African Journal of Business Management Vol. 6(1), pp. 347-360, 11 January, 2012
Available online at http://www.academicjournals.org/AJBM
DOI: 10.5897/AJBM11.2430
ISSN 1993-8233 ©2012 Academic Journals

Full Length Research Paper

The single-allocation hierarchical hub median location problem with fuzzy demands

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Accepted 27 October, 2011

Although, many papers have appeared in the literature of hub location problem, most of them deal with the problem in a crisp environment. In this paper, the single-allocation hierarchical hub median problem (SA-H-MP) with fuzzy demands is addressed. The structure of the model is derived from Yaman (2009) and consists of a three-level network of demand nodes, non-central hubs, and central hubs. It has been assumed that the demands are not known precisely and are estimated using fuzzy variables. In order to solve the problem, a simulation-embedded variable neighborhood search (VNS) is applied. The results of running the proposed approach on the well-known CAB dataset verify that it is able to solve test problems with less than one percent of error.

Key words: Hierarchical network, *p*-hub median, fuzzy variables, variable neighborhood search, simulation.

INTRODUCTION

Hubs are special facilities that serve as switching, transshipment and sorting points in many-to-many distribution systems (Alumur and Kara, 2008). Due to their significance in reality, the study of hub location problem (HLP) has been a focal area of interest among scholars since its very beginning in the late 1960s and various extensions to the classical models have been presented so far. During the last two decades, hub-and-spoke network design problems have received increasing attention in a wide range of application areas such as transportation, telecommunications, computer networks, postal delivery, less-than-truck loading (LTL) and supply chain management (Gelareh et al., 2010). One distinctive feature of HLP is that direct flows between demand nodes are not allowed in a hub network. In other words, to reach a destination, a flow is satisfied through the shortest possible path which should pass through one or more hub nodes. In a nutshell, HLP deals with two different tasks: hub selection, where some nodes are selected to be hubs, and spoke allocation, where an assignment of spoke nodes to hubs is made (Meyer et al., 2009). Hub networks bring about two major benefits in design of a network: (a) the number of links in the network can be dramatically less than a complete network, where direct flows are possible; (b) since the flow between hub nodes is discounted by a discount factor (normally shown as α), hub networks can be more economical for a large variety of applications. Basically, two types of allocation schemes are possible in a HLP as single-allocation and multiple-allocation. While in a single-allocation scheme, a demand node is allocated to one and only one hub node, a multiple-allocation network can be established by allocation of demand nodes to more than a single hub node.

Hierarchical facility location (HFL) problems are one of the facility location variants which have been studied in the literature. Sahin and Süral (2007) identified some applications of HFL problems in waste management, production-distribution, telecommunication, health-care, emergency medical services, etc. One of the special types of HFL problems is the hierarchical hub location problem which is a variant of the classical HLP. Figure 1 illustrates a sample hierarchical hub network which is comprised of three node types. In this figure, nodes 1 to 6 represent demand nodes, nodes 7 to 10 depict hub nodes, and nodes 11 to 14 are central hubs. While the network between central hubs is a complete network, the

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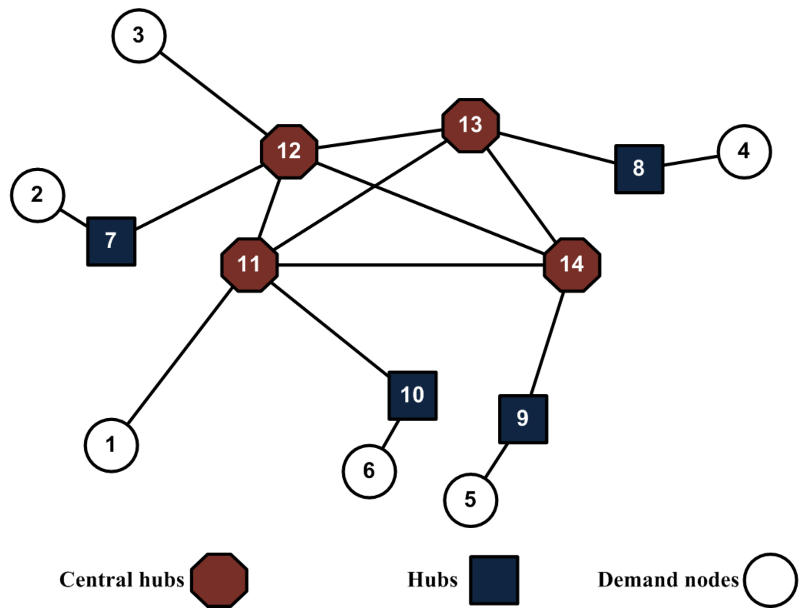

Figure 1. A sample hierarchical network of 14 nodes.

other layers of the network are incomplete graphs. Moreover, each demand node can be allocated to a central hub directly or via a hub node.

Similar to many real-world problems, a location problem may encounter vagueness. For instance, the travel time between two cities can be stated as “between 10 and 11 h”, or the demand of a new product is estimated to be “more than 20000 per year”. Although uncertainty is ubiquitous in location problems, it has received relatively little attention in the literature. There are a number of theories to model and solve problems under uncertainty of which probabilistic and possibilistic approaches are more dominant. Using probabilistic approaches can be beneficial in myriad of applications. However, when there is not enough data, or when there is a need to invest an exorbitant amount of money to garner reliable data, using the possibilistic approach is more reasonable. The possibilistic approach of uncertainty is to a large extent less expressive than probability, but also less demanding in information (Bubois et al., 2004). Hence, using fuzzy logic and possibility theory deserves to be considered as a valuable approach in modeling and solution of many uncertain problems. Perez et al. (2004) introduced four distinct categories of a fuzzy location problem as: location problems with fuzzy vertices, location problems with fuzzy edges, location problems with fuzzy weights, and location problems with fuzzy lengths. The problem in this paper is of the third type, since we assumed that there is some uncertainty in estimation of demands between the network nodes which is handled using fuzzy variables and the knowledge of expert(s). It should be noted that the method to elicit the expert knowledge is beyond the scope of this paper.

The contributions of our paper to the literature are twofold. We put forward an efficient and rigorous methodology to solve SA-H-MP and also present a fuzzy version of the proposed solution procedure to solve SA-H-MP under uncertainty.

The outline of this paper is as follows: The paper proceeds with a review of the literature of the HLP, with a focus on some recent advances. Additionally, here we present a brief overview of using credibility theory in solving some mathematical programming problems. Then, the mathematical formulation of the problem is given hereafter. Next, some basic issues regarding fuzzy variables are discussed. The proposed solution approach is elaborated. Moreover, numerical experiments are presented. Finally, conclusions and some outlooks for future research are presented.

