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Optimal decision-making for supplier-buyer’s maximum profit in a two echelon supply chain

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Supply chain management (SCM) is becoming extremely important to achieve competitiveness in the current business environment. Moreover, the supply chain partnership between suppliers and buyers in SCM has had a significant impact on supply chain performance. In this study, we developed a quantitative model based on the supply chain partnership under several assumptions. For this purpose, we first proved several basic theorems which verify the relationship between supplier and buyer’s total profit for two cases, with and without supply chain partnership. We then proved that there exists the supplier’s selling price per unit which makes the maximum total profits for both supplier and buyer with supply chain partnership greater than those for any given supplier’s selling price per unit without supply chain partnership. Finally, we presented the graphical interpretation of our results by using a linear demand function.

Key words: Supply chain management, partnership, profit sharing, pricing policy, win-win strategy.

INTRODUCTION

Recently the paradigm for corporate management has been rapidly shifted from the competition between individual firms to the competition between supply chains. It is because most of the firms have realized that, to survive in a competitive global business environment, it is not enough to have only the competitiveness of individual firms, but essential to share their win-win strategies together through their close relationships. Consequently, supply chain management (SCM) that enhances supply chain performance by minimizing the total cost or maximizing the total profit as well as the service quality for customers has become one of key issues in the current business environment.

In particular, together with the spread of SCM, the relationship between suppliers and buyers has been changed significantly. In fact, recently, suppliers and buyers have been trying to establish the supply chain partnership which enables to coordinate together so that they are able to obtain the global optimum at which they maximize the supply chain’s total profit, whereas suppliers and buyers in the past just tried to get the local optimum at which they maximize their individual profit independently without supply chain partnership. As a result, many researchers have recognized the supply chain partnership as one of key factors of SCM that has to be continuously maintained to achieve competitive priorities for both suppliers and buyers, and have been actively working on several issues related to supply chain partnerships.

Recent conceptual and empirical studies regarding supply chain partnership have been mainly concentrated on the factors influencing partnerships and the performance of partnerships. It turned out that the main factors that affect supply chain partnership would include trust (Lee and Mellat-Parast, 2009; Mohammed et al., 2009; Corsten and Kumar, 2005; Dapiran and Hogarth-Scott, 2003), information sharing (Larson and Kulchitsky, 2008; Hsu et al., 2008; Li et al., 2005), interdependence (Ryu et al., 2009; Laaksonen et al., 2008; Krishnan et al., 2006; Yilmaz et al., 2005), and cooperation (Haday and Cassivi, 2007; Matopoulos et al., 2007; Fynes et al., 2005). Moreover, Krause et al. (2007) emphasized the importance of a long-term partnership relationship...
between buyers and suppliers in order to improve performance in the supply chain. Fynes et al. (2008) showed that suppliers would be able to improve supply chain performance by engaging in deep partnership types of supply chain relationships. Vachon and Klassen (2006) explored the outcome, in terms of operational performance, of green project partnership in the supply chain. Sodhi and Son (2009) also modeled the strategic as well as the operational dimension of performance of supplier–retailer partnerships in terms of various factors for partnership performance identified in the literature.

As is stated above, methodologies used in many researches in the literature could be classified into qualitative models rather than quantitative models. However, it is essential to utilize various quantitative methodologies to provide the supply chain specific action plans for achieving the win-win situation. For example, it is very important to figure out what the optimal order quantities or the optimal selling prices should be in order to obtain the global optimum of a supply chain based on supply chain partnership. In fact, several researchers such as Sucky (2006); Chan and Kingsman (2007); Van den Heuvel et al. (2007) and Dai and Qi (2007) worked on problems how to maximize savings and enhance profit for the whole supply chain where demand rate is considered fixed. In contrast, the fixed demand assumption was relaxed in some researches where joint lot sizing and pricing decisions are used to determine the optimal price and order quantity for maximization of the firm’s profit (Abad, 1994; Kim and Lee, 1998; Jung and Klein, 2001; Jung and Klein, 2005; Abolhasanpour et al., 2009). Peidro et al. (2009) also provided a review of quantitative models related to supply chain planning methods under uncertainty. In this study, we will develop a quantitative model for the supply chain partnership under several assumptions by extending the results obtained by Van der Veen and Venugopal (2000) and Cachon (1999).

