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A new ranking method based on cross-efficiency in data envelopment analysis

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Data envelopment analysis (DEA) models compute efficiency score for decision making units (DMUs) and discriminate between efficient and inefficient DMUs. Therefore, they rank DMUs except when multiple DMUs have an efficiency score of 1. This paper proposes a new method for complete ranking of DMUs that is based on cross-efficiency evaluation method. One of the drawbacks of the cross-efficiency evaluation method is the existence of multiple cross-efficiency scores due to the presence of alternative optimal solutions of the dual multiplier model. Hence choosing weights between alternative optimal solutions as part of a procedure for ranking DMUs is problematic. Liang et al. (2008) introduced alternative secondary goals in cross-efficiency evaluation. However, this paper finds that their approach is problematic in some situations. As a result, this paper seeks to introduce a new secondary objective function in cross-efficiency evaluation for removing their difficulties. Numerical demonstration reveals the validity of the proposed method by using a real data set of a case study which consists of 20 Iranian bank branches.

Key words: Data envelopment analysis (DEA), ranking, cross-efficiency.

INTRODUCTION

Data envelopment analysis (DEA) is a mathematical programming technique to evaluate the relative efficiency of decision making units (DMUs) with multiple input-output. The efficiency measure obtained by DEA can be used for ranking DMUs, but this ranking can not be applied to efficient units. One of the interesting research subjects is to discriminate between efficient DMUs. Several authors proposed methods for ranking efficient DMUs (Andersen and Petersen, 1993; Jahanshahloo et al., 2007; Alirezaee and Afsharian, 2007). One of the approaches commonly taken is to employ one of the several cross efficiency models. Cross efficiency was first offered by Sexton et al. (1986) and extended first by Doyle and Green (1994) who responded to the argument that Sexton' approach suffers from the existence of alternate optima in solving for each DMUs best efficiency score. The suggested approaches offered by Doyle and Green were later extended by Liang et al. (2008) who

presented three models for choosing among the alternate optima for each DMU. Each new secondary objective function represents an efficiency evaluation criterion. With these new models, one can compare the efficiency scores and obtain a better picture of cross efficiency stability with respect to multiple DEA weights.

The purpose of the current paper is to extend the method of Liang et al. (2008) by introducing a new criterion. We also aim to disclose that the different secondary objective functions in their paper lead to multiple cross efficiency scores and hence the ranking of units is not possible.

While they proposed that each of various forms of secondary goals may be applicable in some circumstances, no attempt was made to supply a solid justification for choosing the most favorable goal. Also, in attempting to present the difficulties of their method, we show that the existence of alternate optimal weights for each secondary model yields various results.

We illustrate our claims with numerical examples. Further, various forms of secondary goals proposed by Liang et al. (2008) are presented. A new secondary objective function in cross-efficiency evaluation is then introduced

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Table 1. Cross-evaluation matrix.

	DMU$_1$	DMU$_2$	.	.	.	DMU$_n$
DMU$_1$	E$_{11}$	E$_{12}$	.	.	.	E$_{1n}$
DMU$_2$	E$_{21}$	E$_{22}$	.	.	.	E$_{2n}$
.	.					
.	.					
.	.					
DMU$_n$	E$_{n1}$	E$_{n2}$	.	.	.	E$_{nn}$

and numerical example is provided to illustrate the power of our proposed ranking methodology.

CROSS-EFFICIENCY EVALUATION

Suppose that we have n DMUs, where each DMU$_j$, j = 1,…,n, produces the same s outputs in (possibly) different amounts y_{rj}, r = 1,…,s, using the same m inputs x_{ij}, i = 1,…,m, also in (possibly) different amounts. All data are assumed to be nonnegative, but at least, one of the components of every input and output vector is positive. Cross-efficiency is often computed in two phases. The first phase is calculated using the CCR (Charnes, Cooper and Rhodes) multiplier model which is presented as follow:

$$E_{PP}^{*} = \max \quad E_{pp} = \frac{\sum_{r=1}^{s} u_{rp} y_{rp}}{\sum_{i=1}^{m} v_{ip} x_{ip}}$$

$$\text{s.t.} \quad E_{pj} = \frac{\sum_{r=1}^{s} u_{rp} y_{rj}}{\sum_{i=1}^{m} v_{ip} x_{ij}} \le 1, \quad j = 1,2,....,n \quad (1)$$

