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A novel financial risk contagion model based on the MGARCH process and its parameter estimation

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This paper proposes a new financial risk contagion model; the contagion-MGARCH model which is based on the multivariate GARCH process. Our measure of risk contagion could characterize the causality of the financial risk contagion, its economic significance, and its determinants by using the contagion equation containing latent variables. Markov Chain Monte Carlo (MCMC) estimation of the parameters in the new contagion model context is also covered.

Key words: Financial risk contagion, MGARCH, latent variable.

INTRODUCTION

It has been frequently observed that financial crises and price volatility simultaneously or successively appear in different financial markets and even in different countries, which is a general financial risk contagion or volatility contagion phenomenon. There exists now a large body of literatures that attempt to theoretically model or empirically study this phenomenon, based on the static or time-varying risk contagion models. Lin and Tamvakis (2001), Forbes and Roberto (2002) and Yang et al. (2004) empirically researched the risk contagion in different financial markets using static models which include the univariate GARCH model, the correlation theory, the VAR model and the impulse response function. Boyer et al. (2006), Cappiello et al. (2006), and Beirne et al. (2010) researched the risk contagion in different financial markets using static models which include the univariate GARCH model, the correlation theory, the VAR model and the impulse response function. Boyer et al. (2006), Cappiello et al. (2006), and Beirne et al. (2010) researched the risk contagion using the AR-DCC-MVGARCH model, the asymmetric dynamics MGARCH model and the multivariate GARCH-in-mean model, respectively, which are time-varying risk contagion models. Of course, there are other methods to model the financial risk contagion, which are different from the above econometric methods. In these models, the time-varying contagion models, based on the MGARCH process, are common methods, and they calculate the financial risk contagion by its time-varying covariance. But, there are still two puzzles, such as the symmetrical covariance in the MGARCH models, which is the same risk contagion values that is inconsistent with the real situation, and equivocal contagion causality about the covariance equation. To deal with these puzzles, this paper proposes a novel and developed time-varying contagion model, that is, the contagion-GARCH model.

THE MGARCH MODELS AND ITS RISK CONTAGION CONNOTATION

The multivariate GARCH models allow the variances and covariances to depend on the information set in a vector ARMA manner and are particularly useful to model multimarket or multivariate financial phenomenon, which includes the financial risk contagion, the market volatility spillover and so on. As the first MGARCH model, the VEC-MGARCH model was introduced by Bollerslev et al. (1988), and it has large number of parameters needed to estimate and the positive definite condition in the covariance matrix, which are two defects. To deal with these two issues, some MGARCH models were developed, such as the BEKK model (Engle and Kroner, 1995), the CCC-MGARCH model (Tse, 2000) and so on.

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These MGARCH models have the same conditional mean equation for output \( Y_t = [y_{t,1}, y_{t,2}, \ldots, y_{t,n}]' \), which can be expressed as follows:

\[
Y_t = M_t + e_t | e_t \sim N(0, \Sigma_t)
\]  

where \( e_t = [e_{t,1}, e_{t,2}, \ldots, e_{t,n}]' \) is a multivariate normal vector with zero mean and a covariance matrix \( \Sigma_t \). The parameterization for \( \Sigma_t \) as a function of the information set \( X_{t-1} \) chosen here allows each element of \( \Sigma_t \) to depend on \( q \) lagged values of the squares and cross-products of \( e_t \), as well as \( p \) lagged values of the elements of \( \Sigma_t \), and a \( J \times 1 \) vector of weakly exogenous variables \( X_t \). The main difference is between various MGARCH models. In the following, we respectively give the covariance matrix vector equations (Equations 2 and 3) of the VEC-GARCH model and the BEKK model, which are two basic MGARCH models.

\[
vec(\Sigma_t) = C_0 + C_1 vec(x_{t-1}') + A(L)vec(e_t^1) + B(L)vec(\Sigma_t) \tag{2}
\]

\[
\Sigma_t = C_0 + C_1 x_{t-1} x_{t-1}' + A(L)e_t^1 e_t^1' + B(L)\Sigma_t \tag{3}
\]

where \( vec(\Sigma) \) is the vector operator that stacks the columns of the matrix, \( vec(\Sigma_t) \) follows a vector ARMAX process in squares and cross-products of the residuals, \( a(L) \) and \( b(L) \) are lag operators, \( C_0 \) is a \( n^2 \times 1 \) parameter vector, \( C_1 \) is a \( n^2 \times J^2 \) parameter matrix, \( A_{ij} \) and \( B_{ij} \) are \( n^2 \times n^2 \) parameter matrices, in Equation 2. \( C_0^* \), \( A_{ik}^* \) and \( B_{jk}^* \) are \( n \times n \) parameter matrices, and \( C_0^* \) is also a triangular matrix, \( C_{ik}^* \) is a \( n^* \times J^* \) parameter matrix, in Equation 3.