Van der Veen and Venugopal (2000) proposed a model of determining the optimal selling price for supplier and buyer in a two echelon supply chain under the assumption that buyer’s demand function is a linear function of its selling price. Cachon (1999) also proposed the buyer’s optimal order quantity models for two cases, with and without supply chain partnership in a two echelon supply chain under the assumption that buyer’s selling price is a function of buyer’s order quantity.

However, it should be noted that Cachon (1999) did not provide how to determine the supplier and buyer’s optimal selling price for two cases, with and without supply chain partnership and Van der Veen and Venugopal (2000) just worked on a too restrictive model by assuming the linearity of the buyer’s demand function. Therefore, in this study, we try to generalize assumptions and extend the results given by Cachon (1999) and Van der Veen and Venugopal (2000) to develop an optimal decision making model for supplier and buyer’s profit sharing and pricing policies based on supply chain partnership in a two echelon supply chain.

We actually assume that the buyer’s demand function is a function of buyer’s selling price and is also a strictly decreasing, concave, and twice differentiable function rather than a linear function. The above assumptions make our model different from those of Cachon (1999) and Van der Veen and Venugopal (2000). Then we will prove the following. Firstly, without supply chain partnership, for a given supplier’s selling price per unit, there exists the buyer’s selling price per unit such that the buyer’s total profit is maximized. Secondly, without supply chain partnership, for a given buyer’s order quantity, there exists the supplier’s selling price per unit such that the supplier’s total profit is maximized. Thirdly, the buyer’s selling price per unit which maximizes the supply chain’s total profit with supply chain partnership is lower than buyer’s selling price per unit which maximizes buyer’s total profit without supply chain partnership. Fourthly, for a given supplier’s selling price per unit, the buyer’s total profit without supply chain partnership is greater than that with supply chain partnership, whereas the opposite case holds for the supplier’s total profit. Finally, there exists the supplier’s selling price per unit which makes the maximum total profits for both supplier and buyer with supply chain partnership greater than those for any given supplier’s selling price per unit without supply chain partnership.

**BASIC ASSUMPTIONS AND MODELS**

As shown in Figure 1, for a given two echelon supply chain that consists of a single supplier and a single buyer, we assume that the buyer orders the customers’ total demand to the supplier and then the supplier immediately delivers the buyer’s order quantity to the buyer so that the buyer can satisfy each customer’s demand within an allowable time requested by customers. Moreover, we assume the followings:

1.) Buyer places an order just once.
2.) Buyer’s selling price per unit is greater than its purchasing price per unit.
3.) Buyer has deterministic demand which is not only a function of buyer’s selling price, but also a strictly decreasing, concave, and twice differentiable function.
4.) Buyer’s order quantity is equal to buyer’s demand.
5.) The sum of supplier’s production cost per unit and supplier’s transportation cost per unit is less than the supplier’s selling price per unit.

We now define several notations to present a quantitative model based on the supply chain partnership.

1) \( p_s \) : supplier’s selling price per unit (that is, buyer’s purchasing price per).
2) \( p_b \): buyer’s selling price per unit.
3) \( c_p \): supplier’s production cost per unit.
4) \( c_t \): supplier’s transportation cost per unit.
5) \( D(p_b) \): buyer’s demand (that is, buyer’s order quantity).
6) \( TP_s \): supplier’s total profit.
7) \( TP_b \): buyer’s total profit.
8) \( TP \): supply chain’s total profit (that is, \( TP = TP_s + TP_b \)).

Without loss of generality, we can assume the sum of supplier’s production cost and transportation cost per unit is a constant value, that is, \( c_p + c_t = c \). Then by following the assumption 5), we have \( c < p_s < p_b \). Since the buyer’s total profit is defined by “(buyer’s selling price per unit - supplier’s selling price per unit) \times buyer’s demand”, \( TP_b \) can be expressed by a function of \( p_b \) and \( p_s \) as follows:

\[
TP_b = TP_b(p_b, p_s) = (p_b - p_s) \times D(p_b)
\]

Similarly, the supplier’s total profit \( TP_s \) can also be expressed by a function of \( p_b \) and \( p_s \) as follows:

\[
TP_s = TP_s(p_b, p_s) = (p_s - c) \times D(p_b)
\]