LITERATURE REVIEW

Since the main goal of this paper is not to review all the pertinent publications to HLP, we will focus on some recent publications in this area, some of the most influential contributions in the history of HLP, and an overview of using credibility theory to solve combinatorial optimization problems. Interested readers can refer to valuable reviews for a wealthier background of HLP, such as Aykin (1995) for continuous HLPs, and Alumur and Kara (2008) for network HLPs.

The first attempts to model a HLP dates back to 1980s when O'Kelly (1987) presented his well-known mathematical formulation for *p*-hub median location problem. Later, Campbell (1994) presented the first linear model for the *p*-hub median location problem. This model has been modified by many scholars, such as Skorin-

Table 1. Some major extensions to the classical hub location problem in the last decade.

Subject	Year	Author(s)
Latest arrival HLP	2001	(Kara and Tansel (2001)
Hub arc location problem	2005	Campbell et al. (2005a, b)
HLP considering congestion at hubs	2005	Elhedhli and Hu (2005)
Latest arrival HLP with stopovers	2007	Yaman et al. (2007)
HLP as a set of *M*/*M*/1 queuing hubs nodes	2007	Rodriguez et al. (2007)
Conditional *p*-hub median location problem	2009	Eiselt and Marianov (2009)
Stochastic *p*-hub center with service level constraints	2009	Sim et al. (2009)
Reliable hub location problem	2009	Kim and O'Kelly (2009)
HLP for time definite transportation	2009	Campbell (2009)
Efficient formulations of incomplete HLP	2009	Alumur and Kara (2009)
HLP with multiple capacity levels	2010	Correia et al. (2010)
Competitive HLP in liner service providers	2010	Gelareh et al. (2010)
Game theoretical model in HLP	2010	Lin and Lee (2010)
Design of an intermodal hub-and-spoke network	2010	Ishfaq and Sox (2010)
A real-world case study of HLP in Morocco	2010	Menou et al. (2010)
Hierarchical HLP network for dual express services	2010	Lin (2010)
Stochastic uncapacitated HLP	2011	Contreras et al. (2011)
HLP with balancing requirements	2011	Correia et al. (2011)
Partitioning-hub-location-routing problem	2011	Catanzaro et al. (2011)
Allocation strategies in HLP	2011	Yaman (2011)
Ordered median hub location problem	2011	Puerto et al. (2011)
HLP with decentralized management	2011	Vasconcelos et al. (2011)
Evolutionary algorithm for capacitated HLP	2011	Kratica et al. (2011)

Kapov et al. (1996), Ernst and Krishnamoorthy (1996), and Ebery et al. (2000).

In the last decade, there has been a considerable increase in the number of publications in the area of HLP. Some of the main contributions to the literature of HLP are given in Table 1. A glance over this table shows that the majority of recent contributions are about presenting new variants of classical HLP, considering game theory concepts, adding partitioning and routing to the location module, etc.

The literature of location problems has witnessed some heuristic and metaheuristic methods to solve variants of HLP. The first heuristic is O'Kelly (1987) which proposed two exhaustive heuristics to solve the *p*-hub median problem. Another heuristic was proposed by Klincewicz (1991) which outperforms the one by O'Kelly (1987). Following these heuristics, a number of metaheuristics have been presented in the literature of HLP, including: Tabu search and GRASP by Klincewicz (1992), simulated annealing by Ernst and Krishnamoorthy (1996), genetic algorithm by Kratica et al. (2007), and variable neighborhood search by Perez-Perez et al. (2007) and Illic et al. (2010).

Using fuzzy variables in modeling mathematical programming problems is rather untouched in the literature. Some of the problems targeted in the literature are reported in Table 2 and their solution procedures are cited for further information. Using credibility theory to solve mathematical programming problems is rather untouched in the literature. Moreover, to the best of our knowledge, there has not been any heuristic or metaheuristic for SA-H-MP. Therefore, this paper addresses an efficient simulation-embedded VNS to solve SA-H-MP in both crisp and fuzzy environments.

PROBLEM DEFINITION

Based on the taxonomy of Klose and Drexl (2005), the following assumptions are made for the problem of this paper which was originally proposed by Yaman (2009):

1. The problem is studied on a network of vertices and edges.
2. The objective is of a minisum type, minimizing the total cost of the flows.
3. There is no capacity restriction in the network.
4. The problem is multi-stage and the decision is to be made for a hierarchy of nodes.
5. There is a single product in the network.
6. Demand in the network is completely inelastic.
7. The problem is static. In other words, the decision is made for a single period.
8. The problem parameters are uncertain and estimated

Table 2. Some problems solved in a fuzzy environment using credibility theory.

Author	Problem	Solution procedure
Peng and Liu (2004)	Parallel machine scheduling	Genetic algorithm
Zhao and Liu (2005)	Standby redundancy optimization	Genetic algorithm
Zheng and Liu (2006)	Vehicle routing problem	Genetic algorithm
Liu and Li (2006)	Quadratic assignment problem	Genetic algorithm
Huang (2007)	Portfolio selection	Genetic algorithm
Yang and Liu (2007)	Fixed charge solid transportation	Tabu search
Zhou and Liu (2007)	Location-allocation problem	Genetic algorithm
Erbao and Mingyong (2009)	Vehicle routing problem	Differential evolution
Lan et al. (2009)	Multi-period production planning	Particle swarm optimization
Liu and Gao (2009)	Multi-job assignment problem	Genetic algorithm
Li et al. (2010)	Portfolio selection	Simulated annealing
Ke and Liu (2010)	Project scheduling	Genetic algorithm
Wen and Kang (2011)	Location-allocation problem	Genetic algorithm
Fazel Zarandi et al. (2011)	Location-routing problem	Simulated annealing
Davari et al. (2011)	Maximal covering location problem	Simulated annealing

using fuzzy variables.
9. There is a single objective in the problem.
10. Both hubs and central hubs are desirable facilities.

The three-layer hierarchical p-hub median location problem was first modeled by Yaman (2009). She presented a mixed integer mathematical programming assuming that triangular inequality holds for costs of the network. The structure of the network is as follows: there is a three-layer hub-and-spoke network in which the first level is composed of central hubs, the second layer is the layer of non-central hubs, and the third level is the demand nodes. Each demand node can host a central or non-central hub and should be allocated to one and only one hub node. Clearly, in satisfying any origin-destination demand, there is a need to visit up to four hub nodes.

