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Full Length Research Paper

Optimal inventory polices with order-size dependent trade credit under delayed payment and cash discount

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In today's competitive business environment, it is very common that suppliers are always willing to provide the purchaser with certain incentives. As a result, suppliers frequently offer two distinct alternatives to increase possible volume of procurement. One is that if the order quantity is greater than or equal to a predetermined quantity, the purchaser obtains a longer permissible delay in payments. The other is that if the order quantity is greater than or equal to a predetermined quantity, the purchaser gets a cash discount with a shorter permissible delay in payments. Viewed from above perspective, this study develops an inventory model with order-size dependent trade credit, in which the supplier provides not only a permissible delay but also a cash discount to the customers. Moreover, an efficient algorithm is provided to obtain the optimal solution, and then an empirical case is investigated to illustrate the theoretical results from both the supplier and customer viewpoints.

Key words: Inventory, deteriorating items, delay in payments, trade credit, cash discount.

INTRODUCTION

The traditional inventory model assumes that inventory is depleted by a constant demand rate. In practice, however, the inventory loss of deterioration items, such as volatile liquids, medicines, electronic components, and frozen seafood products incurred by deterioration should not be neglected. Ghare and Schrader (1963) first analyzed the decaying inventory problem and developed an economic order quantity model with a constant decaying rate. Covert and Philip (1973) derived a revised EOQ model under the assumption of Weibull distribution for deterioration. Philip (1974) then proposed the inventory model with a three-parameter Weibull distribution deterioration rate with no shortages. Shah (1977) extended Philip's (1974) model with consideration to allow shortage. On the other hand, the traditional EOQ model assumes that the customer must pay for goods when supplier upon receipt. However, suppliers often permit their customers to delay payment if the outstanding amount is paid within the fixed settlement period and the order quantity is large. The impact of trade credit on inventory polices has attracted the attention of many researchers. Goyal (1985) derived an EOQ model under the condition of a permissible delay in payment. Aggarwal and Jaggi (1995) then extended Goyal's model to consider deteriorating items. Related studies in this topic can be referred to Jamal et al. (1997), Liao et al. (2000), Chang and Dye (2001), Chang et al. (2001), and Chang et al. (2002). Recently, Ouyang et al. (2006) develop an appropriate model for non-instantaneous deteriorating items when the supplier provides permissible delay in payments. In addition, Liao (2008) developed an EOQ model with exponentially deteriorating items under two-level trade credit.

In the above models, the studies discuss the supplier provides a permissible delay in payments, and did not consider the effects of the cash discount. In some situations, the supplier also may offer a cash discount to encourage retailer to pay for his purchases quickly. Huang and Chung (2003) and Ouyang et al. (2005) proposed an optimal policy for deteriorating items when the supplier offers not only a permissible delay in payment but also a cash discount. Furthermore, Huang and Liao (2008) discussed a simple method to locate

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Table 1. Summary of the different contributions by previous articles.

Author (Reference)	Cash discount	Delay in payment	Deterioration item	trade credit depending on the ordering quantity	Other conditions
Ghare and Schrader (1963)			Yes		
Shah (1977)			yes		allow shortage
Goyal (1985)		Yes			
Aggarwal and Jaggi (1995)		yes	yes		
Jamal et al. (1997)		yes	yes		allow shortage
Chang and Dye (2001)		yes	yes		partial backlogging
Liao et al. (2000)		yes	yes		consider inflation
Chang et al. (2002)		yes	yes		time-value of money
Ouyang et al. (2006)		yes	yes		non-instantaneous deteriorating items
Liao (2008)		yes	yes		Two-levels trade credit.
Huang and Chung (2003)	yes	yes			
Ouyang et al. (2005)	yes	yes			non-instantaneous receipt
Huang and Liao (2008)	yes	yes	yes		
Ho et al. (2008)	yes	yes			integrated supplier–buyer inventory model
Chang et al. (2003)		yes	yes	yes	
Chung and Liao (2004)					
Chang (2004)		yes	yes	yes	Consider Inflation
Chung and Liao (2006)		yes	yes	yes	adopt the discounted cash-flows (DCF) approach
Chang et al. (2010)					
Chang et al. (2009)		yes	yes	yes	Integrated vendor-buyer inventory system
Kreng and Tan (2010)		yes		yes	two-level trade credit.

the optimal solution for exponentially deteriorating items when the supplier permits not only a cash discount but also a permissible delay. Ho et al. (2008) established an integrated supplier–buyer inventory model with the assumption that the market demand is sensitive to the retail price and discuss the trade credit policy including a cash discount and delayed payment. Chung (2010) removed the shortcoming of Ouyang et al. (2005) and presented a solution procedure to search for the entirely optimal order cycles.

The problem of providing a permissible delay in payment for a large order has been considered in literature such as Chang et al. (2003), Chung and Liao (2004), Chang (2004), and Chung et al (2005). In addition, Chung and Liao (2006) adopt the discounted cash-flows (DCF) approach to consider the deteriorating model under the condition of an order-size-dependent. Chang et al (2009) presented a stylized model to determine the optimal strategy for an integrated vendor–buyer inventory system under the condition of trade credit linked to the order quantity. Chang et al. (2010) used the DCF approach to establish an inventory model for deteriorating items with trade credit based on the order quantity. Kreng and Tan (2010) discussed the optimal replenishment decisions under two levels of trade credit policy to take the order quantity into account within the economic order quantity framework

According to current trends in business transactions, the supplier may offer longer credit periods for larger purchase amounts and may also provide cash discounts as the marketing strategies to encourage customers to pay for purchases quickly. In all of the above models, very limited studies have ever tried to consider the supplier providing, both a cash discount and permissible delay in payment, if the buyer commits to ordering a predetermined quantity, however, which is very common in reality. Summary of the different contributions by previous articles are presented in Table 1.

Additionally, previous inventory models of trade credit only focus on the customer viewpoint. However, suppliers often provide cash discounts and trade credit as a marketing strategy to increase sales and reduce inventories. Therefore, optimal inventory policies that consider both the supplier and customer viewpoints are more reasonable than those that consider only from one perspective.

Therefore, this study develops an inventory model with order-size dependent trade credit, in which the supplier provides not only a permissible delay but also a cash dis count to the customers. Moreover, an efficient algorithm is provided to obtain the optimal solution, and then numerical examples are presented to illustrate the theoretical results. Further, an empirical case is investigated to illustrate the theoretical results from both the supplier and customer viewpoints.

Model formulation

Based on the above arguments, the following notation and assumptions are used throughout this paper.

Notation

c unit purchasing cost ($/unit)
A ordering cost ($/order)
h unit stock holding cost, excluding capital opportunity cost ($/(year.unit))
I_e annual simple interest rate that can be earned ($/($.year))
I_p annual simple interest charges for an inventory item ($/($.year))
α cash discount rate, $0 < \alpha < 1$
T replenishment cycle time, in years
$I(t)$ the level of inventory at time t, $0 \leq t \leq T$
D annual demand
Q order size
s unit selling price ($/unit)
T^* the optimal cycle time of $TC(T)$
Q_d quantity at which the cash discount and delay in payments is permitted
θ a positive number representing the inventory deteriorating rate
T_d cycle time beyond which the cash discount and delay in payment are permitted,

$$T_d = \frac{1}{\theta}\ln\left(\frac{\theta}{D}Q_d + 1\right)$$

M_1 the period of cash discount in years
M_2 the period of trade credit in years, $M_1 < M_2$
$TC_1(T)$ the annual total relevant cost, when payment is paid at time M_1
$TC_2(T)$ the annual total relevant cost, when payment is paid at time M_2
$TC(T)$ the annual total relevant cost, which is a function of T

Assumptions

1. Demand is deterministic and constant.
2. Replenishment is instantaneous with a known and constant lead time.
3. Shortages are not allowed, and there is no supplier uncertainty.
4. The time period is infinite.
5. The supplier offers a cash discount when $Q \geq Q_d$ and payment is made with M_1. Otherwise, the full payment is due at M_2.
6. During the time the account is not settled, generated sales revenue is deposited in an interest-bearing account with rate I_e. At the end of the period (i.e., M_1 or M_2), the retailer starts paying for the interest charges for the items in stock with rate I_p.
7. The on-hand inventory deteriorates at a constant rate θ.
8. $s \geq c$ and $I_p \geq I_e$.