Consequently, the supply chain’s total profit \( TP \) can be expressed by the following. Note that \( TP \) is a function of only \( p_b \).

\[
TP = TP(p_b) = TP_s + TP_b = (p_b - c) \times D(p_b)
\]

It is obvious that buyer sells customers the amount of \( D(p_b)(\geq 0) \) with the unit price of \( p_b(\geq 0) \), and there exists the buyer’s maximum selling price per unit of \( \hat{p}_b \) such that \( D(\hat{p}_b) = 0 \). Therefore, we have \( D'(p_b) < 0 \) and \( D''(p_b) < 0 \) for any \( p_b \in [0, \hat{p}_b] \), since \( D(p_b) \) is a strictly decreasing, concave, and twice differentiable function in \([0, \hat{p}_b]\) by the assumption.

**A Decision Making Model Without Supply Chain Partnership**

In this section, we will develop an optimal decision making model without supply chain partnership. We will prove that there exists the supplier and buyer’s selling price per unit such that it maximizes the supplier and buyer’s profit, respectively. For this purpose, we will first prove that the buyer’s total profit function is a concave function of the buyer’s selling price per unit for a given the supplier’s selling price per unit.

**Lemma 1**: For a given \( p_s \), \( TP_b \) is a concave function of \( p_b \) in \([\hat{p}_b, 0]\).

**Proof**: For a given \( p_s \), if we take the derivative of \( TP_b \) with respect to \( p_b \), then we have

\[
TP_b'(p_b, p_s) = D(p_b) + (p_b - p_s) \cdot D'(p_b)
\]

If we take the second derivative of \( TP_b \) with respect to \( p_b \), then we have

\[
TP_b''(p_b, p_s) = 2D'(p_b) + (p_b - p_s) \cdot D''(p_b)
\]

But, by our assumptions, we know that \( D'(p_b) < 0 \), \( p_b - p_s > 0 \), and \( D''(p_b) \leq 0 \). Therefore, it follows that \( TP_b''(p_b, p_s) < 0 \) and it concludes the proof.

Now, by using Lemma 1, we can prove that there exists the buyer’s selling price per unit such that it maximizes the buyer’s total profit for a given supplier’s selling price.

**Lemma 2**: For a given \( p_s \), there exists the buyer’s selling price per unit \( \hat{p}_s \) such that \( TP_b'(p_b, p_s) = 0 \).

**Proof**: By Lemma 1, we know that \( TP_b'(p_b, p_s) < 0 \) for any \( p_b \in [p_s, \hat{p}_b] \). Moreover, since \( TP_b'(p_s, p_s) = D(p_s) > 0 \), \( D(\hat{p}_b) = 0 \), \( p_b - p_s > 0 \), and \( D'(\hat{p}_b) = 0 \), we
Lemma 3: For a given \( q = D(p_b^*) \), \( T\) is a concave function of \( p_b \) in \([c, \hat{p}_b] \). Proof: For a given \( q = D(p_b^*) \), \( T\) is a function of \( p_b \), and if we take the derivative of \( T\) with respect to \( p_b \), then we have

\[
T'_b(p_b, p_s) = D(p_b) + (p_s - p_b) \cdot D'(p_b) < 0.
\]

Therefore, there exists \( p_b \in [c, \hat{p}_b] \) such that \( TP_b^*(p_b^*, p_s) = 0 \).

Moreover, we assume that \( p_b^* \) is a function of \( p_s \), and there exists \( \hat{p}_b \in [c, \hat{p}_b] \). For example, if the buyer's demand function is \( D(p_b) = \alpha - \beta \cdot p_b^2 \), where \( \alpha, \beta > 0 \), then it is not difficult to see that \( p_b^* = \frac{1}{3} \left( 1 + \sqrt{\frac{\beta \rho}{\beta^2 p_s^2 + 3 \alpha \beta}} \right) > 0 \).

Lemma 4: For a given \( q = D(p_b^*) \), there exists the supplier's selling price per unit \( p_s^* \) such that \( TP_b^*(p_b^*, p_s^*) = 0 \).