In the subsequent formulation, the sets I, H, C represent the sets of demand nodes, possible hub nodes, and possible central hubs, respectively (It is to be known that $H \subseteq I$ and $C \subseteq H$). Moreover, Z_{ijl} is a binary variable taking a one value if node $I \in I$ is assigned to hub $j \in H$ and hub j is allocated to central hub $l \in C$. It is worth mentioning that if $j \in H$ becomes a hub node and allocated to the central hub $l \in C$, then the value of Z_{jjl} is equal to 1. Moreover, if node $l \in C$ is a central hub, then the variable Z_{lll} takes a value of 1. g^i_{jl} is the amount of flow which has node $i \in I$ as origin or destination travelling between hub $j \in H$, and central hub $l \in C$. f^i_{kl} denote the amount of traffic of node $i \in I$ as origin travelling from central hub $k \in C$ to central hub $l \in C \setminus \{k\}$. Moreover, p and p_0 are the number of hubs and central hubs to be located. The amount of traffic to be routed from node $i \in I$ to node $m \in I$ is shown as t_{im}. The cost of routing a unit of flow from node $i \in I$ to node $j \in I$ is shown as d_{ij} ($d_{ii} = 0$ and $d_{ij} = d_{ji}$ for all pairs of i and j). The discount factor of routing between hubs and central hubs is shown as α_H and the discount factor of routing between central hubs as α_C. The mathematical formulation of Yaman (2009) is as follows:

$$\min \sum_{i \in I} \sum_{m \in I} (t_{im} + t_{mi}) \sum_{j \in H} d_{ij} \sum_{l \in C} z_{ijl} + \sum_{i \in I} \sum_{j \in H} \sum_{l \in C \setminus \{j\}} \alpha_H d_{jl} g^i_{jl} + \sum_{i \in I} \sum_{j \in H} \sum_{l \in C \setminus \{j\}} \alpha_C d_{jl} f^i_{jl} \quad (1)$$

$$\sum_{j \in H} \sum_{l \in C} z_{ijl} = 1 \quad \forall i \in I \quad (2)$$

$$z_{ijl} \le z_{jjl} \quad \forall i \in I, j \in H \setminus \{i\}, l \in C \quad (3)$$

$$\sum_{m \in H} z_{jml} \le z_{lll} \quad \forall j \in H, l \in C \setminus \{j\} \quad (4)$$

$$\sum_{j \in H} \sum_{l \in C} z_{jjl} = p \quad (5)$$

$$\sum_{l \in C} z_{lll} = p_0 \quad (6)$$

$$\sum_{k \in C \setminus \{l\}} f^i_{lk} - \sum_{k \in C \setminus \{l\}} f^i_{kl} = \sum_{m \in I} t_{im} \sum_{j \in H} (z_{ijl} - z_{mjl}) \quad \forall i \in I, l \in C \quad (7)$$

$$g_{jl}^{i} \geq \sum_{m \in I \setminus \{j\}} (t_{im} + t_{mi})(z_{ijl} - z_{mjl}) \quad \forall i \in I, j \in H, l \in C \setminus \{j\} \quad (8)$$

$$z_{ljl} = 0 \quad \forall j \in H, l \in C \setminus \{j\} \quad (9)$$

$$g_{jl}^{i} \geq 0 \quad \forall i \in I, j \in H, l \in C \quad (10)$$

$$f_{kl}^{i} \geq 0 \quad \forall i \in I, k \in C, l \in C \setminus \{k\} \quad (11)$$

$$z_{ijl} \in \{0,1\} \quad \forall i \in I, j \in H, l \in C \quad (12)$$

Equation 1 is the objective function which calculates the fitness of a solution. This function is the summation of the routing costs between demand nodes and their allocated hub nodes, between hub nodes and their allocated central hubs, and also between the central hubs. Constraint (Equation 2) is used to guarantee that each demand node is allocated to a single hub node. Constraint (Equation 3) ensures that if the demand of node *i* is assigned to the hub *j* and central hub *l*, then the j^{th} node must be a hub. If node *j* is assigned to the central hub *l*, then node *l* must be a central hub. This is ensured using constraint (Equation 4). The number of non-central and central hubs is determined using Equations 5 and 6. The traditional flow balance constraints are modified and stated in constraint (Equation 7). Constraints (Equations 8 and 10) are employed to compute the values of g_{jl}^{i} in terms of the assignment variables. Equation 9 strengthens the LP relaxation of the problem. Finally, constraints (Equations 11 and 12) are employed to ensure that *z* variables are binary and *f* values take positive values.

The most distinctive feature of the problem in this paper is the assumption that demands are fuzzy variables which are not known exactly. One of the applications of this problem is the case where a network is to be built from scratch. Needless to say, there is no historical data in such a case and experts can be regarded as the only reliable source to estimate the uncertain parameter. In such a problem, the mathematical formulation is as Equation 13 in which the expected value of total costs is to be minimized. It should be noted that in Equation 13, demands are fuzzy variables. Owing to these fuzzy values in the objective function, there is no analytical procedure to calculate the expected values. Therefore, a fuzzy simulation algorithm should be designed in order to approximate the fitness of a solution.

$$\min E\Big(\sum_{i \in I}\sum_{m \in I}(t_{im}+t_{mi})\sum_{j \in H} d_{ij}\sum_{l \in C} z_{ijl} + \sum_{i \in I}\sum_{j \in H}\sum_{l \in C \setminus \{j\}} \alpha_H d_{jl} g_{jl}^{i} + \sum_{i \in I}\sum_{j \in H}\sum_{l \in C \setminus \{j\}} \alpha_C d_{jl} f_{jl}^{i}\Big) \quad (13)$$
$$(2)-(12)$$

SA-H-MP is proven to be NP-Hard by Yaman (2009). Knowing that the traditional *p*-hub median problem is NP-hard, this is easily verifiable by relaxing the problem to the traditional *p*-hub median problem assuming $p = p_0$. Hence, for larger sizes of the problem, exact methods are handicapped to reach optimal solutions and there is a need to devise some non-exact procedures. In this paper, an efficient variable neighborhood search (VNS) is presented which shows promising results. Then, a fuzzy simulation is embedded within the proposed VNS to solve the fuzzy test problems.