$$u_{rp} \ge 0, \quad r = 1,...,s$$
$$v_{ip} \ge 0, \quad i = 1,...,m$$

Where u_{rp} and v_{ip} are the assigned weights to the rth output and ith input for DMU$_P$, respectively.
The cross-efficiency of DMU$_t$, using the optimal weights that DMU$_P$ has chosen in model (1), is then:

$$E_{pt} = \frac{\sum_{r=1}^{s} u_{rp}^{*} y_{rt}}{\sum_{i=1}^{m} v_{ip}^{*} x_{it}} \quad p,t = 1,2,...,n \quad (2)$$

Where (*) denotes optimal solutions in model (1). The values obtained from (2) can be organized in a matrix which is called cross-evaluation matrix as shown in Table 1.

For DMU$_t$ (t = 1,…,n), the average of all E_{pt} (p = 1,…,n),

$\overline{E}_t = \frac{1}{n} \sum_{p=1}^{n} E_{pt}$, referred to as the cross-efficiency score

for DMU$_t$. We note that model (1) can be transformed into the following linear programming (LP) problem (Charnes and Cooper, 1962):

$$\max \quad E_{pp} = \sum_{r=1}^{s} u_{rp} y_{rp}$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ip} x_{ip} = 1$$

$$\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \le 0, \quad j = 1,2,...,n \quad (3)$$

$$u_{rp} \ge 0, \quad r = 1,...,s$$
$$v_{ip} \ge 0, \quad i = 1,...,m$$

Model (3) can also be expressed equivalently in the following deviation variable form:

$$\alpha_p^{*} = \min \quad \alpha_p$$

$$\text{s.t.} \quad \sum_{i=1}^{m} v_{ip} x_{ip} = 1$$

$$\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n \quad (4)$$

$$u_{rp} \ge 0, \quad r = 1,...,s$$
$$v_{ip} \ge 0, \quad i = 1,...,m$$
$$\alpha_j \ge 0, \quad j = 1,...,n$$

Where α_p is the deviation variable for DMU$_p$ and α_j the deviation variable for the jth DMU. The deviation variable α_j is referred to as the p-inefficiency of DMU$_j$ by Liang et al. (2008). Liang et al. (2008) proposed that the optimal weights obtained from model (3) (or model (4)) are usually not unique and hence, the cross efficiency defined in (2) is arbitrarily generated, depending on the optimal solution arising from the particular software in use. To resolve this ambiguity, they examined various forms of secondary goals for aiding in cross evaluation. The following model was first considered where the secondary goal is to minimize the sum of inefficiencies:

$$\min \quad \sum_{j=1}^{n} \alpha_j$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1 \tag{5}$$

$$\sum_{r=1}^{s} u_{rp} y_{rp} = 1 - \alpha_p^{*}$$

$$u_{rp} \geq 0, \quad r = 1,...,s$$
$$v_{ip} \geq 0, \quad i = 1,...,m$$
$$\alpha_j \geq 0, \quad j = 1,...,n$$

For the purpose of minimizing the maximal p-inefficiency, Liang et al. (2008) considered as a secondary goal, solving the model:

$$\min \max_{j=1,...,n} \alpha_j$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1 \tag{6}$$

$$\sum_{r=1}^{s} u_{rp} y_{rp} = 1 - \alpha_p^{*}$$

$$u_{rp} \geq 0, \quad r = 1,...,s$$
$$v_{ip} \geq 0, \quad i = 1,...,m$$
$$\alpha_j \geq 0, \quad j = 1,...,n$$

Model (6) can be expressed equivalently in the following form:

$$\min \quad \theta$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1 \tag{7}$$

$$\sum_{r=1}^{s} u_{rp} y_{rp} = 1 - \alpha_p^{*}$$

$$\theta - \alpha_j \geq 0, \quad j = 1,...,n$$

$$u_{rp} \geq 0, \quad r = 1,...,s$$
$$v_{ip} \geq 0, \quad i = 1,...,m$$
$$\alpha_j \geq 0, \quad j = 1,...,n$$

The stated model derives a multiplier bundle that affords the maximum possible efficiency score to the worse-performing DMU. In so doing, the resulting efficiencies of the other DMUs may be forced to be closer in value. Specifically, in attempting to show this worst-performing DMU in its best possible light, the scores of the other (better-performing) DMUs may decrease, hence leading to DMU performance levels that display less variation than was previously the case.