Proof by Lemma 3, we know that \( TP_b^*(p_b^*, p_s) < 0 \) for a given \( q = D(p_b^*) \) and \( p_s \in [c, \hat{p}_s] \). Moreover, we have

\[
T'_b(p_b^*, c) = D(p_b^*) > 0
\]

and \( T'_b(p_b^*, \hat{p}_b) = D(\hat{p}_b) + (\hat{p}_s - c) \cdot D(p_b^*) < 0 \). Therefore, there exists \( p_b^* \in [c, \hat{p}_b] \) such that \( TP_b^*(p_b^*, p_s^*) = 0 \).

Consequently, it can be easily obtained by Lemma 2 and Lemma 4 that the maximum total profit of supply chain without supply chain partnership is \( TP = TP(p_b^*, p_s^*) = (p_b^* - c) \cdot D(p_b^*) \).

A DECISION MAKING MODEL WITH SUPPLY CHAIN PARTNERSHIP

In this section, we will develop an optimal decision making model with supply chain partnership. If we refer to proofs of Lemmas in section 3, we can easily prove that \( TP \) is a concave function of \( p_b \) in \([c, \hat{p}_b] \) and there exists the buyer's selling price per unit \( p_b^* \in [c, \hat{p}_b] \) such that it maximizes \( TP \) with supply chain partnership.

Therefore, the maximum total profit of supply chain with supply chain partnership is

\[
TP = TP(p_b^*, c) = (p_b^* - c) \cdot D(p_b^*)
\]

and the following theorem holds.

Theorem 1: If \( p_s > c \), then \( p_b^* < p_b^* \).

Proof: since \( p_s > c \) and \( D'(p_b^*) < 0 \), it follows that

\[
T'_b(p_b^*, p_s) = (p_b^* - p_s) \cdot D(p_b^*) + (p_b^* - c) \cdot D'(p_b^*) < 0.
\]

By the definition of \( p_b^* \), we have

\[
TP'(p_b^*, p_s) = (p_b^* - c) \cdot D'(p_b^*) + D(p_b^*) = 0
\]

and, by the definition of \( p_b^* \), we have \( TP'_b(p_b^*, p_s) = 0 \). Therefore, it follows by (1) that \( TP'(p_b^*, p_s) = TP'(p_b^*, p_s) > 0 \). But, since \( TP_b^* < 0 \), we know that \( TP_b^* \) is a strictly decreasing function of \( p_b \). Therefore, it follows that \( p_b^* < p_b^* \).

Corollary 1: If \( p_s = c \), then \( p_b^* = p_b^* \).

Proof: If \( p_s = c \), then we have
It should be noted from Theorem 1 that the buyer’s selling price per unit which maximizes the total profit of supply chain without supply chain partnership is less than or equal to the supplier’s total profit without supply chain partnership, and vice versa for the buyer’s total profit. Consequently, in this case, it is not guaranteed to get a buyer’s cooperation which is essential to maintain the supply chain partnership.

However, we can prove that there exists the supplier’s selling price per unit which makes both of the supplier and buyer’s maximum total profit in the case with supply chain partnership greater than those for a given the supplier’s selling price per unit used in the case without supply chain partnership. This implies that the supply chain partnership guarantees the win-win situation for both supplier and buyer in a two echelon supply chain.

**Lemma 5:** For a given \( p_s \), if

\[
TP_b(p_s^*, \bar{p}_s) \geq TP_b(p_s^*(p_s), p_s),
\]

then we have \( \bar{p}_s \leq p_s \).

Proof: suppose not, then, since \( TP_b \) is a decreasing function of \( p_s \), it follows that \( TP_b(p_s^*, \bar{p}_s) > TP_b(p_s^*(p_s), p_s) \), and thus \( TP_b(p_s^*, \bar{p}_s) > TP_b(p_s^*(p_s), p_s) \) holds. It contradicts to Theorem 2.

