Optimal ordering policy for fast deteriorating items

Ping-Hui Hsu

Department of Business Administration, De Lin Institute of Technology, Tucheng, New Taipei 236, Taiwan, R.O.C.
E-mail: pinghuihsu@gmail.com. Tel: +886 2 22733567.

Accepted 27 May, 2011

Facing the demand of periodic pattern and expiration date, how to make an optimal ordering policy by the retailer of fast deteriorating items is the key problem nowadays. In this study, we propose a model for fast deteriorating items with periodic pattern demand and expiration date. An algorithm is presented to derive an optimal replenishment cycle, shortage period and order quantity such that the unit time profit is maximized. The coordination policy between the retailer and the supplier improves the efficiency of the ordering policy especially when the deterioration rate is high. Numerical examples and sensitivity analysis are provided to illustrate the theory.

Key words: Fast deteriorating items, expiration date, coordination, periodic pattern demand.

INTRODUCTION

This study is motivated by a real life problem faced by a florist. Most florists usually make frequent replenishments due to fast deterioration of fresh flowers. The high deteriorated cost and the frequent ordering cost have affected the flower retailer’s benefit. The problem facing a florist is how to develop an ordering policy that maximizes the profit. In general, profit is a function of the sales revenue, the purchasing cost, the lost sale cost, the processing cost, the inventory holding cost, the ordering cost, the production cost, and the shipment cost.

Flower, fruit, and seafood are common fast deteriorating items. These products will deteriorate expeditiously with time resulting in fast decreasing utility or price from the original one. The customer demand follows a periodic pattern that repeat itself after a short time interval. Moreover, the customer demand declines when the product is close to its expiration date. The product expiration date indicates the latest time that the product may be used (not the end of the product life cycle time). The loss of profit is caused by deterioration and declining demand. To improve the supply chain efficiency, the coordination between the supplier and the retailer must be considered.

Deteriorating inventory was originally studied by Ghare and Schrader (1963). Since then, it has received much attention from researchers (Wee, 1995; Rau et al., 2003; Yang, 2004; Hsieh and Lee, 2005; Dye et al., 2007; You, 2005; He et al., 2010). Generally, two situations of deteriorating rate are discussed. One is constant (Shah and Jaiswal, 1977; Aggarwal, 1978; Padmanabhan and Vratb, 1995; Bhunia and Maiti, 1999), and the other is not constant [a: Linear increasing function of time (Bhunia and Maiti, 1998; Mukhopadhyay et al., 2004); b: Weibull distributed (Wee, 1999; Mahapatra, 2005; Chakraborty et al., 1998); c: Other function of time (Abad, 2001)]. Ho et al. (2007) considered the effects of deteriorating inventory on lot-sizing in material requirements planning systems. They presented the effect on the relevant cost due to various deterioration rates. Hsu et al. (2007) addressed a deteriorating inventory replenishment model with expiration date and uncertain lead time. Goyal and Gupta (1989), Weng (1995), Fites (1996), Zimmer (2002), Sucky (2005), considered the coordination between the suppliers and the retailers in order to improve the performance of the supply chains. Hsu et al. (2010) proposed to invest on preservation technology to decrease the deterioration rate of items. However, researches on the influence of expiration date and products with fast deterioration rate have received little attention.

In this study, the customer demand periodic pattern was assumed. The retailer obtains the product from the supplier for sale to the customers. The constant deteriorating rate product has an expiration date. An
algorithm with coordination policy is developed to determine the replenishment and backordering decision of the deteriorating items with expiration date and periodic pattern demand.

ASSUMPTIONS AND NOTATION

The following notation is used throughout this paper. The general parameters are:

- \( T \) Length of a periodic interval
- \( N \) Discrete number; \( NT \) denotes the expiration date of product
- \( K \) Constant deterioration rate of on-hand-stock, \( 0 \leq K < 1 \)
- \( \theta(j) \) The fraction that customers are willing to purchase the item under the condition that they receive their order after \( \eta \) units of time

The decision variables are:

- \( n \) Discrete number; decision variable, \( nT \) denotes the replenishment cycle, \( n \leq N \)
- \( V \) Critical time at which inventory level reaches zero, decision variable

The parameters related to the retailer are:

- \( n_R \) Discrete number; \( n_R T \) denotes the retailer’s optimal replenishment cycle
- \( v_R \) Retailer’s optimal critical time
- \( p \) Retailer’s selling price per unit
- \( p_b \) Retailer’s selling price per unit when shortages occur
- \( h \) Unit inventory holding cost per unit time
- \( c \) Retailer’s wholesale purchase price per unit
- \( c_o \) Retailer’s ordering cost per replenishment cycle
- \( r \) Retailer’s penalty cost per unit of a lost sale including loss of profit
- \( \mu \) Retailer’s processing cost including making an inventory and deteriorated items per period
- \( Q \) Retailer’s order quantity each replenishment
- \( Q_1 \) Retailer’s sales amount without backordering over replenishment cycle
- \( Q_2 \) Retailer’s backordered quantity at the end of replenishment cycle
- \( F_R \) Unit time profit for the retailer

The parameters related to the supplier are:

- \( c_m \) Supplier’s production cost per unit, \( c_m < c \)
- \( s \) Supplier’s shipment cost per replenishment
- \( F_S \) Unit time profit for the supplier

The other related parameters are as follows:

- \( n_T \) Discrete number; \( n_T T \) denotes the supplier-retailer joint optimal replenishment cycle
- \( v_T \) Supplier-retailer joint optimal critical time
- \( I(0) \) Maximum inventory level at the start of a cycle
- \( F \) Unit time system profit, that is, the supplier-retailer joint total profit; \( F=F_R+F_S \)

In developing the model, the following assumptions are made:

(i) The retailer’s selling price per unit \( p \) and backorder price \( p_b \) are predetermined such that: \( p_b=\lambda p > c \), where \( 0<\lambda<1 \).
(ii) Demand for the product is influenced by periods. That is, \( d_j \) is the demand rate at the \( j \)-th period such that \( d_j = dw(j), j = 1,2,...,N \),

where \( w(j) = \frac{N-j+1}{N} \) is a conserved function, and the value of \( d \) is a known constant with \( d>0 \), which denotes the demand rate of the first period. This means that the customers’ demand is less when it is nearer to the product expiration date. Note: The demand at the \( n \)-th period includes two parts, that is, \( [0,v_T] \) and \( [v_T,T] \). Since the retailer is willing to wait for backorders of new items during stockout, the demand during \( [0,v_T] \) is based on \( d_1 \), on the other hand, the demand rate during \( [v_T,T] \) is based on \( d_{in} \), on the other hand, the demand rate during \( [v_T,T] \) is based on \( d_{in} \).
(iii) Demand during the stock out period is partially lost due to impatient customers.
(iv) Backlogged demand is satisfied at the beginning of each replenishment.
(v) Shortage time is less than the length of a periodic interval \( T \).
(vi) The fraction of customers’ backordered is assumed to be linearly decreasing with waiting time \( \eta \) and is assumed to be \( \theta(\eta)=1-\eta/T, 0 \leq \eta < T \).
(vii) The capacity of the warehouse is unlimited.
(viii) There is no replacement or repair of deteriorated items during a given cycle.

MODELING AND ANALYSIS

In this section, a supply chain with the retailer and the supplier is assumed. The retailer obtains the products from the supplier for sale to the customers. The study consider the products of fast deteriorating items with constant deterioration rate \( k \) and expiration date \( NT \). (For example, the retailer places an order of some flowers in bud, the flowers will deteriorate till fade for 7 days \( (N=7, T=1 \text{ days}) \).) The customers’ periodic pattern demand is \( d_j, j=1, 2, 3...N \). Which means the demand is decreasing due to fast deterioration. Shortage backorder is allowed. Backlogged demand is satisfied at the beginning of each replenishment. Placing an optimal order before the selling period of the product is vital to the retailer. The aim of this
study is to maximize the unit time profit by determining (1) the retailer’s replenishment cycle (2) the duration of the shortages, and (3) the retailer’s order quantity \( Q \). Two polices are developed to illustrate our study: (i) Without coordination (ii) With coordination.

The inventory system of the retailer during a given cycle is depicted in Figure 1. Suppose the retailer’s replenishment cycle is set at \( nT \). The study derives the model by backward deduction. Let \( I_n(t), n \leq N \), be the inventory level during the \( n \)th period. The differential equation governing the transition of the system during the period interval is;

\[
\frac{dI_n(t)}{dt} = -d_1, \quad \nu \leq t \leq T. 
\]

(In the \( n \)th period, since the customers who are willing to backorder will need new items, therefore, the demand rate in the \( n \)th period divides into two parts, that is, \( d_n \) in \( 0 \leq t \leq \nu \), and \( d_1 \) in \( \nu \leq t \leq T \).)

\[
\frac{dI_n(t)}{dt} = -kI_n(t) - dw(n), \quad 0 \leq t \leq \nu. 
\]  

with initial condition \( I_n(\nu) = 0 \). From (1), one has

\[
I_n(t) = \frac{dw(n)}{k} \left[ e^{k(\nu-t)} - 1 \right], \quad 0 \leq t \leq \nu. 
\]  

Let \( I_{n-1}(t) \) be the inventory level during the \((n-1)\)th period, then

\[
\frac{dI_{n-1}(t)}{dt} = -kI_{n-1}(t) - dw(n-1), 
\]

with initial condition \( I_{n-1}(T) = I_n(0) \). From (3), one has

\[
I_{n-1}(t) = \frac{d}{k} \left[ w(n-1)e^{k(T-t)} - 1 \right] + w(n)\left( e^{k\nu} - 1 \right) e^{k(T-t)} \right], \quad 0 \leq t \leq T. 
\]

Similarly, the inventory level during the \( j \)th period is

\[
I_j(t) = \frac{d}{k} \left[ w(j)e^{k(T-t)} - 1 \right] + \sum_{i=j+1}^{n-1} w(i)e^{kT} - 1 \right] e^{k[(i-j)T-t]} 
\]

\[
+ w(n)(e^{k\nu} - 1)e^{k[(n-j)T-t]}, \quad 0 \leq t \leq T, \quad j=1, 2, \ldots, n-1. 
\]

Next, the study deduces the retailer’s and the supplier’s objective functions. The objective functions include:

The sales revenues \( R(n, \nu) \),

The purchasing cost \( C(n, \nu) \),

The lost sale cost \( L(n, \nu) \),

The processing cost \( B(n, \nu) \) (including inventory and deteriorated items),

The inventory holding cost \( H(n, \nu) \), and

The ordering cost, \( c_o \).