In the spirit of seeking to minimize the variation among the efficiencies of the DMUs, Liang et al. (2008) proposed formalizing this concept through:

$$\min \quad \frac{1}{n} \sum_{j=1}^{n} \left| \alpha_j - \bar{\alpha} \right|$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1 \tag{8}$$

$$\sum_{r=1}^{s} u_{rp} y_{rp} = 1 - \alpha_p^{*}$$

$$u_{rp} \geq 0, \quad r = 1,...,s$$
$$v_{ip} \geq 0, \quad i = 1,...,m$$
$$\alpha_j \geq 0, \quad j = 1,...,n$$

Table 2. Data for five DMUs.

DMU	A	B	C	D	E
Input1	2	2.5	3	6	7.5
Input2	6	7.5	3	2	2.5
Output	1	1	1	1	1

Where, $\bar{\alpha} = \dfrac{1}{n}\sum_{j=1}^{n} \alpha_j$.

With defining $a_j = \dfrac{1}{2}(|\alpha_j - \bar{\alpha}| + (\alpha_j - \bar{\alpha}))$ and

$b_j = \dfrac{1}{2}(|\alpha_j - \bar{\alpha}| - (\alpha_j - \bar{\alpha}))$, model (8) can be easily converted to a linear form as follows:

$$\min \quad \frac{1}{n}\sum_{j=1}^{n}(a_j + b_j)$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} + \alpha_j = 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} v_{ip} x_{ip} = 1 \qquad\qquad (9)$$

$$\sum_{r=1}^{s} u_{rp} y_{rp} = 1 - \alpha_p^{*}$$

$$a_j - b_j = \alpha_j - \frac{1}{n}\sum_{j=1}^{n}\alpha_j, \quad j = 1,2,...,n$$

$u_{rp} \geq 0, \quad r = 1,...,s$
$v_{ip} \geq 0, \quad i = 1,...,m$
$a_j, b_j, \alpha_j \geq 0, \quad j = 1,...,n$

Liang et al. (2008) calculated the new cross-efficiency score for any DMU$_t$ (t=1,...,n) using n optimal weight vectors obtained from the above models.

METHODOLOGY

It is possible that the previous secondary models have alternate optima and this leads to multiple cross-efficiency scores for a given DMU. In other words, the Liang et al. (2008) approach is problematic when there are alternative optimal solutions for each of the stated models ((5), (7) and (9)). Our claim will be further proved by the illustration of a simple example.

To elaborate the problem in their approach, we present a numerical example with the data set as in Table 2. By using model (3), we can see that DMUs A, C and D are CCR efficient. The initial results are given in Table 3, where in the lines we have efficient DMUs and in the columns we have the optimal weights of models (5), (7) and (9).

In order to rank these DMUs, models (5) and (9) are employed to compute the cross-efficiency scores as illustrated in Tables 4 and 5. In the case of each model, the existence of alternative optimal weights for DMU$_c$ leads to different ranking results. That is, each model provides some relevant cross-efficiency scores corresponding to its alternative optimal solutions.

For example, using the weight vectors optimized for DMU$_c$ in model (5), that is, (1, 0.25, 0.08333333) and (1, 0.08333333, 0.25), we obtain the ranking scores of DMU$_A$ as 0.866666674 and 0.733333334, respectively, as documented in the first row of Table 4. Correspondingly, based upon these weight vectors, the ranking related scores of DMU$_D$ are 0.733333334 and 0.866666674.