**Theorem 4:** For a given \( p_s \), there exists the supplier’s selling price per unit \( \bar{p}_s^* \in [c, p_s] \) such that

\[
TP_b(p_s^*, \bar{p}_s^*) \geq TP_b(p_s^*(p_s), p_s) \quad \text{and} \quad TP_s(p_s^*, \bar{p}_s^*) \geq TP_s(p_s^*(p_s), p_s).
\]

Proof: for a given \( p_s \), we have

i) \( TP_b(p_s^*, \bar{p}_s) \geq TP_b(p_s^*(p_s), p_s) \)

\[
\iff (p_s^* - \bar{p}_s) \cdot D(p_s^*) \geq (p_s^*(p_s) - p_s) \cdot D(p_s^*(p_s))
\]

\[
\iff \bar{p}_s \leq p_s^* \iff (p_s^*(p_s) - p_s) \cdot D(p_s^*(p_s)) = (p_s^* - c) \cdot D(p_s^*) \equiv \bar{p}_s^*.
\]

ii) \( TP_s(p_s^*, \bar{p}_s) \geq TP_s(p_s^*(p_s), p_s) \)

\[
\iff (\bar{p}_s - c) \cdot D(p_s^*) \geq (p_s^*(p_s) - c) \cdot D(p_s^*(p_s))
\]

\[
\iff \bar{p}_s^* \geq p_s^* - c \cdot D(p_s^*(p_s)) \equiv \bar{p}_s^*.
\]

But, by the definition of \( p_s^* \), we have

\[
\bar{p}_s^* - \bar{p}_s = [p_s^* - c \cdot D(p_s^*)] - [p_s^*(p_s) - c \cdot D(p_s^*(p_s))] = D(p_s^*(p_s)) - D(p_s^*) \geq 0 \quad (\text{ie.} \bar{p}_s \leq \bar{p}_s^*)
\]

By the definition of \( p_s^*(p_s) \), we also have

\[
\bar{p}_s^* - \bar{p}_s = [p_s^*(p_s) - c \cdot D(p_s^*(p_s))] \iff [p_s^* - c \cdot D(p_s^*)] \iff [p_s^* - c \cdot D(p_s^*(p_s))] \geq 0 \quad (\text{ie.} \bar{p}_s \leq \bar{p}_s^*)
\]
Moreover, we have \( c \leq \tilde{p}_s \) by our assumption. Therefore, for a given \( p_s \), there exists \( \tilde{p}_s \in [\tilde{p}_s^- , \tilde{p}_s^+] \subseteq [c, p_s] \) such that \( TP_b(p_b^*, \tilde{p}_s) \geq TP_b(p_b^*(p_s), p_s) \) and 
\[
TP_s(p_s^*, \tilde{p}_s) \geq TP_s(p_s^*(p_s), p_s) .
\]

It should be noted that if \( \tilde{p}_s < p_s \) then the supplier has less profit per unit, but more buyer’s order quantity, since \( \tilde{p}_s^*(\tilde{p}_s) < p_b^*(p_s) \) and \( D(p_b^*(\tilde{p}_s)) > D(p_b^*(\tilde{p}_s)) > D(p_b^*(p_s)) \). This implies that the supplier can sell more quantity with cheaper price to the buyer so that the supplier’s total profit with supply chain partnership becomes higher than that without supply chain partnership.

**GRAPHICAL INTERPRETATION OF RESULTS**

In this section, we will graphically illustrate by using an example that our results given in Theorem 2 and 3 hold.

For this purpose, we assume that \( D(p_b) \) is a linear function of \( p_b \) (that is, \( D(p_b) = \alpha - \beta p_b \geq 0, \alpha, \beta \geq 0 \)). Since \( TP_b(p_b^*(p_s), p_s) = (p_b - p_s) \cdot (\alpha - \beta p_b) \) can be considered as a function of \( p_b \), we can get \( p_b^* = \frac{\alpha + \beta p_s}{2\beta} \) by taking the derivative of \( TP_b \) with respect to \( p_b \) and solving \( TP_b'(p_b, p_s) = 0 \). Then immediately follows that \( D(p_b^*) = \frac{1}{2}(\alpha - \beta p_b) \).

Consequently, \( TP_b(p_b^*(p_s), p_s) \) and \( TP_s(p_s^*(p_s), p_s) \) can be expressed as follows:
\[
TP_b(p_b^*(p_s), p_s) = \frac{1}{4\beta}(\alpha - \beta p_b)^2 \\
TP_s(p_s^*(p_s), p_s) = \frac{1}{2}(p_s - c) \cdot (\alpha - \beta p_b) \\
= -\frac{\beta}{2}(p_s - \frac{\alpha + \beta c}{2\beta})^2 + \frac{(\alpha - \beta c)^2}{8\beta} .
\]

Note that \( TP_b(p_b^*(p_s), p_s) \) and \( TP_s(p_s^*(p_s), p_s) \) are not only functions of \( p_s \), but also parabolas.