The replenishment cycle and shortage length are set at \( nT \) and \( T-\nu \) units of time, respectively. Then Retailer’s unit time profit
\[ H(n, \nu) = \sum_{j=1}^{n-2} H_j^T + H_{n-1}^T + H_n^V, \quad n \geq 3. \] (14)

\[ H(1, \nu) = \frac{d}{k} \left( e^{k\nu} - \frac{1}{k} - \nu \right). \] (15)

\[ H(2, \nu) = \frac{d}{k} \left( e^{2k\nu} - \frac{1}{k} - T + \frac{N-1}{N} \left( e^{k\nu} - \frac{1}{k} - \nu \right) \right) + \frac{d}{k} \left( e^{k\nu} - \frac{1}{k} - \nu \right). \] (16)

where (Appendix A)

\[ H_j^T = h \int_0^T I_j(t) dt. \] (17)

\[ H_{n-1}^T = h \int_0^T I_{n-1}(t) dt. \] (18)

\[ H_n^V = h \int_0^V I_n(t) dt. \] (19)

For \( k\nu < 1, \ e^{k\nu} \) is replaced by \( 1 + k\nu + \frac{k^2\nu^2}{2} \) (Taylor series approximation). One has

\[ F_k(1, \nu) = \frac{1}{nT} \left[ p d \nu + k p \left( \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) + \frac{d}{k} \left( \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \right) - \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \right] \right], \quad \nu, \mu, \mu_0 \Rightarrow T, \nu < T. \] (20)

\[ F_k(2, \nu) = \frac{1}{nT} \left[ p d \nu + k p \left( \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) + \frac{d}{k} \left( \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \right) - \frac{d}{k} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \right] \right], \quad \nu, \mu, \mu_0 \Rightarrow T, \nu < T. \] (21)
\[ F_R(n, \nu) = \frac{1}{nT} \left\{ p \left( \frac{d(n-1)(2N+2-n)T}{2N} + \frac{d(N+1-n)\nu}{N} + T \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \right) \right\} \]

\[ - \alpha_1 \ln \left( \frac{N+1-n}{N} \left( k \nu + \frac{k^2 \nu^2}{2} \right) e^{(\alpha_1-1)\nu} \right) + \frac{d}{T} \left( \frac{T^2}{2} - \frac{\nu^2}{2} \right) \]

\[ + \sum_{i=1}^{n-1} w_i \left( e^{(\alpha_1-1)\nu} - 1 \right) \]

\[ + \frac{N-n+2}{N} \left( e^{\frac{\nu}{k}} - 1 \right) + \frac{N-n+1}{N} \left( k \nu + \frac{k^2 \nu^2}{2} \right) - \frac{e^{(\alpha_1-1)\nu} - 1}{k} \]

\[ + \frac{N-n+1}{N} \left( \frac{k \nu + \frac{k^2 \nu^2}{2}}{k} \right) \]

Where

\[ \alpha_1 = \nu(1-e^{\frac{\nu}{k}}) - \sum_{i=2}^{n-1} w_i \left( e^{(\alpha_1-1)\nu} - 1 \right) \]

\[ \alpha_2 = \left[ \frac{e^{\frac{\nu}{k}} - 1}{k} - T \right] \left[ \frac{n-2}{2N} \left( 2N-n \right) + \frac{e^{\frac{\nu}{k}} - 1}{k} \right]. \]

Max: \( F_R(n, \nu) \)

Subject to: \( 1 \leq n \leq N, \quad 0 < \nu \leq T. \)

From Equations 20 to 22, the retailer's unit time profit \( F_R(n, \nu) \) is a function of two variables \( n \) and \( \nu \), where \( \nu \) is a real number and \( n \) is a discrete variable.

**Theorem 1**

\( F_R(n, \nu) \) is concave in \( \nu \).

**Proof (Appendix B)**

From Theorem 1, for given \( n \), we can derive the optimal \( \nu_R^*(n) \) by solving the equation, \( \partial F_R(n, \nu) / \partial \nu = 0 \).

Equating Equation 27 with respect to \( \nu \) to zero, one can derive the retailer's critical time.
\[
\nu_R(n) = \frac{N-n+1}{N} \left[ pd + \frac{dh}{k} - (cd + \frac{dh}{k}) e^{(n-1)kT} \right] + \frac{(cd + \frac{dh}{k})}{T} e^{(n-1)kT}.
\]

(28)

Let \( \nu_R^*(n) = \min \{ \nu_R(n), T \} \). Since the integer variable \( n \) cannot be found by an analytic method, the following solution search procedure is used.

**Solution search procedure**

Step 1. Set \( n = 1, \ nu_R = 0, \ nu_R^* = 0 \) and \( T = 0 \).

Step 2. While \( n \leq N \) do Step 3-5.

Step 3. Solve \( \nu_R(n) \) and \( \nu_R^*(n) \) using (28).

Step 4. Calculate \( F_R(n, \ nu_R(n)) \) using (20)-(22).

Step 5. If \( F_R(n, \ nu_R(n)) > F_R \), let \( F_R = F_R(n, \ nu_R(n)) \), \( n_R = n \) and \( \nu_R = \nu_R(n) \).