The inventory level $I(t)$ gradually decreases mainly to meet demands and partly due to deterioration. Hence, the variation of inventory with respect to time can be described by the following differential equation

$$\frac{dI(t)}{dt} + \theta I(t) = -D, \quad 0 \leq t \leq T, \qquad (1)$$

with the boundary conditions $I(T) = 0$. The solution of Equation (1) is given by

$$I(t) = \frac{D}{\theta}(e^{\theta(T-t)} - 1), \qquad 0 \leq t \leq T.$$

Hence, the order quantity for each replenishment cycle is

$$Q = I(0) = \frac{D}{\theta}(e^{\theta T} - 1).$$

Noted that the annual total relevant cost consists of (a) ordering cost, (b) the annual purchasing cost, (c) the stock holding cost (excluding interest charges), (d) the interest charges for unsold items after the permissible delay M_1 or M_2, and (e) the interest earned from sales revenue during the permissible period $[0, M_1]$ or $[0, M_2]$.

The first three elements from annual total cost are obtained as follows:

Annual order cost = A/T

Annual stock holding cost =
$$\frac{h}{T}\int_0^T I(t)dt = \frac{h}{T}\int_0^T \left[\frac{D}{\theta}(e^{\theta(T-t)} - 1)\right]dt = hD(e^{\theta T} - \theta T - 1)/\theta^2 T$$

In addition, there are two situations to occur in purchasing cost per year.

1. $Q < Q_d$ the retailer must pay the amount of purchasing cost as soon as the items were received, or $Q \geq Q_d$ the fixed trade credit period M_2 is permitted. Hence,

Annual purchasing cost = $c\dfrac{Q}{T}=\dfrac{cD(e^{\theta T}-1)}{\theta T}$

2. $Q\geq Q_d$ and the payment is paid at M_1 to get the cash discount. Hence,

Annual purchasing cost = $c(1-\alpha)\dfrac{Q}{T}=\dfrac{c(1-\alpha)D(e^{\theta T}-1)}{\theta T}$

We noted that the T_d can be determined uniquely by Equation (3) and the inequality $Q<Q_d$ holds if and only if $T<T_d$ where $T_d=\dfrac{1}{\theta}\ln(\dfrac{\theta}{D}Q_d+1)$. As a result, there are three situations to occur: (A) $T_d<M_1<M_2$, (B) $M_1<T_d<M_2$ and (C) $M_1<M_2<T_d$.

(A) $T_d<M_1<M_2$

On the other hand, the capital opportunity cost refers to the interest payable on stock held beyond the credit period less interest earned during the credit period. Hence, there are five cases to discuss the annual capital opportunity cost.

Case 1: For $0<T<T_d$, the retailer must pay the amount of purchasing cost as soon as the items were received, which is similar to the classical EOQ model. As a result, there is no interest earned, however, the interest charged for all unsold items starts at the initial time. Therefore,

Annual capital opportunity cost = $cI_p\int_0^T[\dfrac{D}{\theta}(e^{\theta(T-t)}-1)]dt/T=\dfrac{cI_pD}{\theta^2T}(e^{\theta T}-\theta T-1)$.

Case 2: $T_d\leq T<M_1$**.** For this case of $T_d\leq T<M_1$, the cash discount period M_1 is longer than the replenishment cycle time T. As a result, there is no interest charged. However, the customer can sell the items and earn interest with rate I_e until the end of the trade M_1. Therefore,

Annual capital opportunity cost = $-\dfrac{sI_e[\int_0^T Dtdt+DT(M_1-T)]}{T}=sI_eD(\dfrac{T}{2}-M)$

Case 3: $T_d\leq M_1\leq T$. For $T_d\leq M_1\leq T$, the replenishment cycle time T is longer than or equal to both T_d and M_1. Therefore, the cash discount is permitted and the total relevant cost includes both the interest charged and interest earned. Therefore,

Annual capital opportunity cost

$$=\frac{c(1-\alpha)I_p\int_{M_1}^T[\frac{D}{\theta}(e^{\theta(T-t)}-1)]dt-sI_e\int_0^{M_1}Dtdt}{T}$$

$$=\frac{c(1-\alpha)I_pD[e^{\theta(T-M_1)}-\theta(T-M_1)-1)}{\theta^2T}-\frac{sI_eDM_1^{\,2}}{2T}$$

Case 4: $T_d\leq T\leq M_2$. For this case of $T_d\leq T\leq M_2$ there is a permissible delay M_2 is longer than the replenishment cycle time T and the delay in payments is permitted. Thus, Case 4 is the similar to Case 2. Therefore,

Annual capital opportunity cost = $sI_eD(\dfrac{T}{2}-M)$.

Case 5: $T_d\leq M_2\leq T$

In this case, the replenishment cycle time T is greater than or equal to both T_d and M_2. Since the payment is paid at time M_2, there is no cash discount in this case. Therefore,

Annual capital opportunity cost

$$=\frac{cI_pD[e^{\theta(T-M_2)}-\theta(T-M_2)-1)}{\theta^2T}-\frac{sI_eDM_2^{\,2}}{2T}$$

We denote $TC_1(T)$ = the annual total relevant cost when payment is paid at M_1 or cash discount is not permitted (i.e., $T<T_d$) and $TC_2(T)$ = the annual total relevant cost when payment is paid at M_2 or delay in payments is not permitted, respectively. Here, the annual total relevant cost consists of ordering cost, purchasing cost, stock holding cost and the capital opportunity cost. Based on above arguments, the annual total relevant cost can be written as

$$TC(T)=\begin{cases}TC_1(T) & \text{payment is paid at } M_1 \text{ or cash discount is not permitted}\\ TC_2(T) & \text{payment is paid at } M_2 \text{ or delay in payments is not permitted}\end{cases}$$

Therefore, we have

$$TC_1(T)=\begin{cases}TC_{11}(T) & \text{if } 0<T<T_d\\ TC_{12}(T) & \text{if } T_d\leq T<M_1\\ TC_{13}(T) & \text{if } T\geq M_1\end{cases} \qquad (4a, b, c)$$

and

$$TC_2(T)=\begin{cases}TC_{21}(T) & \text{if } 0<T<T_d\\ TC_{22}(T) & \text{if } T_d\leq T<M_2\\ TC_{23}(T) & \text{if } T\geq M_2\end{cases}, \qquad (5a, b, c)$$

where

$$TC_{11}(T)=\frac{A}{T}+\frac{cD(e^{\theta T}-1)}{\theta T}+\frac{hD(e^{\theta T}-\theta T-1)}{\theta^2 T}+\frac{cI_pD(e^{\theta T}-\theta T-1)}{\theta^2 T} \quad (6)$$

$$TC_{12}(T)=\frac{A}{T}+\frac{c(1-\alpha)D(e^{\theta T}-1)}{\theta T}+\frac{hD(e^{\theta T}-\theta T-1)}{\theta^2 T}-sI_eD(M_1-\frac{T}{2}) \quad (7)$$

$$TC_{13}(T)=\frac{A}{T}+\frac{c(1-\alpha)D(e^{\theta T}-1)}{\theta T}+\frac{hD(e^{\theta T}-\theta T-1)}{\theta^2 T}+\frac{c(1-\alpha)I_pD[e^{\theta(T-M_1)}-\theta(T-M_1)-1]}{\theta^2 T}-\frac{sI_eDM_1^{\ 2}}{2T} \quad (8)$$

$$TC_{21}(T)=TC_{11}(T) \quad (9)$$

$$TC_{22}(T)=\frac{A}{T}+\frac{cD(e^{\theta T}-1)}{\theta T}+\frac{hD(e^{\theta T}-\theta T-1)}{\theta^2 T}-sI_eD(M_2-\frac{T}{2}) \quad (10)$$

$$TC_{23}(T)=\frac{A}{T}+\frac{cD(e^{\theta T}-1)}{\theta T}+\frac{hD(e^{\theta T}-\theta T-1)}{\theta^2 T}+\frac{cI_pD[e^{\theta(T-M_2)}-\theta(T-M_2)-1]}{\theta^2 T}-\frac{sI_eDM_2^{\ 2}}{2T} \quad (11)$$