Therefore, it is appropriate to apply the novel model as secondary objective function. To do this, we refine the selection of the optimal weights made in the first phase of cross-efficiency by choosing those that maximize the efficiency of the n-1 other DMUs separately. For this purpose, suppose that A_p^{*} (p=1,...,n) be the set of optimal weights for DMU$_p$. The following linear fractional programming selects between the alternative optima (if any) provided by (3) as:

$$E_{pt} = \max \quad \frac{u_p^{t} y_t}{v_p^{t} x_t} \qquad\qquad (10)$$

$$\text{s.t.} \quad (u_p, v_p) \in A_p^{*}$$

Model (10) can be rewritten as follows:

$$E_{pt} = \max \quad \frac{\sum_{r=1}^{s} u_{rp} y_{rt}}{\sum_{i=1}^{m} v_{ip} x_{it}}$$

$$\text{s.t.} \quad \sum_{r=1}^{s} u_{rp} y_{rp} = E_{pp}^{*} \qquad\qquad (11)$$

$$\sum_{r=1}^{s} u_{rp} y_{rj} - \sum_{i=1}^{m} v_{ip} x_{ij} \leq 0, \quad j = 1,2,...,n$$

Table 3. The optimal weights of secondary models.

DMU	Model (5)			Model (7)			Model (9		
	u	v_1	v_2	u	v_1	v_2	u	v_1	v_2
A	1	0.25	0.083333	1	0.25	0.083333	1	0.25	0.083333
C	1	0.25	0.083333	1	0.166667	0.166667	1	0.15	0.183333
	1	0.083333	0.25				1	0.183333	0.15
D	1	0.083333	0.25	1	0.083333	0.25	1	0.083333	0.25

Table 4. Results of cross-efficiency score based on alternative optimal weights of model (5).

DMU		
A	0.866666674	0.733333334
C	1.00000000	1.00000000
D	0.733333334	0.866666674

Table 5. Results of cross-efficiency score on alternative optimal weights of model (9).

DMU		
A	0.771428606	0.796491242
C	1.00000000	1.00000000
D	0.796491242	0.771428606

$$\sum_{r=1}^{m} v_{ip} x_{ip} = 1$$

$$u_{rp} \geq 0, \quad r = 1,..., s$$
$$v_{ip} \geq 0, \quad i = 1,..., m$$

Via the three groups of constraints, we consider all the optimal solutions of (3). Now, define $v_p{}^t x_t = \dfrac{1}{k}$ "

where $k > 0$. This linear fractional problem can be trans-

formed to a linear programming (LP) problem (Charnes and Cooper, 1962), which is as follows:

$$E_{pt} = \max \quad \sum_{r=1}^{s} \bar{u}_{rp} y_{rt}$$

s.t. $\sum_{i=1}^{m} \bar{v}_{ip} x_{it} = 1$

$$\sum_{r=1}^{s} \bar{u}_{rp} y_{rp} - k E_{pp}^{*} = 0$$

(12)

$$\sum_{r=1}^{s} \bar{u}_{rp} y_{rj} - \sum_{i=1}^{m} \bar{v}_{ip} x_{ij} \leq 0, \quad j = 1,2,...,n$$

$$\sum_{i=1}^{m} \bar{v}_{ip} x_{ip} = k$$

$$\bar{u}_{rp} \geq 0, \quad r = 1,..., s$$
$$\bar{v}_{ip} \geq 0, \quad i = 1,..., m$$

Table 6. Real data and their CCR efficiencies.

Branch	Inputs			Outputs			CCR efficiency
	Staff	Computer terminals	Space (m^2)	Deposits	Loans	Charge	
DMU1	0.950	0.700	0.155	0.190	0.521	0.293	1.000
DMU2	0.796	0.600	1.000	0.227	0.627	0.462	0.833
DMU3	0.798	0.750	0.513	0.228	0.970	0.261	0.991
DMU4	0.865	0.550	0.210	0.193	0.632	1.000	1.000
DMU5	0.815	0.850	0.268	0.233	0.722	0.246	0.899
DMU6	0.842	0.650	0.500	0.207	0.603	0.569	0.748
DMU7	0.719	0.600	0.350	0.182	0.900	0.716	1.000
DMU8	0.785	0.750	0.120	0.125	0.234	0.298	0.798
DMU9	0.476	0.600	0.135	0.080	0.364	0.244	0.789
DMU10	0.678	0.550	0.510	0.082	0.184	0.049	0.289
DMU11	0.711	1.000	0.305	0.212	0.318	0.403	0.604
DMU12	0.811	0.650	0.255	0.123	0.923	0.628	1.000
DMU13	0.659	0.850	0.340	0.176	0.645	0.261	0.817
DMU14	0.976	0.800	0.540	0.144	0.514	0.243	0.470
DMU15	0.685	0.950	0.450	1.000	0.262	0.098	1.000
DMU16	0.613	0.900	0.525	0.115	0.402	0.464	0.639
DMU17	1.000	0.600	0.205	0.090	1.000	0.161	1.000
DMU18	0.634	0.650	0.235	0.059	0.349	0.068	0.473
DMU19	0.372	0.700	0.238	0.039	0.190	0.111	0.408
DMU20	0.583	0.550	0.500	0.110	0.615	0.764	1.000

Table 7. Results of cross efficiency score based on alternative secondary goals.

