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A model analysis of optimal pricing and pollution under consumption pollution tax

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This paper considers solving environmental problems through levying a consumption pollution tax. Under a given government tax rate, the manufacturer decides not only the optimal price, but also the optimal pollution content for a unit of product. This paper performs a conversion of consumer demand using a general probability density function. In addition to obtaining mathematical functions for optimal product pricing and optimal product pollution for manufacturers, this paper also provides an extended discussion concerning the nature of optimal solutions.

Key words: Consumer evaluation function, consumption pollution tax, optimization, optimal pricing, optimal pollution.

INTRODUCTION

Low-cost production has been emphasized over the impact of production on environmental safety since the advent of the industrial revolution. With the development of mass production, environmental pollution has become increasingly serious. As economy has become progressively more developed, environmental awareness has also begun to grow. "Green issues" now receive more attention than they did before and an important topic for nations across the globe is how to ensure both economic and environmental sustainable development.

In practice, government environmental policies regarding polluting products can be broadly divided into the following two types: command-and-control systems and environmental tax systems. The first type of policy involves the government demanding by decree that the pollution of products produced by manufacturers must meet certain environmental standards. Manufacturers that do not meet pollution standards are punished. This type of dichotomous punishment policy for manufacturers is prone to debate. For example, at what level should pollution standards be set? In dealing with the polluting

products of manufacturers, there are only two options -- passing standards and not passing standards. Typically speaking, in the first type, once a manufacturer's product pollution content meets the standard, manufacturers have no incentive to improve the environmental standards of their products (Javier et al., 2009). Relative to the first type policy, the second type policy has received more attention and has stimulated widespread discussion.

Intuitively speaking, because the product pollution is of a continuous nature, environmental policy should establish a punitive policy on a continuous scale, applying punishments using environmental taxation. Scholars believe that environmental taxation utilizes economic motives, or market forces, to improve pollution levels and can induce the internalization of externalities, potentially achieving the double-dividend effect of environmental taxation. In other words, aside from reducing pollution, taxation can also be used to reduce other tax sources, achieving the effect of improving the economy (Bosquet, 2000; Bento and Jacobsen, 2007; Glomm et al., 2008; Yunchang and Yophy, 2010).

The traditional method in environment tax is Pigouvian tax scheme. The idea is to levy a tax on an externality generation activity equal to its marginal social damage. This is a first-best remedy which, in the absence of other distortions in the economy (Helmuth et al., 1998; Helmuth

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and Firouz, 2001). However, there many kinds of taxes has existed in every country, in other words, distortions is existing and, it is difficult to measure the "marginal social damage". So, it must have a second –best tax scheme (Ian, 2002).

In Taiwan, the traditional environment tax is emission tax. It levies according to engine size of motor or car. This rule is not fair. Every kinds of product may be has different polluting content. Thus, the tax types must be rethinking. In terms of different taxation types, Albrecht (2006) suggested, based on the European situation, that constant emission taxation should be changed to consumption (pollution) tax. The amount of such consumption tax that is levied is based on the environmental impact of products produced by manufacturers. Such a method cannot only reduce the prices of green products, but can also limit some undesirable product distribution results. Consequently, if the latent pollution of a product can be detected by some technological instrument, then based on product pollution, the government can levy consumption taxes based on product sales to solve environmental pollution problems.

The impact of government tax policies on a manufacturer's decisions have been discussed based on pre- and post-manufacturer taxation product prices and changes in quantity (Peter, 1994; Jian et al., 2000; Amyaz et al., 2003; Dolores, 2008). However, if different tax rates based on different levels of product pollution are considered, then the decision variables of manufacturers include not only product price and quantity; and pollution can be a controllable variable. Consumers are concerned about the cost of purchasing a unit product, and this cost includes the tax to be borne in addition to the product price. Even if the costs borne by consumers are completely identical, manufacturers can still choose different price and pollution combinations.

In addition, on the consumer demand side, this paper deviates from methods traditionally discussed in economics by using a general distribution function to express consumer evaluations of products (Sarah and Roberton, 2004). This function is converted to construct the consumer demand function and discuss consumer behavior. Under this model, the study presented in this paper uses mathematical analysis to discuss the optimal decision problems for manufacturers and consumers under consumption pollution taxation, providing a reference for future policy implementation.

MODEL

There are three decision makers in this model: government (pollution prevention agency), manufacturers (polluting product producers), and consumers. Their decision relationships are described subsequently.