In practice, the value for the deterioration rate θ is usually very small. Utilizing a truncated Taylor series expansion for the exponential term, we have $e^{\theta T}\approx 1+\theta T+(\theta T)^2/2$, as θT is small. Using this approximation, the annual total relevant cost $TC_{ij}(T)$ for $i=1,2,\ j=1,2,3,$ can be rewritten as

$$TC_{11}(T)\approx\frac{A}{T}+\frac{cD(2+\theta T)}{2}+\frac{hDT}{2}+\frac{cI_pDT}{2}, \quad (12)$$

$$TC_{12}(T)\approx\frac{A}{T}+\frac{c(1-\alpha)D(2+\theta T)}{2}+\frac{hDT}{2}-sI_eD(M_1-\frac{T}{2}), \quad (13)$$

$$TC_{13}(T)\approx\frac{A}{T}+\frac{c(1-\alpha)D(2+\theta T)}{2}+\frac{hDT}{2}+\frac{c(1-\alpha)I_pD(T-2M_1+\frac{M_1^{\ 2}}{T})}{2}-\frac{sI_eDM_1^{\ 2}}{2T}, \quad (14)$$

$$TC_{21}(T)=TC_{11}(T), \quad (15)$$

$$TC_{22}(T)\approx\frac{A}{T}+\frac{cD(2+\theta T)}{2}+\frac{hDT}{2}-sI_eD(M_2-\frac{T}{2}) \quad (16)$$

and

$$TC_{23}(T)\approx\frac{A}{T}+\frac{cD(2+\theta T)}{2}+\frac{hDT}{2}+\frac{cI_pD(T-2M_2+\frac{M_2^{\ 2}}{T})}{2}-\frac{sI_eDM_2^{\ 2}}{2T}, \quad (17)$$

We get that $TC_{11}(T)>TC_{12}(T)$ if $T>0$ and

$$TC_{11}(T)>TC_{13}(T) \text{ if } T\geq M_1. \quad (18)$$

$TC_{12}(M_1)=TC_{13}(M_1)$

$TC_{11}(T_d)>TC_{12}(T_d)$. So, $TC_1(T)$ is continuous except T = T_d. Similarly, we can obtain that $TC_{21}(T)>TC_{22}(T)$ for $T>0$, $TC_{21}(T)>TC_{23}(T)$ for $T\geq M_2$, $TC_{22}(M_2)=TC_{23}(M_2)$, and $TC_{21}(T_d)>TC_{22}(T_d)$. Therefore, $TC_2(T)$ is continuous except T = T_d. Equations (12) - (17) yield

$$TC_{11}^{'}(T)=-\frac{A}{T^2}+\frac{D(h+c\theta+cI_p)}{2}, \quad (19)$$

$$TC_{12}^{'}(T)=-\frac{A}{T^2}+\frac{D(h+c(1-\alpha)\theta+sI_e)}{2}, \quad (20)$$

$$TC_{13}^{'}(T)=\frac{-2A+[sI_e-c(1-\alpha)I_p]DM_1^{\ 2}}{2T^2}+\frac{D[h+c(1-\alpha)(\theta+I_p)]}{2}, \quad (21)$$

$$TC_{21}^{'}(T)=-\frac{A}{T^2}+\frac{D(h+c\theta+cI_p)}{2} \quad (22)$$

$$TC_{22}^{'}(T)=-\frac{A}{T^2}+\frac{D(h+c\theta+sI_e)}{2}, \quad (23)$$

$$TC_{23}^{'}(T)=\frac{-2A+(sI_e-cI_p)DM_2^{\ 2}}{2T^2}+\frac{D(h+c\theta+cI_p)}{2}, \quad (24)$$

$$TC_{11}^{''}(T)=\frac{2A}{T^3}, \quad (25)$$

$$TC_{12}^{''}(T)=\frac{2A}{T^3}, \quad (26)$$

$$TC_{13}^{''}(T)=\frac{2A+[c(1-\alpha)I_p-sI_e]DM_1^{\ 2}}{T^3}, \quad (27)$$

$$TC_{21}^{''}(T)=\frac{2A}{T^3} \quad (28)$$

$$TC_{22}^{''}(T)=\frac{2A}{T^3} \quad (29)$$

and

$$TC_{23}^{''}(T) = \frac{2A + (cI_p - sI_e)DM_2^{\ 2}}{T^3}. \qquad (30)$$

From Equations (25) and (26), we have $TC_{11}^{''}(T) > 0$ and $TC_{12}^{''}(T) > 0$ for all T >0. So both TC_{11} (T) and TC_{12} (T) are convex on $(0, \infty)$. However, Equation (26) implies that TC_{13} (T) is convex on T > 0 if $2A + [c(1-\alpha)I_p - sI_e]DM_1^2 > 0$. Furthermore, $TC_{12}^{'}(M_1) = TC_{13}^{'}(M_1)$. Therefore, TC_1 is convex on T > 0 if $2A + [c(1-\alpha)I_p - sI_e]DM_1^2 > 0$.

On the other hand, Equations (28) to (29) implies that TC_{21} (T) and TC_{22} (T) are convex on $T > 0$. However, Equation (30) implies TC_{23} (T) are convex on $T > 0$ if $2A + (cI_p - sI_e)DM_2^2 > 0$. Therefore, TC_2 is convex on T > 0 if $2A + (cI_p - sI_e)DM_2^2 > 0$.

(B) $M_1 < T_d < M_2$

If $M_1 < T_d < M_2$ Equations 4(a)-(c) will be modified as follows:

$$TC_1(T) = \begin{cases} TC_{11}(T) & \text{if } 0 < T < T_d \\ TC_{13}(T) & \text{if } T_d \le T \end{cases}$$

Since $M_1 < T_d < M_2$, Equation (18) yields $TC_{11}(T_d) > TC_{13}(T_d)$. So, $TC_1(T)$ is continuous except T = T_d. In addition, $TC_{11}(T)$ is convex on $T > 0$ and TC_{13} (T) is convex on $T > 0$ if $2A + [c(1-\alpha)I_p - sI_e]DM_1^2 > 0$.

On the other hand, If $M_1 < T_d < M_2$ Equation 5(a)-(c) will be modified as follows:

$$TC_2(T) = \begin{cases} TC_{21}(T) & \text{if } 0 < T < T_d \\ TC_{22}(T) & \text{if } T_d \le T < M_2 \\ TC_{23}(T) & \text{if } T \ge M_2 \end{cases}$$

Hence, $TC_2(T)$ is continuous except T = T_d. TC_{21}(T) and TC_{22}(T) are convex on $T > 0$, however, TC_{23}(T) is convex on $T > 0$ if $2A + (cI_p - sI_e)DM_2^2 > 0$.