From the solution search procedure, if the retailer's optimal solution is \( (n_R, \ nu_R(n_R)) \), then the retailer's optimal unit time profit is \( F_R(n_R, \ nu_R(n_R)) \), the supplier's unit time profit is \( F_S(n_R, \ nu_R(n_R)) \), and the unit time system profit is

\[
F(n_R, \ nu_R(n_R)) = F_R(n_R, \ nu_R(n_R)) + F_S(n_R, \ nu_R(n_R)).
\]

(29)

**Example 1**

The preceding theory can be illustrated by the following numerical example with the following parameters:

- Length of a periodic interval, \( T = 10 \text{ h} \)
- Expiration date, \( N = 10, \ NT = 100 \text{ h} \)
- Retailer's selling price per unit, \( p = $30 \)
- Backorder price, \( p_b = \lambda p = 21 \), where \( \lambda = 0.7 \)
- Retailer's unit inventory holding cost per unit time, \( h = $0.05 \)
- Retailer's wholesale purchase price per unit, \( c_o = $12 \)
- Retailer's penalty cost per unit of a lost sale including loss of profit, \( r = $5 \)
- Processing cost per period, \( \mu = $20 \)
- Retailer's ordering cost per replenishment cycle, \( c_o = $800 \)

Demand rate, \( d_j = d w(j), j = 1, 2, ..., 10 \), where \( w(j) = (11 - j)/10, d = 8 \).

With the deterioration rate \( k = 0.008 \), using the mathematical software MATHCAD and MAPLE, the optimal decision is obtained and the results are as follows: (Table 1) \( n_R = 3, \ nu_R^* = 10, \) the optimal replenishment cycle is \( n_R = 3, \) the length of shortage is \( 0, \) the optimal order quantity \( Q = 110, Q = 242.597, \) the sales amount \( Q_1 + Q_2 = 216, \) the deteriorated quantity per period is \( 26.597, \) the optimal unit profit of retailer is \( F_R = $84.74, \) the unit profit of supplier is \( F_S = $51.606, \) and the unit time system profit is \( F_R + F_S = $136.346. \)

**Sensitivity analysis**

Sensitivity analysis is carried out when a parameter of the fixed set of parameter values \( \Phi = \{ k, N, p, d, T, \lambda, h, c, r, \mu, \) and \( c_o \} \) changes 10, 20 and 30%. The results are shown in Tables 2 to 13. The main conclusions drawn from the sensitivity analysis are as follows:

1) The parameters \( p, d \) and \( c \) are very sensitive to \( PPC, \) the parameters \( k, N, T \) and \( c_o \) have medium degree sensitivity to \( PPC, \) the parameters \( \lambda, h, r, \) and \( \mu \) have low degree sensitivity to \( PPC. \)
2) When the values of \( p, d \) and \( N \) increase, \( PPC \) increases.
3) When the values of \( c, k, \) and \( c_o \) increase, \( PPC \) decreases.
4) When \( T \) increases, \( PPC \) does not increase because the replenishment cycle increases to counteract the profit effect.
5) When \( \lambda \) and \( r \) increases, \( PPC \) maintains nearly constant because the shortage is little.

**With coordination**

If the retailer and the supplier coordinate to determine their order quantity by sharing their production and demand information, that is, to determine \( n_j \) and \( \nu_j (n_j) \), then the unit time system profit is

\[
F(n_j, \ nu_j(n_j)) = F_R(n_j, \ nu_j(n_j)) + F_S(n_j, \ nu_j(n_j)).
\]

(30)

**Theorem 2**

\( F(n, \nu) \) is concave in \( V \).

**Proof (Appendix C)**

Applying the solution search procedure and Theorem 2, the optimal solution of \( n_j \) and \( \nu_j (n_j) \) can be derived. With
Table 1. Solution search results without coordination.

<table>
<thead>
<tr>
<th>n</th>
<th>$v_R(n)$</th>
<th>$F_R(n, v_R(n))$</th>
<th>$R(n, v) / nT$</th>
<th>$C(n, v) / nT$</th>
<th>$L(n, v) / nT$</th>
<th>$c_o / nT$</th>
<th>$B(n, v) / nT$</th>
<th>$H(n, v) / nT$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>56.16</td>
<td>240</td>
<td>99.84</td>
<td>0</td>
<td>80</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>83.482</td>
<td>228</td>
<td>98.642</td>
<td>0</td>
<td>40</td>
<td>2</td>
<td>3.876</td>
</tr>
<tr>
<td>3</td>
<td>10</td>
<td>84.74’</td>
<td>216</td>
<td>97.039</td>
<td>0</td>
<td>26.667</td>
<td>2</td>
<td>5.555</td>
</tr>
<tr>
<td>4</td>
<td>9.239</td>
<td>80.08</td>
<td>203.878</td>
<td>95.022</td>
<td>0.029</td>
<td>20</td>
<td>2</td>
<td>6.748</td>
</tr>
<tr>
<td>5</td>
<td>7.675</td>
<td>73.933</td>
<td>192.208</td>
<td>92.525</td>
<td>0.216</td>
<td>16</td>
<td>2</td>
<td>7.533</td>
</tr>
<tr>
<td>6</td>
<td>6.304</td>
<td>67.373</td>
<td>181.044</td>
<td>89.657</td>
<td>0.455</td>
<td>13.333</td>
<td>2</td>
<td>8.226</td>
</tr>
<tr>
<td>7</td>
<td>5.154</td>
<td>60.823</td>
<td>170.166</td>
<td>86.454</td>
<td>0.671</td>
<td>11.429</td>
<td>2</td>
<td>8.79</td>
</tr>
<tr>
<td>8</td>
<td>4.26</td>
<td>54.542</td>
<td>159.428</td>
<td>82.878</td>
<td>0.824</td>
<td>10</td>
<td>2</td>
<td>9.184</td>
</tr>
<tr>
<td>9</td>
<td>3.661</td>
<td>48.733</td>
<td>148.702</td>
<td>78.831</td>
<td>0.893</td>
<td>8.889</td>
<td>2</td>
<td>9.356</td>
</tr>
<tr>
<td>10</td>
<td>3.41</td>
<td>43.579</td>
<td>137.842</td>
<td>74.153</td>
<td>0.869</td>
<td>8</td>
<td>2</td>
<td>9.241</td>
</tr>
</tbody>
</table>