FUZZY VARIABLES

The concept of fuzzy sets was first introduced by Zadeh in the mid 1960s and Kaufmann (1975) coined the term fuzzy variable. To measure a fuzzy event, various measures have been proposed so far. One of the most well-known measures is the possibility measure which was first proposed by Zadeh (1975, 1978). Later, Dubois and Prade (1988) published a considerable amount of publications regarding the theoretical foundations of possibility theory. Due to some restrictions of the possibility theory, Liu and Liu (2002) presented the credibility measure. Traditional measures of uncertainty such as belief measure ((Dempster, 1967) and (Shafer, 1976)), possibility measure (Zadeh, 1978), and necessity measure (Zadeh, 1979) do not assume the self-duality property. Therefore, they are inconsistent with the law of contradiction and law of excluded middle. However, the credibility measure is self-dual and satisfies these two laws. This is a considerable advantage of this measure compared with the other types of measures. Since this paper deals with fuzzy variables, this part is devoted to introducing some basics of credibility theory. For more information, one may consult with valuable sources such as Liu (2009).

Definition 1 (Liu, 2009). Let Θ be a nonempty set, and *P* the power set of Θ, and Cr a credibility measure. Then, the triplet (Θ, *P*, Cr) is called a credibility space

Definition 2 (Liu, 2009). A fuzzy variable is a measurable function from a credibility space (Θ, *P* (Θ), Cr) to the set of real numbers.

Definition 3 (Liu, 2009). Let Θ_k be nonempty sets on which Cr_k are credibility measures, $k = 1, 2, \ldots, n$, respectively, and $\Theta = \Theta_1 \times \Theta_2 \times \ldots \Theta_n$. Then

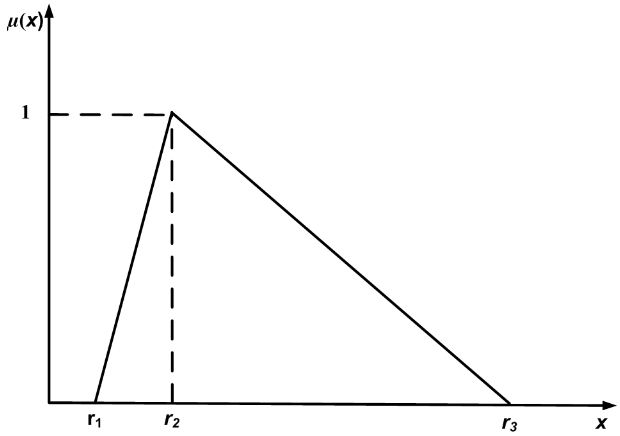

Figure 2. A triangular fuzzy variable.

$$Cr\{(\theta_1,\theta_2,...,\theta_n)\} = Cr_1\{\theta_1\} \wedge Cr_2\{\theta_2\} \wedge ... \wedge Cr_n\{\theta_n\} \quad (14)$$

For each $(\theta_1,\theta_2,\ldots,\theta_n)\in \Theta$.

Definition 4 (Liu, 2009). An n-dimensional fuzzy vector is defined as a function from a credibility space (Θ, P, Cr) to the set of n-dimensional real vectors.

Definition 5 (Liu, 2009). Let ξ be a fuzzy variable defined on the credibility space (Θ, P, Cr). Then, its membership function is derived from the credibility measure by

$$\mu(x) = (2\mathrm{Cr}\{\xi = x\}) \wedge 1, \quad x \in \Re \quad (15)$$

Definition 6 (Liu and Liu, 2002). Let ξ be a fuzzy variable. Then the expected value of ξ is defined by:

$$E[\xi] = \int_0^{+\infty} Cr\{\xi \ge r\}dr - \int_{-\infty}^{0} Cr\{\xi \le r\}dr \quad (16)$$

Definition 7. Let (Θ, P (Θ), Pos) be a possibility space, and A be a set in P (Θ) and the membership function μ (u) of fuzzy variable ξ is given as μ. Then:

$$\mathrm{Pos}\{\xi \le \mathrm{r}\} = \sup_{u\le r} \mu(u) \quad (17)$$

$$\mathrm{Nec}\{\xi \le \mathrm{r}\} = 1 - \sup_{u>r} \mu(u) \quad (18)$$

$$\mathrm{Cr}\{\xi \le \mathrm{r}\} = \frac{1}{2}(Pos\{\xi \le \mathrm{r}\} + Nec\{\xi \le \mathrm{r}\}) \quad (19)$$

To show how an event can be measured using fuzzy measures, a triangular fuzzy variable ξ = (r_1, r_2, r_3) is shown in Figure 2. From the definitions of possibility, necessity and credibility, it is easy to obtain:

$$Pos\{\xi \ge r\} = \begin{cases} 1 & \text{if } r \le r_2 \\ \dfrac{r_3 - r}{r_3 - r_2} & \text{if } r_2 \le r \le r_3 \\ 0 & \text{if } r \ge r_3 \end{cases} \quad (20)$$

$$Nec\{\xi \ge r\} = \begin{cases} 0 & \text{if } r \le r_1 \\ \dfrac{r_2 - r}{r_2 - r_1} & \text{if } r_1 \le r \le r_2 \\ 0 & \text{if } r \ge r_2 \end{cases} \quad (21)$$

$$Cr\{\xi \ge r\} = \begin{cases} 1 & \text{if } r \le r_1 \\ \dfrac{2r_2 - r_1 - r}{2(r_2 - r_1)} & \text{if } r_1 \le r \le r_2 \\ \dfrac{r_3 - r}{2(r_3 - r_2)} & \text{if } r_2 \le r \le r_3 \\ 0 & \text{if } r \ge r_3 \end{cases} \quad (22)$$

SOLUTION PROCEDURE

Here, we deal with the elaboration of the proposed VNS algorithm, its modules, and how it is used to solve the crisp and fuzzy versions of the problem. The proposed approach can be considered as a juxtaposition of two modules: fuzzy simulation, and variable neighborhood search. These modules are elaborated subsequently.

Variable neighborhood search

Variable neighborhood search (VNS) is a relatively recent metaheuristic based on the simple idea of changing neighborhood within a local search to identify better local optima (Mladenovic and Hansen, 1997). It has been used in various fields such as scheduling (Liao and Cheng, 2007), supply chain management (Lejeune, 2006), and routing (Fleszar et al., 2009). VNS exploits systematically the following observations: (i) a local minimum with respect to one neighborhood structure is not necessarily the same with respect to another; (ii) a global minimum is a local minimum with respect to all possible neighborhood structures; (iii) for many problems local minima with respect to one or several neighborhoods are relatively close to each other (Mladenovic et al., 2010).