Similarly, by taking the derivative of \( TP_b \) with respect to \( p_b \) and solving \( TP_b'(p_b) = 0 \), we can easily find out \( p_b^{**} = \frac{\alpha + \beta c}{2\beta} \) and \( D(p_b^{**}) = \frac{1}{2}(\alpha - \beta c) \).

Consequently, \( TP_b(p_b^{**}, p_s) \) and \( TP_s(p_s^{**}, p_s) \) can be also expressed as follows:
\[
TP_b(p_b^{**}, p_s) = -\frac{1}{2}(\alpha - \beta c)p_s + \frac{\alpha^2 - (\beta c)^2}{4\beta} \\
TP_s(p_s^{**}, p_s) = \frac{1}{2}(\alpha - \beta c)(p_s - c) .
\]

Note that \( TP_b(p_b^{**}, p_s) \) and \( TP_s(p_s^{**}, p_s) \) are linear functions of \( p_s \).

Moreover, it should be noted that the equation of the tangent line of \( y = TP_b(p_b^*(p_s), p_s) \) at \( p_s = c \) can be obtained as follows:
\[
y - TP_b(c) = TP_b'(c) \cdot (p_s - c) \Rightarrow y = -\frac{1}{2}(\alpha - \beta c)(p_s - c) + \frac{1}{4\beta}(\alpha - \beta c)^2 \\
= -\frac{1}{2}(\alpha - \beta c)p_s + \frac{\alpha^2 - (\beta c)^2}{4\beta} = TP_b(p_b^{**}, p_s) .
\]

Therefore, we can see that \( y = TP_b(p_b^{**}, p_s) \) is the tangent line of \( y = TP_b(p_b^*(p_s), p_s) \) at \( p_s = c \) and \( TP_b(p_b^{**}, p_s) \leq TP_b(p_b^*(p_s), p_s) \) as shown in Figure 2.

Similarly, the equation of the tangent line of \( y = TP_s(p_s^*(p_s), p_s) \) at \( p_s = c \) can be obtained as follows:
\[
y - TP_s(c) = TP_s'(c) \cdot (p_s - c) \Rightarrow y = -\beta(c - \frac{\alpha + \beta c}{2\beta})(p_s - c) = \frac{1}{2}(\alpha - \beta c)(p_s - c) = TP_s(p_s^{**}, p_s) .
\]

Therefore, we can also see that \( y = TP_s(p_s^{**}, p_s) \) is the tangent line of \( y = TP_s(p_s^*(p_s), p_s) \) at \( p_s = c \) and \( TP_s(p_s^{**}, p_s) \geq TP_s(p_s^*(p_s), p_s) \) as shown in Figure 3.

**Conclusion**

In this study, we showed that the global optimization paradigm with the supply chain partnership which is one of the key issues in SCM dominates the local optimization paradigm without supply chain partnership, under the
several assumptions.

We eventually proved that there exists a supplier’s selling price per unit such that the supply chain partnership guarantees the win-win situation for both supplier and buyer, which means that the maximum total profits for both supplier and buyer at that price with supply chain partnership is greater than those for any given supplier’s selling price per unit without supply chain partnership. Moreover, by using a linear demand function, we graphically illustrated that Theorem 2 and 3 hold. In particular, we proved that if the supplier’s selling price per unit used in the case without supply chain partnership is also equally used in the case with supply chain partnership then the buyer’s and the supplier’s total profit functions with supply chain partnership are actually the tangent lines of the buyer’s and the supplier’s total profit functions without supply chain partnership, respectively. Several researchers have been working on more extended models under their own assumptions than what we presented in this study. For example, Cachon (2003) discussed the supply chain coordination with the contract mechanisms and analyzed which contracts coordinated the supply chain. Cachon and Lariviere (2005) not only demonstrated that revenue sharing coordinated a supply chain with a single retailer and arbitrarily allocated the supply chain’s profit, but also compared revenue sharing...
References