Table 2. Sensitivity analysis for sensitive parameter $k$.

<table>
<thead>
<tr>
<th>$k$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0056</td>
<td>3</td>
<td>10</td>
<td>88.233</td>
<td>4.1</td>
</tr>
<tr>
<td>0.0064</td>
<td>3</td>
<td>10</td>
<td>87.086</td>
<td>2.8</td>
</tr>
<tr>
<td>0.0072</td>
<td>3</td>
<td>10</td>
<td>85.922</td>
<td>1.4</td>
</tr>
<tr>
<td>(0.008)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>0.0088</td>
<td>3</td>
<td>10</td>
<td>83.54</td>
<td>-1.4</td>
</tr>
<tr>
<td>0.0096</td>
<td>3</td>
<td>10</td>
<td>82.322</td>
<td>-2.9</td>
</tr>
</tbody>
</table>

1. PPC denotes percent profit change. 2. The value in {} is the parameter of Example 1.

Table 3. Sensitivity analysis for sensitive parameter $N$.

<table>
<thead>
<tr>
<th>$N$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>2</td>
<td>10</td>
<td>80.792</td>
<td>-4.7</td>
</tr>
<tr>
<td>8</td>
<td>2</td>
<td>10</td>
<td>81.913</td>
<td>-3.3</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>10</td>
<td>83.448</td>
<td>-1.5</td>
</tr>
<tr>
<td>(10)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>10</td>
<td>85.797</td>
<td>1.2</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>10</td>
<td>86.678</td>
<td>2.3</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>10</td>
<td>87.423</td>
<td>3.2</td>
</tr>
</tbody>
</table>

Table 4. Sensitivity analysis for sensitive parameter $p$.

<table>
<thead>
<tr>
<th>$p$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>21</td>
<td>3</td>
<td>10</td>
<td>19.94</td>
<td>-76.5</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>10</td>
<td>41.54</td>
<td>-51</td>
</tr>
<tr>
<td>27</td>
<td>3</td>
<td>10</td>
<td>63.14</td>
<td>-25.5</td>
</tr>
<tr>
<td>(30)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>33</td>
<td>3</td>
<td>10</td>
<td>106.34</td>
<td>25.5</td>
</tr>
<tr>
<td>36</td>
<td>2</td>
<td>10</td>
<td>129.082</td>
<td>52.3</td>
</tr>
<tr>
<td>39</td>
<td>2</td>
<td>10</td>
<td>151.882</td>
<td>79.2</td>
</tr>
</tbody>
</table>
Table 5. Sensitivity analysis for sensitive parameter $d$.

<table>
<thead>
<tr>
<th>$d$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.6</td>
<td>3</td>
<td>10</td>
<td>50.718</td>
<td>-40.1</td>
</tr>
<tr>
<td>6.4</td>
<td>3</td>
<td>10</td>
<td>62.059</td>
<td>-26.8</td>
</tr>
<tr>
<td>7.2</td>
<td>3</td>
<td>10</td>
<td>73.399</td>
<td>-13.4</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>9.6</td>
<td>2</td>
<td>10</td>
<td>108.578</td>
<td>28.1</td>
</tr>
<tr>
<td>10.4</td>
<td>2</td>
<td>10</td>
<td>121.126</td>
<td>42.9</td>
</tr>
</tbody>
</table>

Table 6. Sensitivity analysis for sensitive parameter $T$.

<table>
<thead>
<tr>
<th>$T$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>3</td>
<td>8.293</td>
<td>78.249</td>
<td>-7.7</td>
</tr>
<tr>
<td>8</td>
<td>3</td>
<td>9.245</td>
<td>81.495</td>
<td>-3.8</td>
</tr>
<tr>
<td>9</td>
<td>3</td>
<td>10</td>
<td>83.604</td>
<td>-1.3</td>
</tr>
<tr>
<td>10</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>11</td>
<td>3</td>
<td>10</td>
<td>85.162</td>
<td>0.5</td>
</tr>
<tr>
<td>12</td>
<td>3</td>
<td>10</td>
<td>85.125</td>
<td>0.45</td>
</tr>
<tr>
<td>13</td>
<td>3</td>
<td>10</td>
<td>84.783</td>
<td>0.05</td>
</tr>
</tbody>
</table>

Table 7. Sensitivity analysis for sensitive parameter $\lambda$.

