\sigma} = \sigma_1$, then these volatilities were standardized to the level of the at-the-money volatility as follows:

$$\sigma_s = 100\left(\frac{\sigma_{\beta_{mh}S_{tj}}}{\sigma_{S_{tj}}}\right) \quad (11)$$

There are two motivations for the decision to express the strike prices in standard deviation terms and the volatility relative to the at-the-money volatility. One is that, it can show the implied volatilities level relative to the at-the-money implied volatilities. Another reason is that, expressing strike prices and implied volatilities in this way allows us have comparisons to earlier study worked by Tompkins and D'Ecclesia (2006), which examined implied volatility surfaces for options on S&P 500 index and standardized them in the similar way. Other previous works studied by Tompkins (2001) and Goncalves and Guidolin (2006), they also express the strike prices and volatilities like this way, and they find out that the shape of volatility surfaces display regularity and stability over time. Figure 4 presents the relative implied volatility

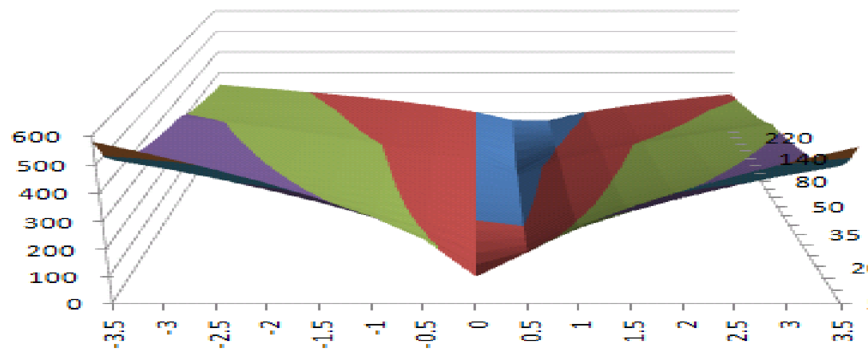

Figure 4. Implied volatility surfaces on Shenzhen composite index.

surface for options on the Shenzhen composite index. It is interesting that this implied volatility curve is similar to that which have appeared in numerous previous empirical studies.

Finally, we compare the average option payoffs obtained by these approaches with original option payoffs. We use the root mean squared pricing error (RMSE) which is suggested by Figlewski (2002) to show which simulation model would have smaller RMSE relied on examining the difference of RMSE between the MPRD, adjusted MPRD and Monte Carlo approaches.

OPTION VALUATION OF THE MPRDS AND MONTE CARLO APPROACH

The proposed approaches will generate alternative price paths that retain the same statistics and time series properties as the original price path. In order to have a comparison with Mixing the Pseudo Random Disturbances (MPRDs) and the common Monte Carlo method, we estimate the payoff values of options from an obvious benchmark, which would be observed on the Shenzhen composite index for the period from 2005 to 2009.

Firstly, we generated 200 paths and estimated T-τ_h option payoffs for each path using the MPRDs approach. Take for example, for a single of the moneyness/maturity combinations, we estimated 243,000 stock prices (1215 prices for 200 times simulations) and determined up to 242,000 different option payoffs in 5 trading days expiration, and we computed the average payoffs and standard deviation over the 200 paths. Therefore, there exist 22,650 option payoffs and as a complete data 67,950,000 simulated option payoffs using MPRDs approaches in a moneyness. Then we compared the average payoffs obtained from using the MPRDs approaches with the original option payoffs by utilizing equation (5) and equation (6).

Furthermore, we compared the average payoffs obtained from using the Monte Carlo approach with the original option payoffs by utilizing equation (7). We assumed that the price process follows the geometric Brownian motion and choose a random draws from a normal distribution to project the terminal price of the underlying asset. We substituted volatilities as the implied volatilities estimated by Black (1976) for every moneyness/maturity (15x20) combination (data defined in Figure. 4) from each individual option contracts into the specific model. To be consistent with the mixing approaches we ran in parallel, for each of the 300 moneyness/maturity combinations, 1215 prices were simulated using Monte Carlo approach for 200 times.

For showing the trend of these results, we summarised our table by only selecting the results from original and simulation paths for several moneyness, K_m=-3.5 (Deep In The Money), -2 (In The Money), 0 (At The Money), 2 (Out of The Money), 3.5 (Deep Out of The Money), and expiration dates, τ_h=5, 20, 40, 60, 120, 250 trading days, such as average payoff values, standard deviation, *p*-value and RMSE is as shown in Table 3.