(C) $M_1 < M_2 < T_d$

If $M_1 < M_2 < T_d$ Equation 4(a) to (c) will be modified as follows:

$$TC_1(T) = \begin{cases} TC_{11}(T) & \text{if } 0 < T < T_d \\ TC_{13}(T) & \text{if } T_d \le T \end{cases}$$

Since $M_1 < M_2 < T_d$, Equation (18) yields $TC_{11}(T_d) > TC_{13}(T_d)$. So, $TC_1(T)$ is continuous except T = T_d. In addition, $TC_{11}(T)$ is convex on $T > 0$ and TC_{13} (T) is convex on $T > 0$ if $2A + [c(1-\alpha)I_p - sI_e]DM_1^2 > 0$.

On the other hand, If $M_1 < M_2 < T_d$ Equation 5(a)-(c) will be modified as follows:

$$TC_2(T) = \begin{cases} TC_{21}(T) & \text{if } 0 < T < T_d \\ TC_{23}(T) & \text{if } T_d \le T \end{cases}$$

Hence, $TC_2(T)$ is continuous except T = T_d. TC_{21} (T) and TC_{22} (T) are convex on $T > 0$, and TC_{23} (T) is convex on $T > 0$ if $2A + (cI_p - sI_e)DM_2^2 > 0$.

Decision rule for optimal cycle time and optimal payment policy

Inventory models are developed under a condition in which a cash discount and a permissible payment delay are offered depending on order quantity. When the order quantity is less than the fixed quantity ($Q < Q_d$), the cash discount and delay in payment are not permitted, and the payment must be made immediately. Otherwise, either the payment is made at the time of M_1 with a cash discount or the payment is made at the time of M_2 without a cash discount. Therefore, three situations are possible: (A) $T_d < M_1 < M_2$; (B) $M_1 < T_d < M_2$; and (C) $M_1 < M_2 < T_d$.

(A) $T_d < M_1 < M_2$

Let $T_{11}^{\ *} = T_{21}^{\ *} = \sqrt{\dfrac{2A}{D[h + c\theta + cI_p]}}$,

$$T_{12}^{\ *} = \sqrt{\frac{2A}{D[h + c(1-\alpha)\theta + sI_e]}} \text{ and}$$

$$T_{22}^{\ *} = \sqrt{\frac{2A}{D[h + c\theta + sI_e]}}$$

Then, $TC_{11}'(T_{11}^{\ *}) = TC_{12}'(T_{12}^{\ *}) = TC_{21}'(T_{21}^{\ *}) = TC_{22}'(T_{22}^{\ *}) = 0$. By the convexity of $TC_{1i}(T)$ for i =1, 2, and $TC_{2i}(T)$, for i =1, 2.

We see $$TC_{1i}^{'}(T) \begin{cases} < 0 & \text{if } T < T_{1i}^{\ *} \\ = 0 & \text{if } T = T_{1i}^{\ *} \\ > 0 & \text{if } T > T_{1i}^{\ *} \end{cases} \quad i=1,2 \qquad (31a,b,c)$$

and

$$TC_{2i}^{'}(T)\begin{cases}<0 \text{ if } T<T_{2i}^{*}\\ =0 \text{ if } T=T_{2i}^{*}\\ >0 \text{ if } T>T_{2i}^{*}\end{cases} \quad i=1,2 \qquad (32\text{ a,b,c})$$

However, equations (27) and (30) implies that $TC_{13}(T)$ and $TC_{23}(T)$ are convex on $T>0$ if $2A+[c(1-\alpha)I_p-sI_e]DM_1^2>0$ and $2A+(cI_p-sI_e)DM_2^2>0$, respectively.

Let $T_{13}^{*}=\sqrt{\dfrac{2A+[c(1-\alpha)I_p-sI_e]DM_1^2}{D[h+c(1-\alpha)(\theta+I_p)]}}$ and

$$T_{23}^{*}=\sqrt{\frac{2A+(cI_p-sI_e)DM_2^2}{D(h+c\theta+cI_p)}}\text{, then}$$

$TC_{13}'(T_{13}^{*})=TC_{23}'(T_{23}^{*})=0$. Therefore,

$$TC_{i3}^{'}(T)\begin{cases}<0 \text{ if } T<T_{i3}^{*}\\ =0 \text{ if } T=T_{i3}^{*}\\ >0 \text{ if } T>T_{i3}^{*}\end{cases} \quad i=1,2 \qquad (33\text{ a,b,c})$$

We find that Equations (31 a,b,c) imply that $TC_{1i}(T)$ is decreasing on $(0,T_{1i}^{*}]$ and increasing on $[T_{1i}^{*},\infty)$ for i=1, 2. Equations (32 a,b,c) imply that $TC_{2i}(T)$ is decreasing on $(0,T_{2i}^{*}]$ and increasing on $[T_{2i}^{*},\infty)$ for i=1, 2. Therefore, equations (19) to (24) give:

$$TC_{11}^{'}(T_d)=TC_{21}^{'}(T_d)=\frac{-2A+(h+c\theta+cI_p)DT_d^2}{2T_d^2}, \quad (34)$$

$$TC_{12}^{'}(T_d)=\frac{-2A+[h+c(1-\alpha)\theta+sI_e]DT_d^2}{2T_d^2}, \quad (35)$$

$$TC_{12}^{'}(M_1)=TC_{13}^{'}(M_1)=\frac{-2A+[h+c(1-\alpha)\theta+sI_e]DM_1^2}{2M_1^2}, \quad (36)$$

$$TC_{22}^{'}(T_d)=\frac{-2A+(h+c\theta+sI_e)DT_d^2}{2T_d^2}, \quad (37)$$

and

$$TC_{22}^{'}(M_2)=TC_{23}^{'}(M_2)=\frac{-2A+(h+c\theta+sI_e)DM_2^2}{2M_2^2} \quad (38)$$

Furthermore, we let

$$\Delta_1=-2A+(h+c\theta+cI_p)DT_d^2\text{ ,} \quad (39)$$

$$\Delta_2=-2A+[h+c(1-\alpha)\theta+sI_e]DT_d^2\text{,} \quad (40)$$

$$\Delta_3=-2A+[h+c(1-\alpha)\theta+sI_e]DM_1^2\text{,} \quad (41)$$

$$\Delta_4=-2A+(h+c\theta+sI_e)DT_d^2 \quad (42)$$

and

$$\Delta_5=-2A+(h+c\theta+sI_e)DM_2^2\text{.} \quad (43)$$

Equations (40) - (43) imply that $\Delta_5>\Delta_3>\Delta_2$ and $\Delta_5>\Delta_4>\Delta_2$. Then, the following lemmas can be obtained.

Lemma 1. Suppose that the payment is paid at time M_1 ($T_d<M_1$) and $2A+[c(1-\alpha)I_p-sI_e]DM_1^2>0$. Then, the following results can be obtained:

	Situations	The optimal annual total cost
1.	$\Delta_1\ge 0$, $\Delta_2\ge 0$ and $\Delta_3\ge 0$	$TC(T^*)=Min\{TC_{11}(T_{11}^{*}),TC_{12}(T_d)\}$
2.	$\Delta_1\ge 0$, $\Delta_2<0$ and $\Delta_3\ge 0$	$TC(T^*)=Min\{TC_{11}(T_{11}^{*}),TC_{12}(T_{12}^{*})\}$
3.	$\Delta_1\ge 0$, $\Delta_2<0$ and $\Delta_3<0$	$TC(T^*)=Min\{TC_{11}(T_{11}^{*}),TC_{13}(T_{13}^{*})\}$
4.	$\Delta_1<0$, $\Delta_2\ge 0$ and $\Delta_3\ge 0$	$TC(T^*)=Min\{TC_{11}(T_d),TC_{12}(T_d)\}$
5.	$\Delta_1<0$, $\Delta_2<0$ and $\Delta_3\ge 0$	$TC(T^*)=Min\{TC_{11}(T_d),TC_{12}(T_2^{*})\}$
6.	$\Delta_1<0$, $\Delta_2<0$ and $\Delta_3<0$	$TC(T^*)=Min\{TC_{11}(T_d),TC_{13}(T_3^{*})\}$