<table>
<thead>
<tr>
<th>$\lambda$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.49</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>0.56</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>0.63</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>0.7</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>0.77</td>
<td>3</td>
<td>9.654</td>
<td>84.769</td>
<td>0.03</td>
</tr>
<tr>
<td>0.84</td>
<td>3</td>
<td>8.618</td>
<td>85.243</td>
<td>0.6</td>
</tr>
<tr>
<td>0.91</td>
<td>3</td>
<td>7.783</td>
<td>86.167</td>
<td>1.7</td>
</tr>
</tbody>
</table>

Table 8. Sensitivity analysis for sensitive parameter $h$.

<table>
<thead>
<tr>
<th>$h=0.05$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.035</td>
<td>3</td>
<td>10</td>
<td>86.406</td>
<td>2</td>
</tr>
<tr>
<td>0.04</td>
<td>3</td>
<td>10</td>
<td>85.851</td>
<td>1.3</td>
</tr>
<tr>
<td>0.045</td>
<td>3</td>
<td>10</td>
<td>85.295</td>
<td>0.65</td>
</tr>
<tr>
<td>0.05</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>0.055</td>
<td>3</td>
<td>10</td>
<td>84.184</td>
<td>-0.66</td>
</tr>
<tr>
<td>0.06</td>
<td>3</td>
<td>10</td>
<td>83.629</td>
<td>-1.3</td>
</tr>
<tr>
<td>0.065</td>
<td>3</td>
<td>10</td>
<td>83.074</td>
<td>-2</td>
</tr>
</tbody>
</table>
### Table 9. Sensitivity analysis for sensitive parameter $c$.

<table>
<thead>
<tr>
<th>$c$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.4</td>
<td>3</td>
<td>10</td>
<td>113.852</td>
<td>34.4</td>
</tr>
<tr>
<td>9.6</td>
<td>3</td>
<td>10</td>
<td>104.148</td>
<td>22.9</td>
</tr>
<tr>
<td>10.8</td>
<td>3</td>
<td>10</td>
<td>94.444</td>
<td>11.5</td>
</tr>
<tr>
<td>(12)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>13.2</td>
<td>3</td>
<td>10</td>
<td>75.036</td>
<td>-11.5</td>
</tr>
<tr>
<td>14.4</td>
<td>3</td>
<td>10</td>
<td>65.332</td>
<td>-22.9</td>
</tr>
<tr>
<td>15.6</td>
<td>3</td>
<td>10</td>
<td>55.628</td>
<td>-34.4</td>
</tr>
</tbody>
</table>

### Table 10. Sensitivity analysis for sensitive parameter $r$.

<table>
<thead>
<tr>
<th>$r$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>4.5</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>(5)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>5.5</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>6</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
<tr>
<td>6.5</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>0</td>
</tr>
</tbody>
</table>

### Table 11. Sensitivity analysis for sensitive parameter $\mu$.

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>14</td>
<td>3</td>
<td>10</td>
<td>85.34</td>
<td>0.71</td>
</tr>
<tr>
<td>16</td>
<td>3</td>
<td>10</td>
<td>85.14</td>
<td>0.47</td>
</tr>
<tr>
<td>18</td>
<td>3</td>
<td>10</td>
<td>84.94</td>
<td>0.24</td>
</tr>
<tr>
<td>(20)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>22</td>
<td>3</td>
<td>10</td>
<td>84.54</td>
<td>-0.24</td>
</tr>
<tr>
<td>24</td>
<td>3</td>
<td>10</td>
<td>84.34</td>
<td>-0.47</td>
</tr>
<tr>
<td>26</td>
<td>3</td>
<td>10</td>
<td>84.14</td>
<td>-0.71</td>
</tr>
</tbody>
</table>

### Table 12. Sensitivity analysis for sensitive parameter $c_o$.

<table>
<thead>
<tr>
<th>$c_o$</th>
<th>$n_R^*$</th>
<th>$v_R^*$</th>
<th>$F_R^*$</th>
<th>PPC (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>560</td>
<td>2</td>
<td>10</td>
<td>95.482</td>
<td>12.7</td>
</tr>
<tr>
<td>640</td>
<td>2</td>
<td>10</td>
<td>91.482</td>
<td>8</td>
</tr>
<tr>
<td>720</td>
<td>2</td>
<td>10</td>
<td>87.482</td>
<td>3.2</td>
</tr>
<tr>
<td>(800)</td>
<td>3</td>
<td>10</td>
<td>84.74</td>
<td>--</td>
</tr>
<tr>
<td>880</td>
<td>3</td>
<td>10</td>
<td>82.073</td>
<td>-3.1</td>
</tr>
<tr>
<td>960</td>
<td>3</td>
<td>10</td>
<td>79.407</td>
<td>-6.3</td>
</tr>
<tr>
<td>1040</td>
<td>3</td>
<td>10</td>
<td>76.74</td>
<td>-9.4</td>
</tr>
</tbody>
</table>
If the joint optimal solution is $Q^+ \in \{15, 16\}$, Figures 2 and 3. The

\[ F(n_j, v_j(n_j)) = F_R(n_j, v_j(n_j)) + F_S(n_j, v_j(n_j)). \]  

It is obviously the optimal unit time system profit (33) is better than that of the unit time system profit (29) (Table 14). The coordination policy can be illustrated by the following Example 2.