A standard VNS starts by initializing a solution and defining a set of neighborhoods $k_{\min}$ to $k_{\max}$. In each iteration, a shaking is carried out in the neighborhood N_k (x) to get a new solution x'. Then, a local search is applied from x' to get a local optimum x''. If this local optimum has a better fitness, it replaces x and k = $k_{\min}$, otherwise, k is increased. The same procedure is

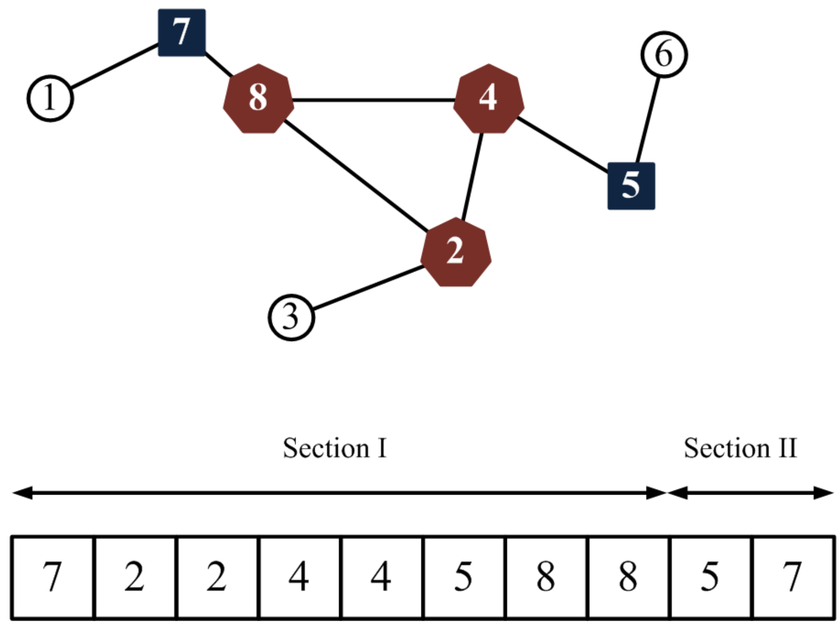

Figure 3. The representation of a sample solution.

followed until the stopping criteria are met.

In this paper, we employ a skewed version of VNS to get better results. In a standard VNS, a shift to a new solution is made only when there is an improvement in the solution quality. However, in a skewed VNS moves which lead to slightly inferior solutions are accepted. In other words, an inferior solution is accepted if the following inequality holds true:

$$\frac{Z_{new} - Z_{inc}}{Z_{inc}} \leq \delta \qquad (23)$$

where Z_{new}, Z_{inc}, and δ represent the fitness of the new solution, the fitness of the current solution, and the maximum tolerable deterioration in the solution quality, respectively. This step is carried out in order to counteract premature convergence. It is to be noted that based on our preliminary experiments, we defined δ = 0.002. Our experiments testified that using a skewed VNS can clearly bring about the ability to escape local optima.

Solution encoding and representation

An efficient solution encoding can considerably contribute to a successful performance of any metaheuristic algorithm. To be resourceful in finding a suitable procedure to encode solutions, a couple of representation schemes were devised. Our experiments using these schemes led us to select a bipartite representation which works as follows. Assume that there are n demand nodes, and the number of hub and central hubs to be located are h and c, respectively. Then, using the proposed representation scheme, each solution contains $n + h$ bits of which n bits are in the first section and the remaining bits are in the second. The first section of a solution shows the allocation of nodes to hubs or central hubs. In other words, the value in the i^{th} bit of the solution string shows the index of the hub/central hub to which the node i is allocated. Moreover, the second section contains the index of hub nodes. Since each central hub is allocated to itself in the first section, there is no need to add the indices of central hubs to the solution string. Hence, the length of the second section is equal to h. Figure 3 shows an example which could be used to explain the encoding scheme further. It should be recalled that the direct allocation of a spoke to a central hub is possible as shown for node 3.

Solution initialization

The initialization of any heuristic or metaheuristic can drastically affect the quality of its solutions. In this paper, a population-based initialization step is carried out as follows: First, a population of initial solutions is generated, each containing a random set of hubs and central hubs. Then, for each solution, demands are allocated to the nearest hub or central hub. The initial solution of the algorithm is the one which has the best solution quality in the initial population of solutions. Although that the nearest-neighbor strategy does not necessarily give optimal solutions for HLP (Alumur and Kara, 2008), it can provide the algorithm with some comparatively good initial

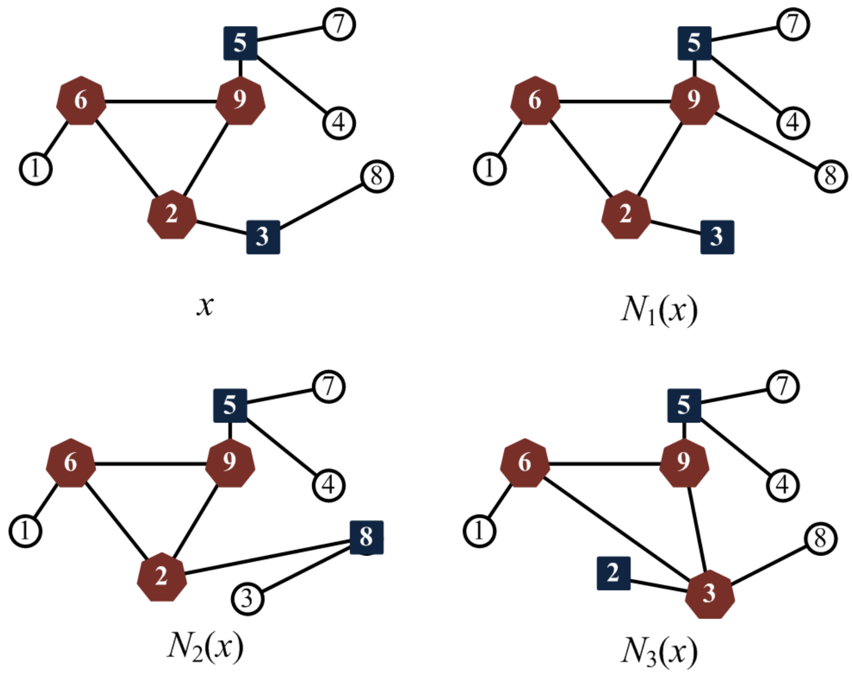

Figure 4. The performance of the three neighborhood search structures on a sample solution.

solutions. It should be noted that in our paper, we defined the size of the population to equal 1000.