Branch	Model (5)	Model (7)	Model (9)
1	0.65172704	0.5926991367	0.5359937936
4	0.8985914917	0.8820317924	0.8018723733
7	0.9655714682	0.9655714653	0.8625539482
12	0.9223182497	0.8959971061	0.804386424
15	0.9999999495	0.8016275456	0.7190537771
17	0.8160075547	0.7898727028	0.6743030822
20	0.7108756472	0.7577520766	0.6865954188

may happen for some ranking methods, do not occur in this method.

$$k \geq \varepsilon$$

Where ε is a non-Archimedean element smaller than a~~ ~ositive real number.

To sum up, we employ a new cross-efficiency of DMU_t in the cross-evaluation matrix. For each efficient DMU_p, the sum of row quantities of other DMUs, $\overline{E}_p = \sum_{\substack{t=1 \\ t \neq p}}^{n} E_{pt}$, is used for ranking.

Hence, DMU_l has a better rank than DMU_k, if both DMUs are the same in efficiency score and $\overline{E}_l \rangle \overline{E}_k$. That is, each DMU that maximizes the sum of the n-1 other DMUs' cross efficiencies has the best rank.

Infeasibility, inability to rank non extreme efficient DMUs, which

NUMERICAL EXAMPLE

Here, we are going to compare the foregoing models by using the inputs and outputs of 20 Iranian bank branches which are presented in Table 6. The data of these branches have been previously analyzed in Amirteimoori and Kordrostami, (2005), and Jahanshahloo et al. (2007). Note that the data are scaled. As can be seen in the last column of Table 6, DMUs 1, 4, 7, 12, 15, 17 and 20 are CCR efficient. Table 7 reports the results of cross efficiency score based upon unique optimal weights obtained from various forms of secondary goals (models

(5), (7) and (9)) that is summarized thus: Optimal weights obtained from model (5):

$$u_{11}=2.119988, u_{21}=1.040665, u_{31}=0.1877675, v_{11}=0.2374179, v_{21}=0, v_{31}=4.996471$$
$$u_{14}=0.7289014, u_{24}=0.2977297, u_{34}=0.6711569, v_{14}=1.086949, v_{24}=0, v_{34}=0.2847113$$
$$u_{17}=0.8355995, u_{27}=0.9421343, u_{37}=0, v_{17}=0.8490030, v_{27}=0, v_{37}=1.113048$$
$$u_{1\,12}=0.8593443, u_{2\,12}=0.9689064, u_{3\,12}=0, v_{1\,12}=0.8731287, v_{2\,12}=0, v_{3\,12}=1.144677$$
$$u_{1\,15}=0.8583285, u_{2\,15}=0.5407308, u_{3\,15}=0, v_{1\,15}=0, v_{2\,15}=0.9713109, v_{3\,15}=0.1716769$$
$$u_{1\,17}=0.9601902, u_{2\,17}=0.9135829, u_{3\,17}=0, v_{1\,17}=0.5816888, v_{2\,17}=0.3191518, v_{3\,17}=1.106440$$
$$u_{1\,20}=0.9392494, u_{2\,20}=0.3836491, u_{3\,20}=0.8648408, v_{1\,20}=1.400623, v_{2\,20}=0, v_{3\,20}=0.3668739$$