**Example 2**

$N=10$, $p=30$, $d=1$, $T=100$, $h=0.005$, $c=10$, $c_m=5$, $r=25$, $s=150$, and $c_o=1200$. With the
deterioration rate $k=0.0025$, using the mathematical software MATHCAD and MAPLE 7, the optimal decision is obtained and the results are as follows (Table 14):

**Sensitivity analysis**

Sensitivity analysis with different deterioration rate $k$ is carried out in Tables 15 and 16, Figures 2 and 3. The main conclusions drawn are as follows:

1. Table 15 and Figure 2 show the changes in $n_R$, $n_V$, the deteriorated quantity per period $(I_1(0) - Q_1)/n_R$ and the ordering quantity $I_1(0) + Q_2$ for variable $k$. It is shown that as $k$ increases, the replenishment cycle $n_R$ decreases, but $(I_1(0) - Q_1)/n_R$ increases.

2. Table 16 and Figure 3 show the changes in the retailer's unit time profit $F_R$, the supplier's unit time profit $F_S$ and the unit time system profit $F_R + F_S$ for variable $k$. It is shown that as $k$ increases, $F_R$, $F_R + F_S$ decreases, but $F_S$ increases.

From the above-mentioned analysis, it can be shown that higher deterioration rate leads to lower system profit. However, if the retailer and the supplier coordinate to determine their order quantity, the system profit increases significantly. Even though the overall profit is better, the
The study develops a maximum profit model for coordinating the retailer and the supplier. The study focuses on how to determine the optimal ordering decision for fast deteriorating items with expiration date. The customers’ demand will decrease due to nearness to expiration date. Facing the deterioration and expiration date, how to decide an optimal order quantity is vital to retailer. The study develops a maximum profit model for coordinating the retailer and the supplier. The numerical examples show that the higher deterioration rate results in less the profit. However, the coordination policy between the retailer and the supplier will improve the efficiency of the ordering policy. When the deterioration rate increases, coordination should be considered since the percent profit increases significantly. However, in order to entice the retailer to cooperate, compensation mechanism must be incorporated.

**Conclusion**

This study focuses on how to determine the optimal ordering policy for fast deteriorating items with expiration date. The customers’ demand will decrease due to nearness to expiration date. Facing the deterioration and expiration date, how to decide an optimal order quantity is vital to retailer. The study develops a maximum profit model for coordinating the retailer and the supplier. The numerical examples show that the higher deterioration rate results in less the profit. However, the coordination policy between the retailer and the supplier will improve the efficiency of the ordering policy. When the deterioration rate increases, coordination should be considered since the percent profit increases significantly. However, in order to entice the retailer to cooperate, compensation mechanism must be incorporated (Zimmer, 2002). The results of this study give managerial insights to decision maker developing an optimal ordering decision for deteriorating product with expiration date.

**Acknowledgements**

The author would like to thank the editor and anonymous reviewers for their valuable comments to improve the quality of the paper.
Figure 2. Optimal ordering decisions with various deterioration rates for the retailer.

Figure 3. Percent profit increase due to coordination.

REFERENCES

Appendix A

\[ Q_1 = \sum_{j=1}^{n-1} \int_0^T d_j dt + \int_0^T d_n dt = \frac{d(n-1)(2N + 2 - n)T}{2N} + \frac{d(N + 1 - n)\nu}{N} \quad \text{(A-1)} \]

\[ Q_2 = \int_0^T d_1 \theta(T-t) dt = \frac{d}{T} \left( \frac{T^2}{2} - \nu^2 \right) \quad \text{(A-2)} \]

\[ C(n, \nu) = [I_1(0) + Q_2]c \]

\[ = \left( \frac{d}{k} \right) \{ w(1)(e^{kT} - 1) + \sum_{i=2}^{n-1} w(i)(e^{kT} - 1)e^{k[(i-1)T]} \]

\[ + w(n)(kv + k^2v^2/2)e^{k[(n-1)T]} \} + \frac{d}{T} \left( \frac{T^2}{2} - \nu^2 \right) \} c \]

\[ \approx \left[ \frac{d}{k} \left\{ \alpha_1 + \frac{N+1-n}{N} (kv + k^2v^2/2)e^{k[(n-1)T]} \right\} + \frac{d}{T} \left( \frac{T^2}{2} - \nu^2 \right) \} c \]

\[ , \quad 0 < \nu \leq T, n \geq 3, \]

where \( \alpha_1 = w(1)(e^{kT} - 1) + \sum_{i=2}^{n-1} w(i)(e^{kT} - 1)e^{k[(i-1)T]} \), and

\[ I_1(0) = \begin{cases} \\
\frac{d}{k} (kv + k^2v^2/2), & n = 1.
\end{cases} \quad \text{(A-4)} \]

\[ L(n, \nu) = r \int_0^T d_1 [1 - \theta(T-t)] dt = rd \left( \frac{T}{2} + \frac{\nu^2}{2T} - \nu \right) \quad \text{(A-5)} \]

\[ H_j^T = h \int_0^T I_j(t) dt \]

\[ = \frac{dh}{k} \left\{ \frac{N-j+1}{N} \left( e^{kT} - 1 \right) k - T \right\} + \frac{e^{kT} - 1}{N} \frac{1}{k} \left[ (N-j+1) e^{kT} \frac{1 - e^{(n-j)kT}}{1 - e^{kT}} \right] \]

\[ - \frac{e^{kT} - (n-j)e^{(n-j)kT} + (n-j-1)e^{(n-j+1)kT}}{\left(1-e^{kT}\right)^2} + \frac{N-n+1}{N} (e^{kv} - 1)e^{k(n-j)T} \frac{1 - e^{-kT}}{k} \}