Government decides the t value; t is the pollution taxation for the unit pollution produced after the product is

used by consumers. The pollution products considered by this paper entail that governments cannot apply pollution tax policies as incentives for manufacturers to produce zero-pollution products..... (2.1)

Manufacturers decide P and w values based on the pollution tax rate t decided by the government, maximizing profits within a set period of time. P represents product price, while w is the pollution of a unit product following consumer use. The consumption pollution tax is tw ; consumption pollution tax is levied on consumers for manufacturers to collect. The tax-inclusive price of a product is $P + tw$.

This paper represents consumer evaluations of a unit product using the symbol x . This evaluation is the upper limit for the price consumers are willing to pay to obtain a unit product. x differs according to different consumers. The evaluation x of a minority of consumers may be a negative value (a negative x value represents that, even if the price of the product is reduced to 0, the consumer would still be unwilling to possess the product). This paper supposes that the probability density function of x distribution to be $f(x)$, the mean to be μ , and the variance to be σ^2 . This paper also supposes the latent consumption quantity of a product (consumption when price = 0) to be N .

If the unit product pollution when the government does not levy pollution taxes (when $t = 0$) is represented by \hat{w} , then $g(w), w \in [0, \hat{w}]$ represents the pollution reduction investment cost for the unit product pollution of a manufacturer w . Due to the incremental increase of marginal cost $\{g'(w)\}$, g have the following characteristics:

$$g(w) > 0, \forall w \in [0, \hat{w}], g(\hat{w}) = 0,$$

$$g'(w) \leq 0, g'(\hat{w}) = 0 \text{ \& } g''(w) > 0, \forall w \in [0, \hat{w}] \tag{2.2}$$

$g'(\hat{w}) = 0$ represents that the marginal cost of pollution reduction is 0 at the unit pollution upper limit \hat{w} (when the product is at its maximum pollution, the manufacturer can easily reduce product pollution).

The sufficient conditions for a consumer purchasing a product are: the evaluation x of a consumer for the product must not be less than the price of obtaining the product $P + tw$, meaning that : $x \geq P + tw$.

The problem faced by manufacturers: based on the pollution taxation rate t decided by the government and

after understanding the $f(x)$ distribution in the consumer group, then how manufacturers should decide (P, w) values to maximize the corresponding total profit π ; and the mathematical model is:

$$\max_{(p,w)} \pi = [p - g(w) - c] \cdot N \cdot \int_{p+tw}^{\infty} f(x) dx \quad (2.3)$$

In this expression, c is the production cost of the manufacturer for a unit product.

Optimal Solution for the Model

For a given t value, let $(p^*(t), w^*(t))$ be the optimal solution for model expression (2.3); also cause k^* to be the tax-inclusive product price, meaning that $k^*(t) = p^*(t) + tw^*(t)$.

In examining model (2.3), it is valid that, if $k = k^*(t)$ is taken from the following problem (3.2), then $(p^*(t), w^*(t))$ is the optimal solution for problem (3.2) at the same time. (3.1)

The process for seeking the optimal solution (2.3) is divided into the following two stages for discussion:

Stage 1: For a given t value, consider how manufacturers should decide (P, w) in a situation maintaining the cost of a consumer obtaining a unit product $p + tw$ as a constant k . The mathematical model for the problem of maximizing unit profit $[p - g(w) - c]$ is as follows: (Note: because k is a constant, $N \int_k^{\infty} f(x)$ and the selection of (p, w) are unrelated; consequently, under the constraint of $p + tw \equiv k$, manufacturers pursuing the maximum unit profit H is equivalent to pursuing the maximum total profit π).

$$\begin{cases} \max_{(p,w)} & H = [p - g(w) - c] \\ \text{s.t.} & p + tw \equiv k \end{cases} \quad (3.2)$$

In the previous expression t and k are two given positive numbers.

Given t and k , let (\bar{p}, \bar{w}) be the optimal solution of expression (3.2). From the constraint of (3.2) obtain $p = k - tw$, $w \in [0, \frac{k}{t}]$; substituting it into the objective

function of (3.2), expression (3.2) can be rewritten as:

$$\max_{w \in [0, \frac{k}{t}]} H = [k - tw - g(w) - c] \quad (3.2')$$

Because $H(\frac{k}{t}) = -g(\frac{k}{t}) - c < 0$ (meaning that the unit profit of the manufacturer is negative at $\frac{k}{t}$), the maximum point \bar{w} of H will not occur at the right end of interval $[0, \frac{k}{t}]$, meaning $\bar{w} \neq \frac{k}{t}$.