Optimal weights obtained from model (7):

$$u_{11}=2.119988, u_{21}=1.040665, u_{31}=0.1877675, v_{11}=0.2374179, v_{21}=0, v_{31}=4.996471$$
$$u_{14}=0.9315927, u_{24}=0.3457089, u_{34}=0.6017146, v_{14}=0.7985953, v_{24}=0.5622092, v_{34}=0$$
$$u_{17}=0.9997408, u_{27}=0.9089413, u_{37}=0, v_{17}=0.704374, v_{27}=0.6765715, v_{37}=0.2503206$$
$$u_{1\,12}=0.04200873, u_{2\,12}=1.077825, u_{3\,12}=0, v_{1\,12}=0.7104028, v_{2\,12}=0.3931596, v_{3\,12}=0.6600376$$
$$u_{1\,15}=0.8612717, u_{2\,15}=0.4411961, u_{3\,15}=0.2360703, v_{1\,15}=0.2502842, v_{2\,15}=0.7310307, v_{3\,15}=0.2979469$$
$$u_{1\,17}=0.1132427, u_{2\,17}=0.9898082, u_{3\,17}=0, v_{1\,17}=0.5818870, v_{2\,17}=0.5202538, v_{3\,17}=0.5168812$$
$$u_{1\,20}=1.202371, u_{2\,20}=0.4461934, u_{3\,20}=0.7766102, v_{1\,20}=1.030717, v_{2\,20}=0.7256221, v_{3\,20}=0$$

Optimal weights obtained from model (9):

$$u_{11}=2.119988, u_{21}=1.040665, u_{31}=0.1877675, v_{11}=0.2374179, v_{21}=0, v_{31}=4.996471$$
$$u_{14}=0.4094238, u_{24}=0.3803971, u_{34}=0.6805702, v_{14}=0.9144441, v_{24}=0.2791916, v_{34}=0.2640500$$
$$u_{17}=0.3977990, u_{27}=0.7740786, u_{37}=0.3225277, v_{17}=1.113190, v_{27}=2667018, v_{37}=0.1131304$$
$$u_{1\,12}=0.4321273, u_{2\,12}=1.025838, u_{3\,12}=0, v_{1\,12}=0.8576898, v_{2\,12}=0.1115848, v_{3\,12}=0.9093468$$
$$u_{1\,15}=0.9900377, u_{2\,15}=0.03802422, u_{3\,15}=0, v_{1\,15}=0.2888438, v_{2\,15}=0.7244091, v_{3\,15}=0.2532296$$
$$u_{1\,17}=0.5976200, u_{2\,17}=0.9462142, u_{3\,17}=0, v_{1\,17}=0.4098929, v_{2\,17}=0.8055163, v_{3\,17}=0.5209623$$
$$u_{1\,20}=0.5018829, u_{2\,20}=0.4862679, u_{3\,20}=0.8452070, v_{1\,20}=1.166068, v_{2\,20}=0.3331228, v_{3\,20}=0.2739295$$

It can be seen that models (5), (7) and (9) yield various results and hence the ranking of units is not possible. In other words, the results of cross-efficiency score are not unique or stable. For example, in the second column, DMU15 gets the most ranking related score, whilst in the third column, it has 4th rank amongst DMUs. DMU17, which is located in the sixth position by the last column, gets the fifth rank according to third column and so on. The question is: which of secondary objective functions must be used in this situation. Although Liang et al. (2008) demonstrated that their proposed models with the different objective functions can be applied under different circumstances, but it is difficult in general to choose between different criteria in determining the final cross efficiency.

Using the proposed ranking methodology, we obtain the new cross-efficiency scores of the seven DEA efficient bank branches. The results are given in Table 8, from which it is seen that bank branch 15 has the biggest ranking score and is therefore the best of the 20 bank branches. It can be seen easily that the efficiency ranking is unique, whereas the cross-efficiency evaluation may result in different efficiency rankings, depending on which of the secondary models is utilized. Indeed, the proposed ranking methodology can successfully distinguish between all DMUs and hence makes a new contribution

Table 8. Results of ranking by proposed method.

Branch	Model (12)	The proposed rank
1	11.1033995	7
4	14.278857	2
7	14.1432995	3
12	13.2885239	5
15	14.7564376	1
17	13.8135663	4
20	11.9638897	6

to DEA ranking.

Conclusion

In this paper, we developed a ranking methodology for DMUs by introducing a new criterion. We also compared our method with the method developed by Liang et al. (2008). We showed that our method is superior to this method in removing their deficiencies. What we should point out here is that the computation complexity of our method can increase with increasing the number of DMUs, so, how to provide a better method is an interesting issue for future research.

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