Termination criterion

In our preliminary experiments, we realized that after three minutes, there is virtually no sign of improvement in the solution quality. Therefore, running the procedure for 180 s has been considered as the termination criterion.

Neighborhood search structure (NSS)

The core of any VNS is its neighborhood search structures and how it explores the search space to reach better solutions. The set of neighborhoods used for shaking is at the heart of the VNS. Each neighborhood should strike a proper balance between perturbing the incumbent solution and retaining the good parts of the incumbent solution (Hemmelmayr et al., 2009). In this paper, three structures have been employed which deal with both the allocation and the location sections of a solution. The set of solutions which neighbor the incumbent solution x using the q^{th} NSS is shown as $N_q(x)$. The first mechanism which is shown as $N_1(x)$ deals with changing the allocation of nodes $i \in I \backslash H$ from a hub $j \in H$ to another hub $l \in H$. To put it in simpler terms, $N_1(x)$ contains all the solutions which differ from the current solution in the allocation of π nodes. Our exhaustive experiments showed that the values of π which are greater than 3 are not effective to intensify the search. Thus, we have restricted the value of π to three. The second move $N_2(x)$ is associated with substitution of the role of the node $i \in H$ and another node $j \in I \backslash H$. Clearly, using this move, the roles of a hub and a demand node are exchanged. Finally $N_3(x)$ is carried out when the role of a node $i \in H \backslash C$ and another node $j \in C$ are exchanged without affecting the number of hubs and central hubs located. Figure 4 illustrates a sample solution which is modified using each of the three structures. In this paper, $N_1(x)$ is used as the local search procedure of VNS and the other two are the shaking procedures to be applied.

Simulation-embedded VNS

Since the objective function of the mathematical model has a fuzzy parameter, there is no deterministic procedure to calculate the fitness of a solution and to find the optimal solution. In order to simulate a fuzzy programming model like U: $x \rightarrow E\,[f\,(x, \xi)]$, which is dealt with in this paper, Liu (2009) presented a simulation algorithm as given in Figure 5.

NUMERICAL EXAMPLES

Here, we set out to show the efficiency of the proposed approach. To do so, the problem generation method is elaborated and numerical examples are solved. Moreover, we go into the

- Set $e = 0$.
- Randomly generate θ_k from the credibility space (Θ, P, Cr) and write $\nu_k = (2\ \mathrm{Cr}\{\theta_k\}) \wedge 1$ and produce $\xi_k = \xi(\theta_k)$, $k = 1, 2, \ldots, N$, respectively. Then, randomly generate ξ_k and write $\nu_k = \mu(\xi_k)$ for $k = 1, 2, \ldots, N$, where μ is the membership function of ξ.
- $a = f(x, \xi_1) \wedge f(x, \xi_2) \wedge \ldots f(x, \xi_N)$ and $b = f(x, \xi_1) \vee f(x, \xi_2) \vee \ldots f(x, \xi_N)$.
- For SimItr = 1: N
  - Generate a random number r from the range [a, b].
  - If $r \geq 0$, then $e = e + \mathrm{Cr}\{f(x, \xi) \geq r\}$, else $e = e - \mathrm{Cr}\{f(x, \xi) \leq r\}$
- Endfor
- Return $U(x) = a \vee 0 + b \wedge 0 + e.\ (b - a)/N$

Figure 5. The simulation algorithm of Liu (2009).

performance of the algorithm in detail.

Test problems

Here, we report the results of running the proposed algorithm on a set of test problems. In order to get a better understanding of the performance of the proposed algorithm, first the efficiency of the proposed VNS is attested using a set of test problems. Then, one of the instances is solved using the simulation-embedded VNS assuming fuzzy demands. In generation of the test problems, the well-known CAB dataset has been used, similar to Yaman (2009). It was presumed that $\alpha_C < \alpha_H$. Five sets of test problems were generated totally, each containing problems with different values of p and p_0 up to 6 nodes. These test problems are different in the values of the pair (α_C, α_H) as shown in Tables 3, 4, and 5.

Computer specifications

All the test problems were run on a 2.53 GHz CPU equipped with 4 Gigabytes of RAM, using the CPLEX 12.2 solver. Moreover, the proposed solution algorithm was coded using Visual C++.

RESULTS, VALIDATION AND DISCUSSION

Crisp test problems

In this part of the work, a set of experiments were carried out in order to assess the efficiency of the proposed approach. All the test problems were solved using the CPLEX commercial solver. Then, results of the proposed VNS are compared against the results obtained from CPLEX.

To compare these two outputs, the relative percentage deviation (RPD) has been used which is found as stated in Equation 24.

$$\text{Relative percentage deviation (RPD)} = \frac{\text{Fitness}_{\text{VNS}} - \text{Fitness}_{\text{CPLEX}}}{\text{Fitness}_{\text{CPLEX}}} * 100$$

In which $\text{Fitness}_{\text{VNS}}$ and $\text{Fitness}_{\text{CPLEX}}$ are the outputs from VNS and CPLEX, respectively. To show the performance of the proposed VNS, each test problem was solved five times. Then, the results of the worst, average, and best runs are summarized in Tables 3 through 5. Results show that while the worst performance of the proposed VNS hardly ever exceeds one percent, there are some instances where the optimal solution is found using the proposed VNS. Results show that in 35 out of 36 instances, the optimal solution is obtainable in at least one of the five replications using the proposed procedure. In addition, in 23 instances, the optimal solutions are attained in all the five replications. In other words, the algorithm is able to reach the global optimum in all of the five runs. Figure 6 depicts the average performance of the proposed VNS for problems of various settings.