\[ , \quad j = 1, 2, \ldots, n-2. \]
\[
\sum_{j=1}^{n-2} H_j^T = \frac{dh}{k} \left\{ \left( e^{kT} - \frac{1}{k} - T \right) \frac{(n-2)(2N+3-n)}{2N} + \frac{e^{kT} - 1 - e^{-kT}}{N} \right\} \\
\times \left[ 1 - \frac{e^{kT}}{1-e^{kT}} \right] \left( N - 1 - e^{(n-2)kT} \right) \frac{(N+1)(e^{(n-1)kT}) - 1}{\left( 1 - e^{-kT} \right)^2} \\
- \frac{(n-2)e^{kT}}{(1-e^{kT})^2} + \frac{1}{1-e^{kT}} \left[ n(e^{(n-1)kT} - e^{kT}) - e^{(n-1)kT} - (n-1) + (n-2) \right] \frac{(1 - e^{-kT})^2}{(1-e^{kT})^2} \\
- \frac{1}{1-e^{kT}} \left[ (n-1)(e^{nkT} - e^{2kT}) - (n-1)e^{2kT} + (n-2)e^{kT} \right] + \frac{N-n+1}{N} \left( e^{nkT} - 1 \right) \frac{e^{(n-1)kT} - e^{kT}}{k} \\
\approx \frac{dh}{k} \left\{ \alpha_2 + \frac{N-n+1}{N} \left( kV + k^2V^2 \right) \frac{e^{(n-1)kT} - e^{kT}}{k} \right\},
\]

where

\[
\alpha_2 = \left[ \frac{e^{kT} - 1}{k} - T \right] \frac{(n-2)(2N+3-n)}{2N} + \frac{e^{kT} - 1 - e^{-kT}}{N} \frac{1}{k}.
\]

\[
H_{n-1}^T = h \int_0^T I_{n-1}(t) dt \\
= \frac{dh}{k} \left\{ \frac{N-n+2}{N} \left( \frac{e^{kT} - 1}{k} - T \right) + \frac{N-n+1}{N} \left( e^{kV} - 1 \right) \frac{e^{kT} - 1}{k} \right\} \\
\approx \frac{dh}{k} \left\{ \frac{N-n+2}{N} \left( \frac{e^{kT} - 1}{k} - T \right) + \frac{N-n+1}{N} \left( kV + k^2V^2 \right) \frac{e^{kT} - 1}{k} \right\}.
\]

(A-7)
\[ H_n^\nu = h \int_0^\nu I_n(t) dt = \frac{dh}{k} \frac{N - n + 1}{N} \left( e^{k\nu} - 1 \right) e^{\frac{1}{k}}. \]

\approx \frac{dh}{k} \frac{N - n + 1}{N} \left( k\frac{v^2}{2} \right). \tag{A-9}

\[ H(1,\nu) = \frac{dh}{k} \left( e^{k\nu} - 1 \right). \]

\[ H(2,\nu) = \frac{dh}{k} \left( e^{kT} - 1 \right) \left( -T + \frac{N-1}{N} \left( e^{k\nu} - 1 \right) e^{\frac{kT}{k}} \right) + \frac{dh}{k} \frac{N-1}{N} \left( e^{k\nu} - 1 \right) \]

\approx \frac{dh}{k} \left( e^{kT} - 1 \right) \left( -T + \frac{N-1}{N} \left( k\nu + k\frac{v^2}{2} \right) e^{\frac{kT}{k}} \right) + \frac{dh}{k} \frac{N-1}{N} \left( k\frac{v^2}{2} \right). \tag{A-11}

Appendix B: Proof of Theorem 1.

\[ \frac{\partial^2}{\partial \nu^2} F_R(n,\nu) = \frac{1}{nT} \left[ \frac{d(c-\lambda p-r)}{T} - (cd + \frac{hd}{k}) \frac{N+1-n}{N} e^{(n-1)kT} k \right]. \tag{B-1} \]

Since \( c < \lambda p \), one has \( c - \lambda p - r < 0 \), and \( (cd + \frac{hd}{k}) \frac{N+1-n}{N} e^{(n-1)kT} k > 0 \), which leads to \( \frac{\partial^2}{\partial \nu^2} F_R(n,\nu) / \frac{\partial \nu^2} < 0 \), this completes the proof.

Appendix C: Proof of Theorem 2.

\[ F(n,\nu) = \frac{1}{nT} \left[ R(n,\nu) - (I(0) + Q_s)c_m - L(n,\nu) - B(n,\nu) - H(n,\nu) - c_o - s \right], \quad 0 < \nu \leq T, n \leq N. \tag{C-1} \]

\[ \frac{\partial^2}{\partial \nu^2} F(n,\nu) = \frac{1}{nT} \left[ \frac{d(c_m - \lambda p - r)}{T} - (c_md + \frac{hd}{k}) \frac{N+1-n}{N} e^{(n-1)kT} k \right]. \tag{C-2} \]

Note \( c_m < c < \lambda p \), one has \( c_m - \lambda p - r < 0 \), and \( (c_md + \frac{hd}{k}) \frac{N+1-n}{N} e^{(n-1)kT} k > 0 \), which leads to \( \frac{\partial^2}{\partial \nu^2} F(n,\nu) / \frac{\partial \nu^2} < 0 \), this completes the proof.