Table 3 reports the results of the estimators for the average option payoffs over the 200 paths from the MPRDs and Monte Carlo approaches. The table also shows the actual payoffs, *p* value and RMSE for several moneyness, K_m, and expiration dates, τ_h. In Table 3, estimated option payoff values compare with the actual option payoff values. Most of the *p* values tend to be zero. These results are inconsistent with the results of Tompkins and D'Ecclesia (2006). This study regarded the RMSE of the MPRD and the adjusted MPRD approaches as valid measures because their results are more close to the actual option payoff values. Though, the MC uses the ex-post implied volatility as parameters, the way of random drawing from a normal distribution does not provide estimators close to the benchmark of the option payoff values. The RMSE of the Monte Carlo simulation option payoff values tend to be higher than those of the other two MPRD approach.

The RMSE trend of the MPRD simulations option payoff values tends to be consistent with the adjusted one within different moneyness and time to expirations. Figures 5 and 6 showed the RMSE of different moneyness relative to ATM on different time to expiration in the MPRDs respectively. The figures show a similar trend.

Table 3. Statistics of estimated option payoff values, *p*-value and the RMSE.

	Moneyness (in standard deviation terms)									
Trading days to expiration	**DITM**	**ITM**	**ATM**	**OTM**	**DOTM**	**DITM**	**ITM**	**ATM**	**OTM**	**DOTM**
			5 days					**20 days**		
Original path (%)	16.395	10.092	2.412	0.055	0.000	31.883	20.886	6.331	0.183	0.000
Average mixed paths (%)	16.253	9.914	2.233	0.041	0.000	31.165	20.011	5.122	0.117	0.000
Std dev mixed paths	1.82E-04	2.89E-04	6.35E-04	1.29E-04	7.57E-06	1.45E-03	1.62E-03	3.00E-03	7.34E-04	2.74E-05
p-value of difference	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	0.0018
RMSE	0.00143	0.00180	0.00191	0.00019	0.00001	0.00733	0.00890	0.01245	0.00099	0.00003
Average mixed/adjusted paths (%)	16.359	10.038	2.414	0.065	0.001	31.581	20.447	5.624	0.198	0.002
Std dev mixed/adjusted paths	2.02E-04	3.47E-04	6.81E-04	1.80E-04	2.56E-05	1.51E-03	1.77E-03	3.23E-03	1.02E-03	1.14E-04
p-value of difference	<0.001	<0.001	0.3397	<0.001	<0.001	<0.001	<0.001	<0.001	0.1075	0.393
RMSE	0.00042	0.00064	0.00068	0.00020	0.00003	0.00338	0.00473	0.00777	0.00103	0.00012
Average MC paths (%)	16.290	9.974	0.901	0.168	0.062	20.491	14.298	3.608	3.882	3.865
Std dev MC paths	5.17E-03	4.21E-03	6.57E-04	4.60E-04	3.17E-04	2.64E-02	2.06E-02	5.11E-03	1.02E-02	1.23E-02
p-value of difference	0.0022	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
RMSE	0.00526	0.00436	0.01513	0.00121	0.00070	0.11693	0.06901	0.02770	0.03837	0.04055
			40 days					**60 days**		
Original path (%)	44.492	30.534	10.935	0.646	0.000	54.465	38.924	15.521	1.491	0.001
Average mixed paths (%)	42.764	28.627	8.069	0.240	0.000	51.357	35.436	10.682	0.352	0.001
Std dev mixed paths	4.21E-03	4.33E-03	6.47E-03	1.94E-03	2.58E-05	8.06E-03	8.14E-03	1.07E-02	3.39E-03	7.33E-05
p-value of difference	<0.001	<0.001	<0.001	<0.001	0.0038	<0.001	<0.001	<0.001	<0.001	0.2709
RMSE	0.01778	0.01954	0.02937	0.00450	0.00003	0.03210	0.03581	0.04956	0.01188	0.00007
Average mixed/ adjusted paths (%)	43.611	29.489	8.961	0.395	0.004	52.650	36.740	11.972	0.586	0.010
Std dev mixed/ adjusted paths	4.43E-03	4.62E-03	7.25E-03	2.70E-03	1.87E-04	8.72E-03	8.90E-03	1.20E-02	4.84E-03	5.69E-04
p-value of difference	<0.001	<0.001	<0.001	<0.001	0.0042	<0.001	<0.001	<0.001	<0.001	0.0181
RMSE	0.00986	0.01142	0.02102	0.00368	0.00019	0.02013	0.02357	0.03745	0.01026	0.00057
Average MC paths (%)	27.722	20.418	7.437	9.139	10.522	33.035	26.365	10.964	13.187	16.460
Std dev MC paths	7.00E-02	5.05E-02	1.64E-02	3.26E-02	3.87E-02	1.17E-01	7.97E-02	3.32E-02	5.37E-02	7.83E-02
p-value of difference	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
RMSE	0.18167	0.11302	0.03860	0.09093	0.11208	0.24387	0.14866	0.05634	0.12865	0.18218

Table 3. Contd.