Proof: (Appendix A)

Lemma 2. Suppose that the payment is paid at time M_2 ($T_d<M_2$) and $2A+(cI_p-sI_e)DM_2^2>0$. Then, the following results can be obtained:

	Situations	The optimal annual total cost
1.	$\Delta_1 \geq 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_{21}^*), TC_{22}(T_d)\}$
2.	$\Delta_1 \geq 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_{21}^*), TC_{22}(T_{22}^*)\}$
3.	$\Delta_1 \geq 0$, $\Delta_4 < 0$ and $\Delta_5 < 0$	$TC(T^*) = Min\{TC_{21}(T_{21}^*), TC_{23}(T_{23}^*)\}$
4.	$\Delta_1 < 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_d), TC_{22}(T_d)\}$
5.	$\Delta_1 < 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_d), TC_{22}(T_{22}^*)\}$
6.	$\Delta_1 < 0$, $\Delta_4 < 0$ and $\Delta_5 < 0$	$TC(T^*) = Min\{TC_{21}(T_d), TC_{23}(T_{23}^*)\}$

Proof: (Appendix A)

Lemma 3. Suppose that $2A+[c(1-\alpha)I_p - sI_e]DM_1^2 \leq 0$, then $\Delta_3 > 0$.

Proof: Since $[c(1-\alpha)I_p - sI_e]DM_1^2 \leq -2A$. Therefore,

$$\Delta_3 = -2A + [h + c(1-\alpha)\theta + sI_e]DM_1^2 > [h + c(1-\alpha)\theta + sI_e]DM_1^2 + [c(1-\alpha)I_p - sI_e]DM_1^2$$
$$= [h + c(1-\alpha)\cdot(\theta + I_p)]DM_1^2 > 0$$

Lemma 4. Suppose that $2A+(cI_p - sI_e)DM_2^2 \leq 0$, then $\Delta_5 > 0$.

Proof: Since $(cI_p - sI_e)DM_2^2 \leq -2A$. Therefore,

$$\Delta_5 = -2A + (h + c\theta + sI_e)DM_2^2 > (h + c\theta + sI_e)DM_2^2 + (cI_p - sI_e)DM_2^2$$
$$= [h + c(\theta + I_p)]DM_2^2 > 0$$

Combining Lemma 1, Lemma 2, Lemma 3 and Lemma 4, the following theorem can be obtained.

Theorem 1. If $T_d < M_1 < M_2$, then have the following results

	Situations	T he optimal annual total cost
1.	$\Delta_1 \geq 0$, $\Delta_2 \geq 0$, $\Delta_3 \geq 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_{11}^*), TC_{12}(T_d), TC_{22}(T_d)\}$
2.	$\Delta_1 \geq 0$, $\Delta_2 < 0$, $\Delta_3 \geq 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_{11}^*), TC_{12}(T_{12}^*), TC_{22}(T_d)\}$
3.	$\Delta_1 \geq 0$, $\Delta_2 < 0$, $\Delta_3 \geq 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_{11}^*), TC_{12}(T_{12}^*), TC_{22}(T_{22}^*)\}$
4.	$\Delta_1 \geq 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*) TC_{22}(T_d)\}$
5.	$\Delta_1 \geq 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_1^*), TC_{13}(T_{13}^*), TC_{22}(T_{22}^*)\}$
6.	$\Delta_1 \geq 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 < 0$ and $\Delta_5 < 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*), TC_{23}(T_{23}^*)\}$
7.	$\Delta_1 < 0$, $\Delta_2 \geq 0$, $\Delta_3 \geq 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_d^*), TC_{12}(T_d^*), TC_{22}(T_d)\}$
8.	$\Delta_1 < 0$, $\Delta_2 < 0$, $\Delta_3 \geq 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_d), TC_{12}(T_{12}^*), TC_{22}(T_d)\}$
9.	$\Delta_1 < 0$, $\Delta_2 < 0$, $\Delta_3 \geq 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_d), TC_{12}(T_2^*), TC_{22}(T_{22}^*)\}$
10.	$\Delta_1 < 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 \geq 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_d), TC_{13}(T_3^*), TC_{22}(T_d)\}$
11.	$\Delta_1 < 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 < 0$ and $\Delta_5 \geq 0$	$Min\{TC_{11}(T_d), TC_{13}(T_3^*), TC_{22}(T_{22}^*)\}$
12.	$\Delta_1 < 0$, $\Delta_2 < 0$, $\Delta_3 < 0$, $\Delta_4 < 0$ and $\Delta_5 < 0$	$Min\{TC_{11}(T_d), TC_{13}(T_{13}^*), TC_{23}(T_{23}^*)\}$

(1) Suppose that $2A+[c(1-\alpha)I_p - sI_e]DM_1^2 > 0$. Then, the optimal cycle time and the optimal policy as stated above situations1-12.

(2) Suppose that $2A+[c(1-\alpha)I_p - sI_e]DM_1^2 \le 0$. Then, the optimal cycle time and the optimal policy as stated above situations1-3 and 7-9.

(B) $M_1 < T_d < M_2$

When $M_1 < T_d < M_2$, $TC_1(T)$ can be expressed as followed :

$$TC_1(T) = TC_{13}(T) \text{ if } T_d \le T \quad (44)$$

Obviously, $TC_{11}(T_d) \ne TC_{13}(T_d)$. Hence $TC(T)$ is well-defined and continuous except $T = T_d$. Equation (21) yield that

$$TC_{13}'(T_d) = \frac{-2A+[sI_e - c(1-\alpha)I_p]DM_1^2}{2T_d^2} + \frac{D[h+c(1-\alpha)(\theta+I_p)]}{2} \quad (45)$$

Therefore, let

$$\Delta_6 = -2A+[sI_e - c(1-\alpha)I_p]DM_1^2 + [h+c(1-\alpha)(\theta+I_p)]DT_d^2 \quad (46)$$

Furthermore, the following lemma can be obtained.

Lemma 5. Suppose that the payment is paid at time M_1 ($M_1 < T_d$). Then, the following results can be obtained.

	Situations	The optimal annual total cost
1.	$\Delta_1 \ge 0$, $\Delta_6 \ge 0$	$TC(T^*) = Min\{TC_{11}(T_{11}^*), TC_{13}(T_d)\}$
2.	$\Delta_1 \ge 0$, $\Delta_6 < 0$	$TC(T^*) = Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*)\}$
3.	$\Delta_1 < 0$, $\Delta_6 \ge 0$	$TC(T^*) = Min\{TC_{11}(T_d), TC_{13}(T_d)\}$
4.	$\Delta_1 < 0$, $\Delta_6 < 0$	$TC(T^*) = Min\{TC_{11}(T_d), TC_{13}(T_{13}^*)$

Proof: (Appendix B)

Combining Lemma 2, Lemma 4 and Lemma 5, the following theorem can be obtained.