Considering the differential of H for w in (3.2') yields:

$$H'(w) = -t - g'(w), H''(w) = -g''(w) < 0, \forall w$$

Thus, if $H'(0) = -t - g'(0) \leq 0$, then $H'(0) < 0, \forall w \in [0, \frac{k}{t}]$; consequently, $\bar{w} = 0$. This result contradicts the assumption of (2.1). It can be seen from the foregoing discussion that \bar{w} will not be positioned at the right end of interval $[0, \frac{k}{t}]$, meaning $\bar{w} \in (0, \frac{k}{t})$. Consequently, the derivative of objective function H of (3.2') must be 0 at point \bar{w} , meaning:

$$0 = H'(\bar{w}) = \frac{d}{dw} [k - tw - g(w) - c]_{w=\bar{w}} = -t - g'(\bar{w}), \quad \text{further meaning:} \\ g'(\bar{w}) = -t, \forall (t, k) \quad (3.3)$$

Considering the differential of (3.3) for t and utilizing (2.2) yields:

$$g''(\bar{w}(t)) \cdot \bar{w}'(t) = -1, \text{ meaning } \bar{w}'(t) = \frac{-1}{g''(\bar{w}(t))} < 0 \quad (3.4)$$

Because $\frac{d^2}{dw^2} [k(t) - tw - g(w) - c] = \frac{d}{dw} (-t - g'(w)) = -g''(w) < 0, \forall w \in (0, \hat{w})$, there is exactly one \bar{w} that satisfies expression (3.3), meaning:

$$\bar{w} = g'^{-1}(-t); \text{ therein } g'^{-1} \text{ is the inverse of } g' \quad (3.5)$$

Using (3.1), (3.3), and (3.5) shows that the w^* of optimal solution (p^*, w^*) in Model (2.3) must satisfy:

$$g'(w^*) = -t, \text{ meaning } w^* = g'^{-1}(-t), \forall t \quad (3.6)$$

Stage 2: Using (3.6) allows the mathematical model (2.3) to be rewritten as:

$$\max_p \pi = (p - g(g'^{-1}(-t)) - c) \cdot N \cdot \int_{p^* + tg'^{-1}(-t)}^{\infty} f(x) \quad (2.3')$$

Where t is a given positive number. Expression (2.3') shows that first order necessary conditions for obtaining the optimal solution are:

$$0 = \left. \frac{d\pi}{dp} \right|_{p=p^*} = N \cdot \int_{p^* + tg'^{-1}(-t)}^{\infty} f(x) dx - N \cdot [p^* - g(g'^{-1}(-t)) - c] f'(p^* + tg'^{-1}(-t)) \quad (3.7),$$

Meaning:

$$\int_{p^* + tg'^{-1}(-t)}^{\infty} f(x) dx = [p^* - g(g'^{-1}(-t)) - c] f'(p^* + tg'^{-1}(-t)) \quad (3.8)$$

Where p^* is a function of t , written as $p^* = p^*(t)$. Expression (3.8) can be further rewritten as:

$$[p^* - g(g'^{-1}(-t)) - c] = \frac{\int_{p^* + tg'^{-1}(-t)}^{\infty} f(x) dx}{f'(p^* + tg'^{-1}(-t))} \quad (3.8.1)$$

The second-order necessary conditions for obtaining the optimal solution of (2.3') are that there must exist some deleted neighborhood B in $p^*(t)$, establishing the following inequalities:

$$\left. \frac{d^2\pi}{dp^2} \right|_{p=p^*(t)} = -2f'(p^* + tg'^{-1}(-t)) - [p^* - g(g'^{-1}(-t)) - c] f''(p^* + tg'^{-1}(-t)) < 0, \forall p \in B$$

, meaning:

$$2f'(p^* + tg'^{-1}(-t)) + [p^* - g(g'^{-1}(-t)) - c] f''(p^* + tg'^{-1}(-t)) > 0, \forall p \in B \quad (3.9)$$

It can be known from (3.6) and the previous inferences that, for a given t , the optimal price $p^*(t)$ of a manufacturer is determined by expression (3.8). The optimal unit pollution $w^*(t)$ is determined by function g as shown as follows:

$$w^*(t) = g'^{-1}(-t)$$

NATURE OF OPTIMAL SOLUTION

Discussion 1: The impact of an increase in unit production cost c on optimal price P^* . As can be seen from the right side of expression (3.7), when unit product production cost c increases, the right side of the equation

will increase, meaning that the optimal price P^* will also increase.

Discussion 2: The impact of increasing latent quantity N on optimal price P^* . As can be seen from the right side of expression (3.7), when N increases, the right side of the expression will increase, meaning that the optimal price P^* will also increase

Discussion 3: The impact of increases in unit product pollution taxation rate t on the unit product price $[p^*(t) + tw]$ paid by consumers.