	120 days					250 days				
Original path (%)	81.182	63.460	32.856	6.630	0.563	133.423	114.238	74.624	24.362	0.834
Average mixed paths (%)	70.348	51.691	18.079	0.756	0.002	99.727	79.464	33.722	1.826	0.005
Std dev mixed paths	2.47E-02	2.48E-02	2.84E-02	1.03E-02	1.24E-04	8.58E-02	8.57E-02	8.51E-02	3.03E-02	5.80E-04
p-value of difference	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
RMSE	0.11111	0.12026	0.15046	0.05963	0.00561	0.34765	0.35809	0.41774	0.22738	0.00831
Average mixed/ adjusted paths (%)	73.071	54.419	20.616	1.264	0.024	106.108	85.850	39.632	3.142	0.039
Std dev mixed/ adjusted paths	2.73E-02	2.74E-02	3.12E-02	1.43E-02	1.36E-03	9.38E-02	9.37E-02	9.21E-02	4.05E-02	2.27E-03
p-value of difference	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001	<0.001
RMSE	0.08556	0.09446	0.12630	0.05553	0.00556	0.28874	0.29887	0.36177	0.21601	0.00826
Average MC paths (%)	58.713	42.099	23.003	27.603	39.119	93.926	87.289	51.439	56.194	55.976
Std dev MC paths	3.97E-01	2.61E-01	9.82E-02	1.61E-01	2.83E-01	1.99E+00	1.18E+00	4.09E-01	5.78E-01	8.11E-01
p-value of difference	<0.001	<0.001	<0.001	<0.001	<0.001	0.0027	<0.001	<0.001	<0.001	<0.001
RMSE	0.45509	0.33639	0.13894	0.26411	0.47787	2.02753	1.20934	0.46929	0.65870	0.97939

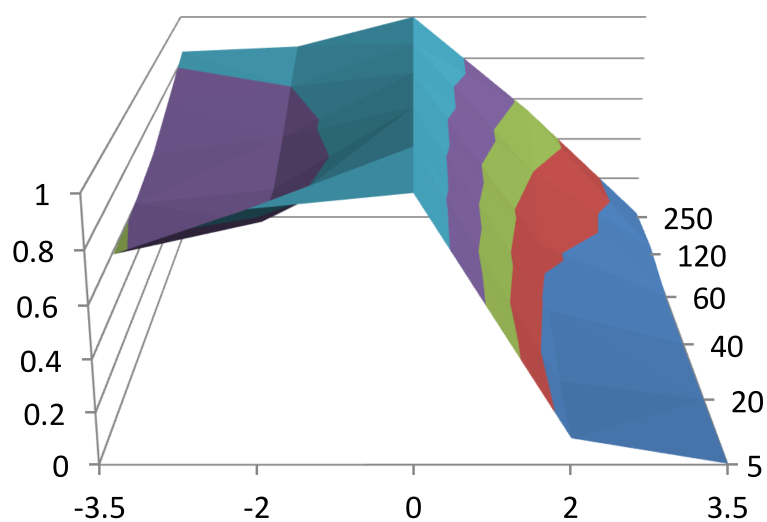

Figure 5. The RMSE of different moneyness relative to ATM on different time of expiration in the MPRD.

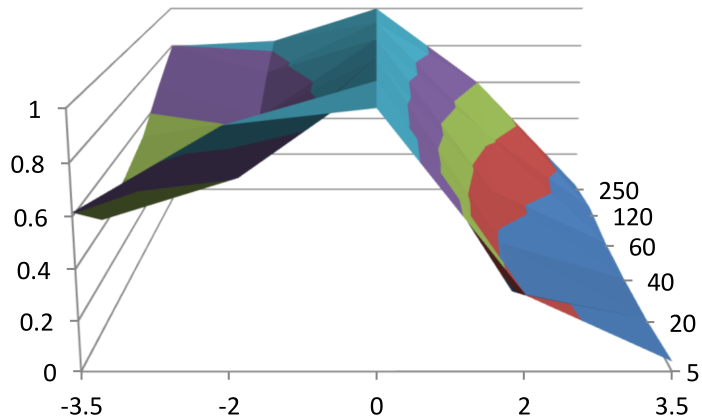

Figure 6. The RMSE of different moneyness relative to ATM on different time to expiration in the adjusted MPRD.