Theorem 2. If $M_1 < T_d < M_2$, then have the following results:

	Situations	The optimal annual total cost
1.	$\Delta_1 \ge 0$, $\Delta_6 \ge 0$, $\Delta_4 \ge 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_d), TC_2$
2.	$\Delta_1 \ge 0$, $\Delta_6 \ge 0$, $\Delta_4 < 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_d), TC_2$
3.	$\Delta_1 \ge 0$, $\Delta_6 \ge 0$, $\Delta_4 < 0$, $\Delta_5 < 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_d), TC_2$
4.	$\Delta_1 \ge 0$, $\Delta_6 < 0$, $\Delta_4 \ge 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*), TC$
5.	$\Delta_1 \ge 0$, $\Delta_6 < 0$, $\Delta_4 < 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*), TC$
6.	$\Delta_1 \ge 0$, $\Delta_6 < 0$, $\Delta_4 < 0$, $\Delta_5 < 0$	$Min\{TC_{11}(T_{11}^*), TC_{13}(T_{13}^*), TC$
7.	$\Delta_1 < 0$, $\Delta_6 \ge 0$, $\Delta_4 \ge 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_d), TC_{13}(T_d), TC_{22}$
8.	$\Delta_1 < 0$, $\Delta_6 \ge 0$, $\Delta_4 < 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_d), TC_{13}(T_d), TC_{22}$
9.	$\Delta_1 < 0$, $\Delta_6 \ge 0$, $\Delta_4 < 0$, $\Delta_5 < 0$	$Min\{TC_{11}(T_d), TC_{13}(T_d), TC_{23}$
10.	$\Delta_1 < 0$, $\Delta_6 < 0$, $\Delta_4 \ge 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_d), TC_{13}(T_{13}^*), TC_2$
11.	$\Delta_1 < 0$, $\Delta_6 < 0$, $\Delta_4 < 0$, $\Delta_5 \ge 0$	$Min\{TC_{11}(T_d), TC_{13}(T_{13}^*), TC_2$
12.	$\Delta_1 < 0$, $\Delta_6 < 0$, $\Delta_4 < 0$, $\Delta_5 < 0$	$Min\{TC_{11}(T_d), TC_{13}(T_{13}^*), TC_2$

(1) Suppose that $2A+(cI_p - sI_e)DM_2^2 > 0$. Then, the optimal cycle time and the optimal policy as stated above situations1-12.

(2) Suppose that $2A+[cI_p - sI_e]DM_2^2 \le 0$. Then, the optimal cycle time and the optimal policy as stated above situations1-2, 4-5, 7-8 and 10-11.

(C) $M_1 < M_2 < T_d$

When $M_1 < M_2 < T_d$, $TC_2(T)$ can be expressed as followed :

$$TC_2(T) = TC_{23}(T) \text{ if } T_d \le T \quad (47)$$

Obviously, $TC_{21}(T_d) \ne TC_{23}(T_d)$. Hence $TC(T)$ is well-defined and continuous except $T = T_d$.

Equation (24) yield that

$$TC_{23}{}'(T_d) = \frac{-2A + (sI_e - cI_p)DM_2{}^2}{2T_d{}^2} + \frac{D(h + c\theta + cI_p)}{2} \quad (48)$$

Therefore, we let

$$\Delta_7 = -2A + (sI_e - cI_p)DM_2{}^2 + [h + c\theta + cI_p]DT_d{}^2 \quad (49)$$

Furthermore, the following lemma can be obtained.

Lemma 6. Suppose that the payment is paid at time M_2 ($M_2 < T_d$). Then, the following results can be obtained:

	Situations	The optimal annual total cost
1.	$\Delta_1 \geq 0$, $\Delta_7 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_{21}{}^*), TC_{23}(T_d)\}$
2.	$\Delta_1 \geq 0$, $\Delta_7 < 0$	$TC(T^*) = Min\{TC_{21}(T_{21}{}^*), TC_{23}(T_{23}{}^*)\}$
3.	$\Delta_1 < 0$, $\Delta_7 \geq 0$	$TC(T^*) = Min\{TC_{21}(T_d), TC_{23}(T_d)\}$
4.	$\Delta_1 < 0$, $\Delta_7 < 0$	$TC(T^*) = Min\{TC_{21}(T_d), TC_{23}(T_{23}{}^*)\}$

Proof: (Appendix B)

Combining Lemma 5 and Lemma 6, the following theorem can be obtained.

Theorem 3. If $M_1 < M_2 < T_d$, then have the following results:

	Situations	The optimal annual total cost
1.	$\Delta_1 \geq 0$, $\Delta_6 \geq 0$, $\Delta_7 \geq 0$	$Min\{TC_{21}(T_{21}{}^*), TC_{13}(T_d), TC_{23}(T_d)\}$
2.	$\Delta_1 \geq 0$, $\Delta_6 \geq 0$, $\Delta_7 < 0$	$Min\{TC_{21}(T_{11}{}^*), TC_{13}(T_d), TC_{23}(T_{23}{}^*)\}$
3.	$\Delta_1 \geq 0$, $\Delta_6 < 0$, $\Delta_7 \geq 0$	$Min\{TC_{21}(T_{11}{}^*), TC_{13}(T_{13}{}^*), TC_{23}(T_d)\}$
4.	$\Delta_1 \geq 0$, $\Delta_6 < 0$, $\Delta_7 < 0$	$Min\{TC_{21}(T_{21}{}^*), TC_{13}(T_{13}{}^*), TC_{23}(T_{23}{}^*)\}$
5.	$\Delta_1 < 0$, $\Delta_6 \geq 0$, $\Delta_7 \geq 0$	$Min\{TC_{21}(T_d), TC_{13}(T_d), TC_{23}(T_d)\}$
6.	$\Delta_1 < 0$, $\Delta_6 \geq 0$, $\Delta_7 < 0$	$Min\{TC_{21}(T_d), TC_{13}(T_d), TC_{23}(T_5{}^*)\}$
7.	$\Delta_1 < 0$, $\Delta_6 < 0$, $\Delta_7 \geq 0$	$Min\{TC_{21}(T_d), TC_{13}(T_{13}{}^*), TC_{23}(T_d)\}$
8.	$\Delta_1 < 0$, $\Delta_6 < 0$, $\Delta_7 < 0$	$Min\{TC_{21}(T_d), TC_{13}(T_{13}{}^*), TC_{23}(T_{23}{}^*)\}$

EMPIRICAL CASE

The company and problem

Gallant Ocean Corporation was established in 1984 as wholesale and resale exporter, importer, seafood processor, and distributor of frozen seafood products in Asia. Over the past few years, the company has been operating not only in Taiwan, but also in China, Vietnam, Thailand, and Myanmar to provide the best products and services to customers. Gallant Ocean Co. constructed its automated low-temperature distribution centers in 2003, which provide efficient backup support as well storage capability at temperatures below zero degrees F (-l8° C) in order to reduce deterioration. Major brokers are the important intermediaries within distribution channels between the importers and consumers of aquaculture products. To facilitate sales of imported aquaculture products and to reduce the costs of stocks and losses, importers offer high price discounts and large flexible payment schedules to major brokers who purchase large quantities of products.

The policy

Frozen seafood is available throughout the year; however, fresh seafood is available only during the fishing season. From the supply side, the profile for supply of aquaculture products during the fishing season

Table 2. Optimal solutions under different parametric values of *s*, *A*, θ, and Q_d .

s	*A*	θ	Q_d	T_d	T^*	Q^*	TC^*
70	5000	0.1	500	0.02497	0.14229	2845	1060276
			3000	0.14889	0.14889	2977	1060349
			5000	0.24693	0.24693	4938	1071272
		0.4	500	0.02488	0.11253	2250	1078939
			3000	0.14567	0.14567	2913	1081928
			5000	0.23828	0.23828	4765	1105249
	7500	0.1	500	0.02497	0.17415	3482	1076077
			3000	0.14889	0.14889	2977	1060302
			5000	0.24693	0.24693	4938	1081397
		0.4	500	0.02488	0.13773	2754	1098918
			3000	0.14567	0.14567	2913	1099090
			5000	0.23828	0.23828	4765	1115741
140	5000	0.1	500	0.02497	0.13724	2744	1057772
			3000	0.14889	0.14889	2977	1057998
			5000	0.24693	0.24693	4938	1069855
		0.4	500	0.02488	0.10854	2170	1075173
			3000	0.14567	0.14567	2913	1079525
			5000	0.23828	0.23828	4765	1103780
	7500	0.1	500	0.02497	0.17005	3400	1074043
			3000	0.14889	0.14889	2977	1068876
			5000	0.24693	0.24693	4938	1079979
		0.4	500	0.02488	0.13449	2689	1096347
			3000	0.14567	0.14567	2913	1096687
			5000	0.23828	0.23828	4765	1114272

can be divided into three phases. In the first phase, catches continually increase; in the second phase, fishing catches begin to peak; in the final stage, stocks starts to decline. Each phase differs and requires distinct marketing strategies. In the early phase of the fishing season, suppliers should attract low-volume and high-price retail market entrants by applying the policy of decreasing Q_d and slightly increasing α , M_1 and M_2 to promote demand. During the rapid growth period, sales pick up rapidly, new buyers enter the market, and previous buyers make repeat orders. The supplier can then use the policy of increasing Q_d and drastically increasing α , M_1 and M_2 to stimulate more demand. In the second phase, quantities of catch begin to accommodate demand, and growth slows precipitously. The supplier can use the policy of fixed Q_d and increasing α , M_1 and M_2 to maintain market share. In the last phase of continuous decline, demand may exceed supply. The supplier can then adopt a policy of sharply decreasing Q_d and decreasing α , M_1 and M_2 to increase profits.