From a runtime point of view, results show that while, in some cases, CPLEX is unable to reach optimal solutions in more than 4 h; our proposed VNS is run in 3 min, regardless of the problem parameters. Interestingly, the results of the proposed approach are not far from optimality. Moreover, the proposed algorithm is able to escape local optima owing to the inherent mechanisms of VNS. Figure 7 depicts a sample trend of solution quality

Table 3. Comparing the results of CPLEX and the proposed VNS for (α_C, α_H) = (0.6, 0.9).

p	p_0	CPLEX		VNS						
		Time	Fitness	Time	Best	Gap (%)	Average	Gap (%)	Worst	Gap (%)
3	1	299.818	10426074560	180	10426074560	0.00	10426074560	0.00	10426074560	0.00
3	2	664.455	9464597766	180	9464597766	0.00	9464597766	0.00	9464597766	0.00
3	3	10.187	8826647392	180	8826647392	0.00	8826647392	0.00	8826647392	0.00
4	2	1469.67	9311789331	180	9311789331	0.00	9320417952	0.09	9333360883	0.2
4	3	2379.343	8606860144	180	8606860144	0.00	8606860144	0.00	8606860144	0.00
4	4	13.338	8020821500	180	8020821500	0.00	8020821500	0.00	8020821500	0.00
5	3	3447.17	8454051709	180	8454051709	0.00	8477251006	0.27	8565711102	1.32
5	4	2183.92	7931288504	180	7931288504	0.00	7935960257	0.06	7954647267	0.29
5	5	19.188	7486046509	180	7486046509	0.00	7486046509	0.00	7486046509	0.00
6	4	3869.621	7862099067	180	7862099067	0.00	7873639054	0.15	7913767875	0.66
6	5	2059.852	7399297863	180	7399297862	0.00	7402520001	0.04	7415408556	0.22
6	6	22.574	7071536179	180	7071536178	0.00	7071536178	0.00	7071536178	0.00

Table 4. Comparing the results of CPLEX and the proposed VNS for (α_C, α_H) = (0.8, 0.9).

p	p_0	CPLEX		VNS						
		Time	Fitness	Time	Best	Gap (%)	Average	Gap (%)	Worst	Gap (%)
3	1	197.232	10426074560	180	10426074560	0.00	10426074560	0.0	10426074560	0.00
3	2	1890.56	10114622268	180	10114622268	0.00	10114622268	0.00	10114622268	0.00
3	3	37.268	9896424156	180	9896424156	0.00	9896424156	0.00	9896424156	0.00
4	2	13435.836	9946414639	180	9946414639	0.00	9950406334	0.04	9956393877	0.10
4	3	8661.222	9618082826	180	9618082826	0.00	9618082826	0.00	9618082826	0.00
4	4	42.042	9288636845	180	9288636845	0.00	9288636845	0.00	9288636845	0.00
5	3	14877.347	9465274391	180	9465274391	0.00	9476001546	0.11	9497283547	0.34
5	4	6583.757	9095608117	180	9095608117	0.00	9095608117	0.00	9095608117	0.00
5	5	121.353	8831244506	180	8831244506	0.00	8831244506	0.00	8831244506	0.00
6	4	7360.922	8974808838	180	8974808838	0.00	9007333312	0.36	9056862401	0.91
6	5	4819.144	8666718166	180	8666718166	0.00	8666718166	0.00	8666718166	0.00
6	6	120.776	8463112374	180	8463112374	0.00	8463112374	0.00	8463112374	0.00

for the case with (α_C, α_H, p, p_0) = (0.6, 0.9, 6, 4). Hence, there are compelling evidences to assert that the proposed procedure is efficient and can be used to solve fuzzy version as well.

Experiments with fuzzy demands

In order to examine the effect of considering demands as fuzzy variables, a sample test problem was generated. It has been assumed that demands are symmetric fuzzy variables (r_1, r_2, r_3) where r_2 equals the original demands of the CAB dataset. Moreover, r_1 and r_3 are values which

Table 5. Comparing the results of CPLEX and the proposed VNS for (α_C, α_H) = (0.8, 0.8).

p	*p*₀	CPLEX		VNS						
		Time	Fitness	Time	Best	Gap (%)	Average	Gap (%)	Worst	Gap (%)
3	1	135.674	9923897797	180	9923897797	0.00	9923897797	0.00	9923897797	0.00
3	2	2132.987	9923897797	180	9923897797	0.00	9923897797	0.00	9923897797	0.00
3	3	25.319	9896424156	180	9896424156	0.00	9896424156	0.00	9896424156	0.00
4	2	3115.464	9528786908	180	9528786908	0.00	9528786908	0.00	9528786908	0.00
4	3	9415.83	9406173571	180	9406173571	0.00	9423345201	0.18	9474656081	0.73
4	4	44.164	9288636845	180	9288636845	0.00	9288636845	0.00	9288636845	0.00
5	3	7570.588	9098003487	180	9098003487	0.00	9128810863	0.34	9150389806	0.58
5	4	10431.382	8962997030	180	8962997030	0.00	8979750668	0.19	9012553142	0.55
5	5	122.399	8831244506	180	8831244506	0.00	8831244506	0.00	8831244506	0.00
6	4	7864.931	8689594212	180	8702839208	0.15	8749267304	0.69	8780392471	1.04
6	5	9567.073	8562974155	180	8562974155	0.00	8584361231	0.25	8593279495	0.35
6	6	129.996	8463112374	180	8463112374	0.00	8463112374	0.00	8463112374	0.00

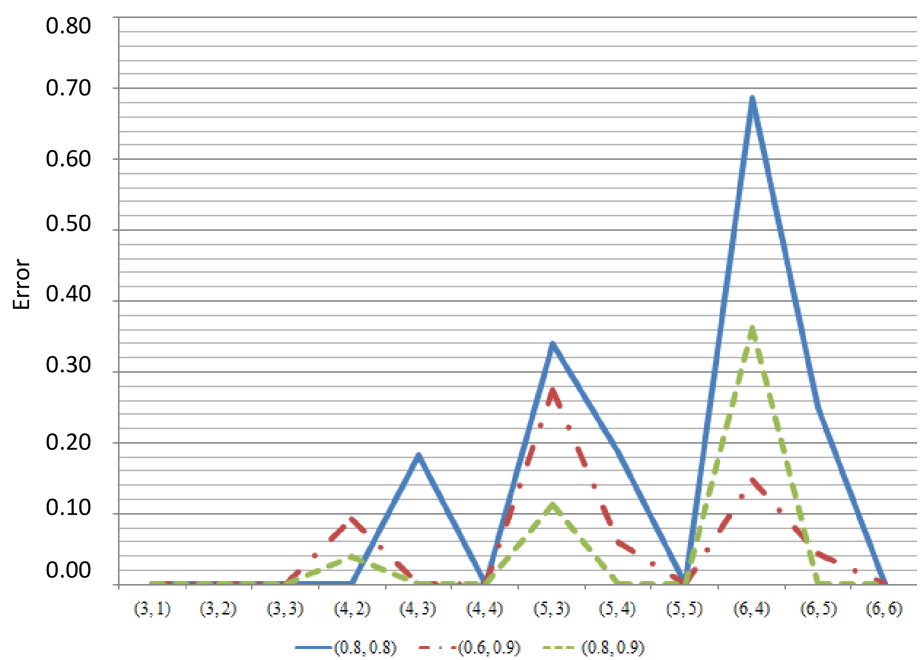

Figure 6. The average error of the proposed VNS for problems with various values of (p, p_0).