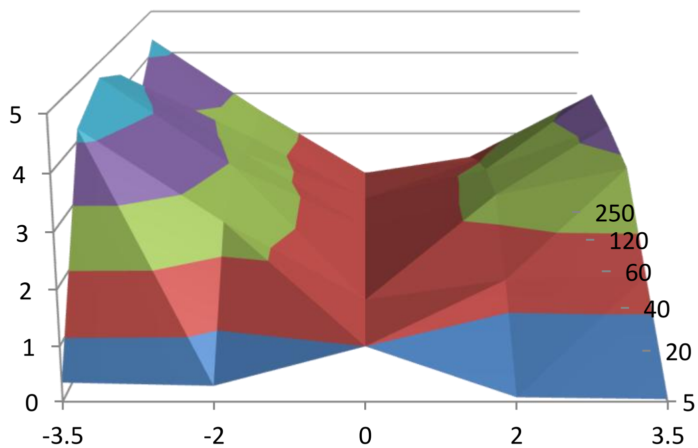

Figure 7. The RMSE of different moneyness relative to ATM on different time to expiration in the MC.

In addition, The effects of MPRD model is similar to the adjusted MPRD model for the pricing option payoff values at the moneyness of deep out of the money within 60 trading days to expiration.

The RMSE of Monte Carlo simulation option payoff values show inconsistence trend on different time to expirations with the MPRDs. Figure 7 shows the RMSE of different moneyness relative to ATM on different time to expiration of the MC. The results show that the RMSE of simulated option payoffs using MPRDs would be lower for the out of the money than in the money and at the money. In addition, this study found that while the time to expiration is longer, the RMSE of simulated option payoff is higher.

Finally, the study uses the root mean squared pricing error (RMSE) suggested by Figlewski (2002) to show which simulation model would have smaller RMSE by examining the RMSE differences between the approaches of the MPRD, adjusted MPRD and Monte Carlo. Table 4 reports the t -Test results of RMSE differences between the MPRDs and Monte Carlo approaches. It shows that the RMSE difference of MPRD1 minus MC presents a significantly negative result. The RMSE difference of MPRD1 minus MC is significantly less than zero, therefore, the RMSE of MPRD1 tends to be less than the RMSE of MC. This study also found similar results while testing the RMSE difference of MPRD2 minus MC and MPRD2 minus MPRD1. The RMSE difference of MPRD2 minus MC and MPRD2 minus MPRD1 also presents a significantly negative result. In other words, the RMSE of MPRD2 tends to be less than the RMSE of MC and MPRD1.

Conclusion

This study discussed whether the historical simulations or the GARCH effect in historical simulations and Monte

Table 4. t-Test of RMSE differences between the MPRDs and Monte Carlo approaches.

Null hypothesis (H_0)(N=30)	RMSE of MPRD1	RMSE of MPRD2	RMSE of MC	Difference of RMSE
RMSE difference of MPRD1-MC 0	0.06806	—	0.28773	-0.21967 ***
RMSE difference of MPRD2-MC 0	—	0.05661	0.28773	-0.23112 ***
RMSE difference of MPRD2- MPRD1 0	0.06806	0. 05661	—	-0.01146 ***

***, ** and * denote significance at the 1, 5 and 10% levels, respectively.

Carlo methods are appropriate to use in simulations. This study proposes an approach to non-parametric and depends on a historical return process. The results of this study differ from the results of the research of Tompkins and D'Ecclesia (2006). The estimated option payoff values using the two MPRDs and Monte Carlo approaches compared with the actual option payoff values. Though the results show that the estimated option payoff values differ significantly from the actual option payoff values, the RMSE of the MPRD and adjusted MPRD approaches are available measures. By contrast, the RMSE of Monte Carlo simulation option payoff values tend to be higher than those of the two MPRDs approaches are.

The study used the MPRDs simulation methods to observe option payoff values more accurately at the moneyness of out of the money, especially for deep out of the money. The RMSE of the simulated option payoffs would be lower for the out of the money (OTM) than in the money (ITM) and at the money (ATM). The study found that the estimated option payoff values lose their accuracy over a long period. The effects of MPRD model is similar to the adjusted MPRD model for the pricing option payoff values at the moneyness of deep out of the money within 60 trading days to expiration.

The study results show that the RMSE of the two MPRD approaches tend to be less than the Monte Carlo simulation on the valuation of option payoffs. The result is consistent with the research of Okten and Eastman (2004). The RMSE of the adjusted MPRD approach tends to be less than the MPRD on the valuation of option payoff values. This result is consistent with Akiray (1989) and Chu and Freund (1996). The reason of MPRD and the adjusted MPRD are better than Monte Carlo simulations in this study may be the statistical features of the disturbances.

For the future lines of this research, these approaches could provide some exotic options valuation from both the original price path and mixture price paths, such as barrier options, look back and Asian options. Also, the MPRDs model could be used for the analysis of the derivatives where mostly parametric models are used and are back-tested on historical data. These approaches could be extended to the simulation of portfolio performance. The single asset case could be considered and the original price paths for multitude assets could be examined in a similar way.

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