Example 1

The empirical case as above-mentioned is investigated to illustrate the theoretical results. In addition, the proposed approach is applied to study the optimal ordering and payment policies issues between Gallant Ocean Co. and major broker. The relative parameter values are listed as follows:

$A = \$5000/\text{order}$,
$c = \$55/\text{unit}$,
$s = \$80/\text{unit}$,
$\alpha = 0.05$,
$M_1 = 0.05$ years,
$M_2 = 0.20$ years,
D=200000 units/year,
$h = \$5/\text{unit/year}$,
$I_e = \$0.2/\$/\text{year}$,
$I_p = \$0.3/\$/\text{year}$, and
Q_d =5000 units/order.

According to the above procedure, Table 2 shows the optimal ordering payment policies by the different values of s, A, θ, and Q_d.

From Table 2, the following inferences can be obtained:

Table 3. Optimal solutions under different parametric values of h, θ and M_2.

		α =0.04, M_1=0.16, M_2=0.25				α =0.04, M_1=0.16, M_2=0.30			
h	θ	Q^*	T^*	TC^*	ΔTC^*	Q^*	T^*	TC^*	ΔTC^*
5	0.25	7649	T_2=0.03824	10309533		7586	T_{22}=0.03793	10303628	
10	0.10	7996	T_2=0.03998	10298119	-0.11%	7968	T_{22}=0.03984	10290998	-0.12%
5	0.30	7368	T_2=0.03684	10319440		7302	T_{22}=0.03651	10313861	
10	0.18	7505	T_2=0.03753	10314473	-0.05%	7463	T_{22}=0.03732	10307955	-0.06%

(a) For fixed A, s, and Q_d, when inventory deteriorating rate θ increases, the optimal replenishment cycle T^* decreases, and the optimal order quantity Q^* also decreases.

(b) For fixed A, θ and $T_d < M_1 < M_2$, when the unit selling price s decreases, the optimal replenishment cycle T^* increases, and the optimal order quantity Q^* increases as well. For fixed A, θ and $M_1 < T_d < M_2$ or $M_1 < M_2 < T_d$, we show that when the unit selling price s decreases, the optimal replenishment cycle T^* does not change, and the optimal ordering quantity Q^* does not change.

(c) For fixed s, θ and $T_d < M_1 < M_2$, when ordering cost A increases, the optimal replenishment cycle T^* increases, and the optimal order quantity Q^* increases as well. For fixed s, θ and $M_1 < T_d < M_2$ or $M_1 < M_2 < T_d$, when ordering cost A increases, the optimal replenishment cycle T^* does not change, and the optimal ordering quantity Q^* does not change.

d) In addition, for fixed s, θ and A, when the minimum order quantity increases, the optimal replenishment cycle T^* increases, and the optimal order quantity Q^* increases as well.

Noticed that if the customer has higher ordering cost, lower unit selling price, or lower inventory deteriorating rate, and if the supplier is willing to provide the lower minimum order quantity, the major broker tends to order higher quantity with longer replenishment cycle. However, when the relative parametric values are identical and the supplier is willing to provide the lower minimum order quantity, then the major broker tends to buy goods more frequently with smaller quantities.

Example 2

To investigate the effects of h and θ on the optimal annual total relevant cost, TC^* is derived by the proposed method with different h and θ. The following observations can be reached from Table 3.

(a) Decisions of θ and h

For fixed α, M_1, M_2, when holding cost h increases and deteriorating rate θ decrease at the same time. The optimal total cost for the supplier depends on the h and θ. In general, the unit holding cost h increases when the inventory-deteriorating rate θ is decreases, and vice versa. In Table 3, when α =0.04, M_1=0.16, M_2=0.25, h increases from 5 to 10, and θ decreases from 0.25 to 0.1, the optimal total cost decreases from 10309533 to 10298119, and the annual total relevant cost decreases by 0.11%. This implies that the supplier is willing to improve storage facilities and efficient backup support as well as flow control to extend product expiry and to reduce deterioration rates.

This study analyzes an empirical case to illustrate the proposed approach both supplier and customer viewpoints. From the perspective of the major broker, the best decision for two distinct alternatives is the trade credit with the lowest total relative cost. From the importer perspective, different cash discounts and trade credit can be offered by considering delayed payments as a marketing strategy to increase sales and reduce current stocks.

Conclusions

The benefits of trade credit have been demonstrated to be an important issue in inventory policies to be competitive. This study incorporates cash discounts and permissible delay in payments to determine the optimal

order quantity, if the order quantity is greater than or equal to a predetermined quantity. Theorems 1-3 provide the solution procedure to determine the optimal replenishment under the consideration to minimize the annual total relevant cost. In addition, the empirical study of frozen seafood products offer the standpoints from both buyers and sellers with the following managerial insights: (1) From the customer perspective, if the customer has higher ordering cost, lower unit selling price, or lower unit stock holding cost, and if the supplier is willing to provide the lower minimum order quantity, the purchaser tends to order higher quantity with longer replenishment cycle. However, when the relative parameters are identical, if the supplier is willing to provide the lower minimum order quantity, the purchaser tends to buy goods more frequently but in smaller quantities. (2) From the supplier perspective, the supplier is willing to improve storage facilities and efficient backup support as well as flow control to extend product expiry and to reduce deterioration rates. (3) The supplier can use the appropriate policy to adjust the minimum order quantity and trade credit that can affect the optimal ordering quantity and the total demand from the buyer.

From the above arguments, the unique contribution of this study is the construction of an economic order quantity model for deteriorating items under the conditions of both cash discount and permissible delay in payment with purchase order quantity. Further, the optimal ordering and payment policies for both the supplier and customer are discussed as well.