It is clear that \( \Sigma_t \) of the BEKK model will be positive definite under very weak conditions and the number of parameters is just the second power of \( n \) less than the fourth power of \( n \) in the VEC-GARCH model. However, the equivocal economic significance of Equation 3 and the variables of the BEKK model is an obvious defect to model the financial risk contagion. Following Tse (2000), we consider the CCC-MGARCH model, in which the covariance matrix \( \Sigma_t \) with a constant correlation \( r_{ij} \).

Then, we can get \( h_{ij,t} = r_{ij} h_{ii,t} h_{jj,t} \). The parameters number of the CCC-MGARCH model will be the second power of \( n \) by this setting. We denote \( D_t \) as a diagonal matrix with diagonal element \( h_{ii,t} \), and \( G_t = \{ r_{ij} \} \) as a correlation matrix. Furthermore, we let \( \Sigma_t \) be \( \Sigma_t = D_t G_t D_t \), then we could meet the positive definite condition only if the matrix \( G_t \) is a positive definite matrix. Therefore, the CCC-MGARCH model may be a more convenient MGARCH model under the condition of the constant correlation.

The time-varying covariances, as a volatility relation between different financial markets, can reflect a dynamics risk contagion phenomenon. Thus, the risk contagion models based on the MGARCH process could be better than the static models and other risk contagion models. However, as mentioned above, there are still two puzzles to model financial risk contagion using the MGARCH models, such as the symmetrical covariance means the same risk contagion values between different financial markets, and equivocally risk contagion causality in \( \Sigma_t \) formed by all variances and covariances.

**THE CONTAGION-MGARCH MODEL**

**The model formulation**

Here, we propose a new time-varying financial risk contagion model, named contagion-MGARCH model, which consists of three parts including the condition mean equation, the volatility equation, and the unique contagion equation.

Let Equation 1 be the condition mean equation of the new financial risk contagion model, which is similar to the MGARCH models. The parameterization for the volatility equation in the contagion-MGARCH model is a function of the information set \( X_{t-1} \) chosen here allows the volatility \( h_{ij,t} \) to follow the GARCH process like the MGARCH models, and depend on \( m \) lagged values of the risk contagion factor \( h_{ij,t} \) calculated by the contagion equation (Equation 4). It is worth noting that \( h_{ij,t} \) is not the conditional covariance. Then, the volatility equation can be expressed as Equation 5.

\[
h_{ij,t} = c_{ij} + g(L)h_{ij,t-1} + b_{ij}(L)h_{ij,t} + b_1(L)h_{ij,t-1} + s_{ij}^2 h_{ij,t-1} \tag{4}
\]
where \( a(L) \), \( b(L) \) and \( g(L) \) are lag operators, and \( j, i \) is the index of the innovation \( h_{ji,j} \), which is a latent variable and could capture the time-varying financial risk contagion, two lagged realized variances \( (h_{ii,j} \text{ and } h_{jj,j}) \) came from different markets, and an independent stochastic process. There are three reasons for this new contagion equation. First, for the reason that the risk contagion will appear in clusters as the conditional volatility which is an intrinsic reason of the financial contagion, we put \( g(L) \times h_{ji,j} \) into Equation 4. Second, because the financial risk contagion comes from one financial market and affect other financial markets, and closely come in contact with their realized volatilities, we put \( b_j(L) \times h_{ji,j} \) and \( b_i(L) \times h_{ji,j} \) into Equation 4. The last, we put \( h_{ji,j} \) into Equation 4 to catch some stochastic information which could not be depicted by other factors. Therefore, the latent variable \( h_{ji,j} \) of the contagion equation could measure the financial risk contagion effect and characterize its causality and determinants.

Furthermore, we could change Equations 4 and 5 to a vector equation (Equation 6) as follows:

\[
\text{vec}(T_t) = C_0 + C \text{vec}(x_{ij}^t) + A(L)\text{vec}(e_{ij}^t) + B(L)\text{vec}(T_t)
\]

(6)

where \( A \) is a \( n \times n \) variance-contagion matrix as \( H_t \), in which the non-diagonal element \( h_{ji,j} \) is the risk contagion factor, \( e_{ij}^t \) is a \( n \times n \) parameter matrix, \( C \), \( A \) and \( B \) are \( n^2 \times n^2 \) parameter matrix, \( b_{xy,j} \) is the elements of \( B_j \) with five kinds cases as follows:

1. If \( x = (k - 1)N + k \) and \( y = (k - 1)N + i \), then \( b_{xy,j} \) is a coefficient of the volatility equation (Equation 5) needed to estimate;
2. If \( x^1 = (k - 1)N + k \) and \( y = (l - 1)N + l \), then \( b_{xy,j} \) is a coefficient of the contagion equation (Equation 4) needed to estimate;
3. If \( x^1 = (k - 1)N + k \) and \( y = (l - 1)N + l \), then \( b_{xy,j} \) is a coefficient of the contagion equation (Equation 4) needed to estimate;
4. If \( x^1 = (k - 1)N + k \) and \( y = x \), then \( b_{xy,j} \) is a coefficient of the contagion equation (Equation 4) needed to estimate;
5. Else, \( b_{xy,j} \) is set to zero;

where, \( t = 1, 2, L , n \), \( k = 1, 2, L , N \), \( l = [x \div N] \), \([g]\) is a ceiling-int operator, \( a_{ij} = s_{ij}^2 \). Then, Equations 1, 4 and 5 (or vector equation 6) could construct the contagion-MGARCH model, and be further optimized by taking the following methods:

1. For the covariance \( h_{ij,j} \) just needs weaker condition in the new model, we let the correlation in \( H_t \) be a constant coefficient \( r_{ij} \) as the CCC-MGARCH model. Then, we can get the corresponding covariance \( r_{ij} \sqrt{h_{ii,j}h_{jj,j}} \), and a new financial risk contagion model, that is, Contagion-CCC-MGARCH model. The number of parameters of this new risk contagion model is just the second power of \( n \) which is the same as the BEKK model;
2. By referencing the SV model (Taylor, 1994), we let the conditional variances be \( \mathbb{E}^{0.5 h_{ii,j}} \), and the risk contagion be \( \mathbb{E}^{0.5 h_{ij,j}} \). Then, the positive definite condition of matrix \( H_t \) could be easily meet.

Based on the preceding two settings, the contagion equation (Equation 4) and the volatility equation (Equation 5) of the contagion-MGARCH model can be expressed as follows:

\[
h_{ji,j} = c_{ji} + g(L)h_{ii,j} + b_j(L)h_{ji,j} + b_i(L)h_{ij,j} + s_{ji}^2 h_{ij,j}
\]

(7)

\[
h_{ii,j} = c_{ii} + b(L)h_{ii,j} + \sum_{j=1}^{N} g_j(L)h_{ij,j} + a(L)\log(e_{ij}^2)
\]

(8)
To illustrate in the bivariate case, the contagion-MGARCH model is simplified as follows:

$$Y_t = M_t + e_t, \quad e_t | \chi_{t-1} : \mathcal{N}(0, H_t),$$

$$H_t = \begin{bmatrix} \exp(h_{11t}) & \exp(h_{12t}) \\ \exp(h_{21t}) & \exp(h_{22t}) \end{bmatrix},$$

$$e_t^2 \sim \text{IG}(2.5, 0.025),$$

where $h_{ijt}$ are the latent variables. Therefore, in this paper, we investigate the Markov Chain Monte Carlo (MCMC) approach to estimate the parameters and latent variables in the new financial risk contagion model, and the basic steps are described as follows:

**Step 1**

Calculate the joint distributions (Equations 20 and 21) of $y_{ijt}$, and the joint posterior distribution (Equations 22 and 23) of the latent variables $h_{jjt}$.

**Step 2**

Assume the parameters $u$, $a$, $b$, $c$ and are prior independent, then, we empirically play a slightly informative prior for $u : \mathcal{N}(0, 0.001)$ and $c : \mathcal{N}(0, 0.001)$, and set the same priors parameters as Meyer (2000), that is, set Beta $(a, b)$ prior distribution be $a$ and $b$, where $a = 2a^2 - 1$ and $b = 2b^2 - 1$, with $a = 20$ and $b = 1.5$, which could give a prior mean of 0.86. Then, we let $s^2$ be a conjugate inverse-gamma prior distribution, that is, $s^2 : \text{IG}(2.5, 0.025)$, which gives a prior mean of 0.0167 and prior standard deviation of 0.0236.

**Step 3**

Estimate parameters based on the aforementioned MCMC method, which could calculate by the Gibbs sampling method including updating parameters and generating new values from posterior distributions, and get posterior sample mean of the parameters and latent variables. Here, we suggest directly calculating this model via Gibbs sampling method with WinBUGS or Matlab.

**Conclusions**

In this paper, we have proposed a new econometric
model to calculate and analyze the financial risk contagion phenomenon, that is, the Contagion-MGARCH model (or the Contagion-CCC-MGARCH model), which has a natural interpretation about the financial risk contagion and is relatively parsimonious. The model parameters can be estimated without too much difficulty by the MCMC approach, which is also studied in detailed. Meanwhile, we think and believe that this new financial risk contagion model could capture common movements of financial risk contagion and volatility spillover in the financial markets, as mentioned before, and extend the multivariable GARCH model. It is a pity that we have not taken the empirical analysis and study using this new model and real financial data, definitely, which will be further investigated and studied in near future.

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