Expression (3.7) differentiates t and uses the proof in Appendix 1 and expression (3.9), obtaining:

$$\begin{aligned} \frac{d[p^*(t) + tw^*]}{dt} &= [p^{*'}(t) + w^*(w) + tw^{*'}] \\ &= \frac{f(p^*(t) + tw^*)w^*}{2f(p^*(t) + tw^*) + [p^*(t) - g(w^*) - c] f'(p^*(t) + tw^*)} > 0 \end{aligned} \quad (4.1)$$

Discussion 4: Assume a situation where the consumer group evaluation function $f(x)$ is an exponential distribution.

$$f(x; \theta) = \begin{cases} \frac{1}{\theta} e^{-\frac{x}{\theta}} & \text{for } x > 0 \\ 0 & \text{elsewhere} \end{cases}$$

Assume

where θ is the mean value of $f(x)$, as an exponential distribution; after calculation, expression (3.9) can obtain the optimal unit profit for the manufacturer $[p^* - g(g'^{-1}(-t)) - c]$ as the mean θ of the evaluation of the consumer group for the product. The proof is as follows: substituting $f(x; \theta)$ into expression (3.8) obtains:

$$\int_{p^* + tg'^{-1}(-t)}^{\infty} \frac{1}{\theta} e^{-\frac{x}{\theta}} dx = [p^* - g(g'^{-1}(-t)) - c] \frac{1}{\theta} e^{-\frac{p^* + tg'^{-1}(-t)}{\theta}}$$

$$e^{\frac{p^* + tg'^{-1}(-t)}{-\theta}} = [p^* - g(g'^{-1}(-t)) - c] \frac{1}{\theta} e^{\frac{p^* + tg'^{-1}(-t)}{-\theta}}$$

Thus,

$$\text{Obtaining } p^* = g(g'^{-1}(-t)) + c + \theta \quad (4.2)$$

Consequently, p^* will increase proportionally to c, θ ; it will decrease with the increase of t . p^* and N are unrelated.

RESULTS AND DISCUSSION

This paper considers how to solve environmental pollution problems using governmental levying of consumption pollution taxes and produces a specific mathematical model to be discussed from the decision-making processes between government, manufacturers, and consumers under this tax system.

In the discussion of manufacturer decision behaviors in this model, this paper adds unit optimal pollution selection to previous settings of manufacturer product price and quantity. In consumer settings, an attempt is made to discuss consumer evaluations of products using a probability density function, aiming to use this expanded model to more clearly understand manufacturer decision behaviors under this consumption tax system.

This model uses a mathematical solution to obtain the optimal unit pollution for manufacturers under a set pollution tax rate, as in expression (3.6). It can be seen from this expression that the optimal pollution is primarily determined by the pollution reduction cost function. In addition, the optimal pricing for manufacturers is represented by expression (3.8). With this tax method, as shown in expression (3.4), it is found that the optimal unit product pollution will decrease along with increases in tax rates, meeting the ideal of this method for solving environmental pollution issues.

Further, it can be obtained from expression (3.8.1) that unit profit of manufacturers is equal to the Hessian rate at the tax-inclusive prices of product evaluation distributions of consumer groups. In terms of increases in tax rates, based on expression (4.1), the changes in unit profit due to increases in tax-inclusive prices for consumers are based on the Hessian rate at that price point. The distribution of consumer evaluations can be known through actual market investigation; further calculation of changes in the Hessian rate can allow for understanding of changes in manufacturer unit profits, providing a reference for manufacturers.

Consequently, this paper constructs consumer behaviors based on consumer evaluations of products, providing more specific descriptions than seen in previous economics studies. Based on this method, the distribution function of consumers is related to the unit profit of manufacturers in addition to market demand; specific results can be obtained through market investigation.

In terms of setting the consumer evaluation function, an exponential distribution was used to obtain the optimal pricing for manufacturers, as shown in expression (4.2). It can be seen from this expression that the optimal unit price for manufacturers is equal to the mean of consumer product evaluations and that factors affecting optimal

pricing include product unit production cost, the mean of consumer product evaluations, and unit product pollution tax rate; optimal pricing is unrelated to latent consumption. In addition, increases in product costs and latent consumption quantity have the effect of increasing optimal pricing according to this study.

Future studies can be extended in a number of directions: first, determination of government taxation objectives and the optimal tax rate; second, examination of specific forms of pollution reduction cost functions; and third, empirical studies the forms of consumer evaluation functions. These are directions worthy of consideration for future study.

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