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Appendix A. Proof of Lemma 1

(1) If $\Delta_1 \geq 0$, $\Delta_2 \geq 0$ and $\Delta_3 \geq 0$, then $TC_{11}^{'}(T_d) \geq 0$, $TC_{12}^{'}(T_d) \geq 0$ and $TC_{12}^{'}(M_1) = TC_{13}^{'}(M_1) \geq 0$. Equations (39) to (41) imply that

(a) $TC_{11}(T)$ is decreasing on $(0, T_1^*]$ and increasing on $[T_1^*, T_d]$
(b) $TC_{12}(T)$ is increasing on $[T_d, M_1]$
(c) $TC_{13}(T)$ is increasing on $[M_1, \infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the minimum value at $T^* = T_1^*$ on $(0, T_d]$ and $TC(T)$ has the minimum value at $T = T_d$ on $[T_d, \infty)$. Hence
$TC(T^*) = \min \{TC_{11}(T_1^*), TC_{12}(T_d)\}$

(2) If $\Delta_1 \geq 0$, $\Delta_2 < 0$ and $\Delta_3 \geq 0$, then $TC_{11}^{'}(T_d) \geq 0$, $TC_{12}^{'}(T_d) < 0$ and $TC_{12}^{'}(M_1) = TC_{13}^{'}(M_1) \geq 0$. Equations (39) to (41) imply that

(a) $TC_{11}(T)$ is decreasing on $(0, T_{11}^*]$ and increasing on $[T_{11}^*, T_d]$
(b) $TC_{12}(T)$ is decreasing on $[T_d, T_{12}^*]$ and increasing on $[T_{12}^*, M_1]$
(c) $TC_{13}(T)$ is increasing on $[M_1, \infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the minimum value at $T^* = T_{11}^*$ on $(0, T_d]$ and $TC(T)$ has the minimum value at $T = T_{12}^*$ on $[T_d, \infty)$. Hence
$TC(T^*) = \min \{TC_1(T_1^*), TC_2(T_2^*)\}$

(3) If $\Delta_1 \geq 0$, $\Delta_2 < 0$ and $\Delta_3 < 0$, then $TC_1^{'}(T_d) \geq 0$, $TC_2^{'}(T_d) < 0$ and $TC_2^{'}(M_1) = TC_3^{'}(M_1) < 0$. Equations (39) to (41) imply that

(a) $TC_1(T)$ is decreasing on $(0, T_{11}^*]$ and increasing on $[T_{11}^*, T_d]$
(b) $TC_2(T)$ is decreasing on $[T_d, M_1]$
(c) $TC_3(T)$ is decreasing on $[M_1, T_{13}^*]$ and increasing on $[T_{13}^*, \infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the minimum value at $T^* = T_{11}^*$ on $(0, T_d]$ and $TC(T)$ has the minimum value at $T = T_{13}^*$ on $[T_d, \infty)$. Hence
$TC(T^*) = \min \ TC_1(T_1^*), TC_3(T_3^*)$

(4) If $\Delta_1 < 0$, $\Delta_2 \geq 0$ and $\Delta_3 \geq 0$, then $TC_1^{'}(T_d) < 0$, $TC_2^{'}(T_d) \geq 0$ and $TC_2^{'}(M_1) = TC_3^{'}(M_1) \geq 0$. Equations (39) to (41) imply that

(a) $TC_1(T)$ is decreasing on $(0, T_d]$
(b) $TC_2(T)$ is increasing on $[T_d, M_1]$
(c) $TC_3(T)$ is increasing on $[M_1, \infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the minimum value at $T^* = T_d$ on $(0, T_d]$ and $TC(T)$ has the minimum value at $T^* = T_d$ on $[T_d, \infty)$. Hence
$TC(T^*) = \min \{TC_1(T_d), TC_2(T_d)\}$

(5) If $\Delta_1 < 0$, $\Delta_2 < 0$ and $\Delta_3 \geq 0$, then $TC_1^{'}(T_d) < 0$, $TC_2^{'}(T_d) < 0$ and $TC_2^{'}(M_1) = TC_3^{'}(M_1) \geq 0$. Equations (39) to (41) imply that

(a) $TC_1(T)$ is decreasing on $(0, T_d]$
(b) $TC_2(T)$ is decreasing on $[T_d, T_{12}^*]$ and increasing on $[T_{12}^*, M_1]$
(c) $TC_3(T)$ is increasing on $[M_1, \infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the minimum value at $T^* = T_d$ on $(0, T_d]$ and $TC(T)$ has the minimum value at $T^* = T_2^*$ on $[T_d, \infty)$. Hence
$TC(T^*) = \min \ TC_1(T_d), TC_2(T_2^*)$

(6) If $\Delta_1 < 0$, $\Delta_2 < 0$ and $\Delta_3 < 0$, then $TC_1^{'}(T_d) < 0$, $TC_2^{'}(T_d) < 0$ and $TC_2^{'}(M_1) = TC_3^{'}(M_1) < 0$. Equations (39) to (41) imply that

(a) $TC_1(T)$ is decreasing on $(0, T_d]$
(b) $TC_2(T)$ is decreasing on $[T_d, M_1]$
(c) $TC_3(T)$ is decreasing on $[M_1, T_{13}^*]$ and increasing

on $[T_{13}^{\ *},\infty)$

Combining (a)-(c), we conclude that $TC(T)$ has the

minimum value at $T^* = T_d$ on $(0,T_d]$ and $TC(T)$ has the minimum value at $T^* = T_3^{\ *}$ on $[T_d,\infty)$. Hence

$$TC(T^*) = \min\{TC_1(T_d), TC_3(T_3^{\ *})\}$$

Appendix B. Proof of Lemma 5

(1) If $\Delta_1 \geq 0$ and $\Delta_6 \geq 0$, then $TC_1^{\ '}(T_d) \geq 0$, $TC_3^{\ '}(T_d) \geq 0$.

Equations (39) and (46) imply that (a) $TC_1(T)$ is decreasing on $(0,T_1^{\ *}]$ and increasing on $[T_1^{\ *},T_d]$

(c) $TC_3(T)$ is increasing on $[T_d,\infty)$

Combining (a) and (b), we conclude that $TC(T)$ has the minimum value at $T^* = T_1^{\ *}$ on $(0,T_d]$ and $TC(T)$ has the minimum value at $T = T_d$ on $[T_d,\infty)$. Hence

$$TC(T^*) = \min\{TC_1(T_1^{\ *}), TC_3(T_d)\}$$

(2) If $\Delta_1 \geq 0$ and $\Delta_6 < 0$, then $TC_1^{\ '}(T_d) \geq 0$, $TC_3^{\ '}(T_d) < 0$. Equations (39) and (46) imply that

(a) $TC_1(T)$ is decreasing on $(0,T_1^{\ *}]$ and increasing on $[T_1^{\ *},T_d]$ (c) $TC_3(T)$ is decreasing on $[T_d,T_3^{\ *}]$ and increasing on $[T_3^{\ *},\infty)$

Combining (a) and (b), we conclude that $TC(T)$ has the minimum value at $T^* = T_1^{\ *}$ on $(0,T_d]$ and $TC(T)$ has the minimum value at $T = T_3^{\ *}$ on $[T_d,\infty)$. Hence

$$TC(T^*) = \min\{TC_1(T_1^{\ *}), TC_3(T_3^{\ *})\}$$

(3) If $\Delta_1 < 0$ and $\Delta_6 \geq 0$, then $TC_1^{\ '}(T_d) < 0$, $TC_3^{\ '}(T_d) \geq 0$.

Equations (39) and (46) imply that

(a) $TC_1(T)$ is decreasing on $(0,T_d]$

(c) $TC_3(T)$ is increasing on $[T_d,\infty)$

Combining (a) and (b), we conclude that $TC(T)$ has the minimum value at $T^* = T_d$ on $(0,T_d]$ and $TC(T)$ has the minimum value at $T = T_d$ on $[T_d,\infty)$. Hence

$$TC(T^*) = \min\{TC_1(T_d), TC_3(T_d)\}$$

(4) If $\Delta_1 < 0$ and $\Delta_6 < 0$, then $TC_1^{\ '}(T_d) < 0$, $TC_3^{\ '}(T_d) < 0$.

Equations (39) and (46) imply that

(a) $TC_1(T)$ is decreasing on $(0,T_d]$

(c) $TC_3(T)$ is decreasing on $[T_d,T_3^{\ *}]$ and increasing on $[T_3^{\ *},\infty)$

Combining (a) and (b), we conclude that $TC(T)$ has the minimum value at $T^* = T_d$ on $(0,T_d]$ and $TC(T)$ has the minimum value at $T = T_3^{\ *}$ on $[T_d,\infty)$. Hence

$$TC(T^*) = \min\ TC_1(T_d), TC_3(T_3^{\ *})$$