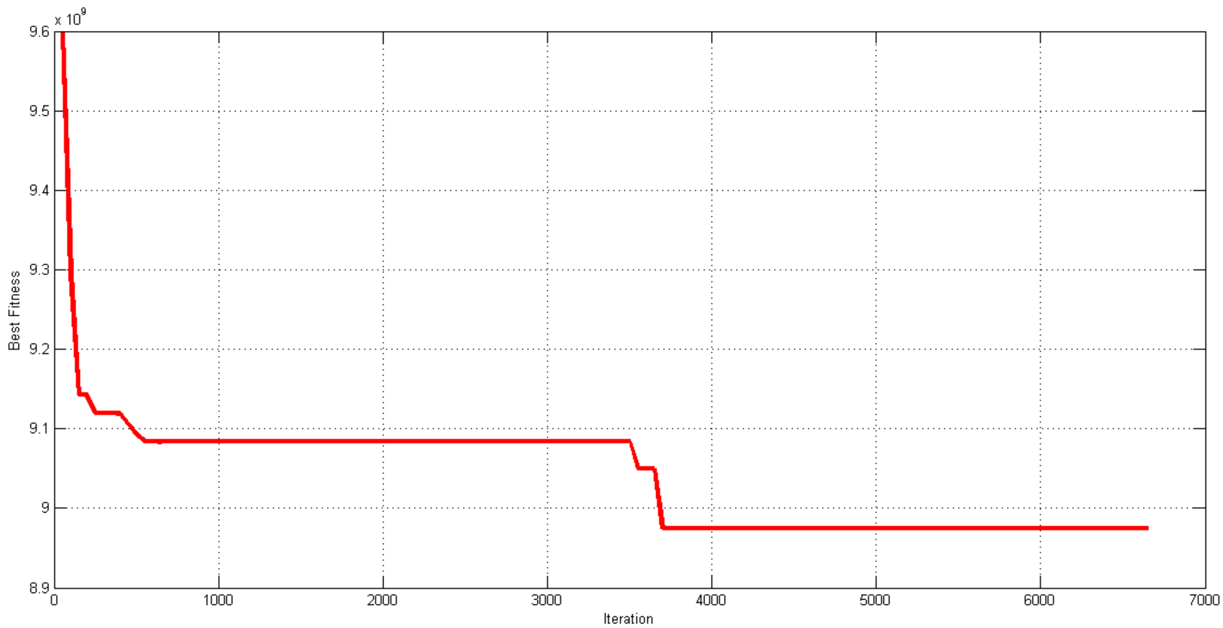

Figure 7. The performance of the algorithm in improving the solution quality.

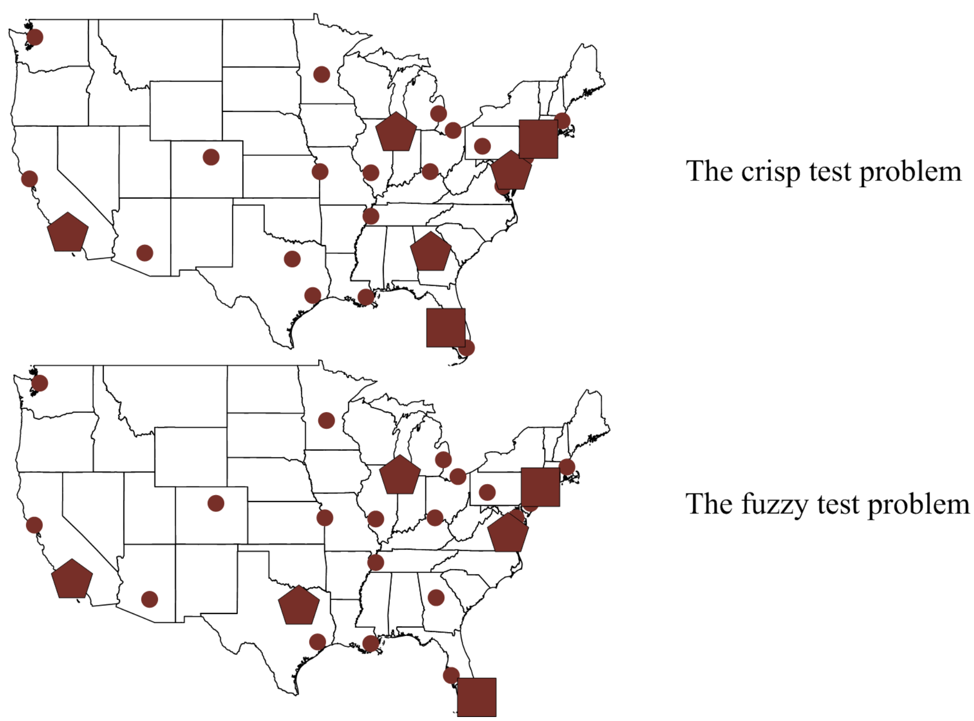

Figure 8. The crisp and fuzzy versions for the problem with (α_C, α_H, p, p_0) = (0.6, 0.9, 6, 4).

are ω% below and above r_1, respectively. To generate the demand of each pair of nodes, the value of ω was considered to be one of the values 2, 5 and 10% for each pair of origin-destination nodes. To simulate each solution, 10000 iterations were used. Figure 8 shows the results of running the proposed solution algorithm for the case (α_C, α_H, p, p_0) = (0.6, 0.9, 6, 4) and the fuzzy test problem with the same parameters. The outputs of the problem for the crisp and fuzzy version are depicted in Figure 8, which differ in one non-central hub and a central hub. Clearly, the added uncertainty to the problem parameters can account for such a change in the solution. Apparently, using other levels of uncertainty can lead to other solutions which can be considerably different.

CONCLUSION AND FUTURE RESEARCH AREAS

To bring this paper to a close, we summarize the main points of the paper and some research directions are proposed. This paper considers the fuzzy version of SA-H-MP and presented an efficient simulation-embedded VNS to solve it. The results of running the algorithm for the crisp test problems showed that the proposed algorithm is able to solve problems in an efficient and effective way and with errors not more than one percent. Furthermore, the procedure is robust to solve fuzzy test problem. However, there are still potential future research directions to be followed. For instance, the hierarchical hub location problem can be considered for some other types of the hub location problem such as covering or center problems. Moreover, there is still a need to fill the gap of assuming random variables in the literature of hierarchical hub location problems. Another future research area can be the inclusion of capacities for hub nodes in the network.

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