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Full Length Research Paper

Optimal ordering policy for fast deteriorating items

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Facing the demand of periodic pattern and expiration date, how to make an optimal ordering policy by the retailer of fast deteriorating items is the key problem nowadays. In this study, we propose a model for fast deteriorating items with periodic pattern demand and expiration date. An algorithm is presented to derive an optimal replenishment cycle, shortage period and order quantity such that the unit time profit is maximized. The coordination policy between the retailer and the supplier improves the efficiency of the ordering policy especially when the deterioration rate is high. Numerical examples and sensitivity analysis are provided to illustrate the theory.

Key words: Fast deteriorating items, expiration date, coordination, periodic pattern demand.

INTRODUCTION

This study is motivated by a real life problem faced by a florist. Most florists usually make frequent replenishments due to fast deterioration of fresh flowers. The high deteriorated cost and the frequent ordering cost have affected the flower retailer's benefit. The problem facing a florist is how to develop an ordering policy that maximizes the profit. In general, profit is a function of the sales revenue, the purchasing cost, the lost sale cost, the processing cost, the inventory holding cost, the ordering cost, the production cost, and the shipment cost.

Flower, fruit, and seafood are common fast deteriorating items. These products will deteriorate expeditiously with time resulting in fast decreasing utility or price from the original one. The customer demand follows a periodic pattern that repeat itself after a short time interval. Moreover, the customer demand declines when the product is close to its expiration date. The product expiration date indicates the latest time that the product may be used (not the end of the product life cycle time). The loss of profit is caused by deterioration and declining demand. To improve the supply chain efficiency, the coordination between the supplier and the retailer must be considered.

Deteriorating inventory was originally studied by Ghare and Schrader (1963). Since then, it has received much attention from researchers (Wee, 1995; Rau et al., 2003; Yang, 2004; Hsieh and Lee, 2005; Dye et al., 2007; You, 2005; He et al., 2010). Generally, two situations of deteriorating rate are discussed. One is constant (Shah and Jaiswal, 1977; Aggarwal, 1978; Padmanabhana and Vratb, 1995; Bhunia and Maiti, 1999), and the other is not constant [a: Linear increasing function of time (Bhunia and Maiti, 1998; Mukhopadhyay et al., 2004); b: Weibull distributed (Wee, 1999; Mahapatra, 2005; Chakrabarty et al., 1998); c: Other function of time (Abad, 2001)]. Ho et al. (2007) considered the effects of deteriorating inventory on lot-sizing in material requirements planning systems. They presented the effect on the relevant cost due to various deterioration rates. Hsu et al. (2007) addressed a deteriorating inventory replenishment model with expiration date and uncertain lead time. Goyal and Gupta (1989), Weng (1995), Fites (1996), Zimmer (2002), Sucky (2005), considered the coordination between the suppliers and the retailers in order to improve the performance of the supply chains. Hsu et al. (2010) proposed to invest on preservation technology to decrease the deterioration rate of items. However, researches on the influence of expiration date and products with fast deterioration rate have received little attention.

In this study, the customer demand periodic pattern was assumed. The retailer obtains the product from the supplier for sale to the customers. The constant deteriorating rate product has an expiration date. An algorithm with coordination policy is developed to determine the replenishment and backordering decision of the deteriorating items with expiration date and periodic pattern demand.

ASSUMPTIONS AND NOTATION

The following notation is used throughout this paper.
The general parameters are:

T *Length* of a periodic interval
N *Discrete* number; *NT* denotes the expiration date of product
K Constant deterioration rate of on-hand-stock, $0 \leq k < 1$
$\theta(\eta)$ The fraction that customers are willing to purchase the item under the condition that they receive their order after η units of time

The decision variables are:

n) Discrete number; decision variable, *nT* denotes the replenishment cycle, $n \leq N$
ν) Critical time at which inventory level reaches zero, decision variable

The parameters related to the retailer are:

n_R*) Discrete number; n_R^*T denotes the retailer's optimal replenishment cycle
v_R*) Retailer's optimal critical time
p) Retailer's selling price per unit
p_b) Retailer's selling price per unit when shortages occur
h) Unit inventory holding cost per unit time
c) Retailer's wholesale purchase price per unit
c_o) Retailer's ordering cost per replenishment cycle
r) Retailer's penalty cost per unit of a lost sale including loss of profit
μ) Retailer's processing cost including making an inventory and deteriorated items per period
Q) Retailer's order quantity each replenishment
Q_1) Retailer's sales amount without backordering over replenishment cycle
Q_2) Retailer's backordered quantity at the end of replenishment cycle
F_R) Unit time profit for the retailer

The parameters related to the supplier are:

c_m) Supplier's production cost per unit, $c_m < c$
s) Supplier's shipment cost per replenishment
F_S) Unit time profit for the supplier

The other related parameters are as follows:

n_J*) Discrete number; n_J*T denotes the supplier-retailer joint optimal replenishment cycle
v_J*) Supplier-retailer joint optimal critical time
$I_1(0)$ Maximum inventory level at the start of a cycle
F) Unit time system profit, that is, the supplier-retailer joint total profit; $F=F_R+F_S$

In developing the model, the following assumptions are made:

(i) The retailer's selling price per unit *p* and backorder price p_b are predetermined such that: $p_b=\lambda p > c$, where $0<\lambda<1$.
(ii) Demand for the product is influenced by periods. That is, d_j is the demand rate at the *j*-th period such that
$$d_j = dw(j),\ j = 1,2,...N,$$
where $w(j) = \frac{N-j+1}{N}$ is a conserved function, and the value of *d* is a known constant with $d>0$, which denotes the demand rate of the first period. This means that the customers' demand is less when it is nearer to the product expiration date. Note: The demand at the *n*-th period includes two parts, that is, $[0,\nu]$ and $[\nu,T]$. Since the retailer is willing to wait for backorders of new items during stockout, the demand during $[\nu,T]$ is based on d_1, on the other hand, the demand rate during $[0,\nu]$ is d_n.
(iii) Demand during the stock out period is partially lost due to impatient customers.
(iv) Backlogged demand is satisfied at the beginning of each replenishment.
(v) Shortage time is less than the length of a periodic interval *T*.
(vi) The fraction of customers' backordered is assumed to be linearly decreasing with waiting time η and is assumed to be
$\theta(\eta)=1-\eta/T$, $0 \leq \eta < T$.
(vii) The capacity of the warehouse is unlimited.
(viii) There is no replacement or repair of deteriorated items during a given cycle.

MODELING AND ANALYSIS

In this section, a supply chain with the retailer and the supplier is assumed. The retailer obtains the products from the supplier for sale to the customers. The study consider the products of fast deteriorating items with constant deterioration rate *k* and expiration date *NT*. (For example, the retailer places an order of some flowers in bud, the flowers will deteriorate till fade for 7 days (*N*=7, *T*=1 days).) The customers' periodic pattern demand is d_j, *j*=1, 2, 3…*N*. Which means the demand is decreasing due to fast deterioration. Shortage backorder is allowed. Backlogged demand is satisfied at the beginning of each replenishment. Placing an optimal order before the selling period of the product is vital to the retailer. The aim of this

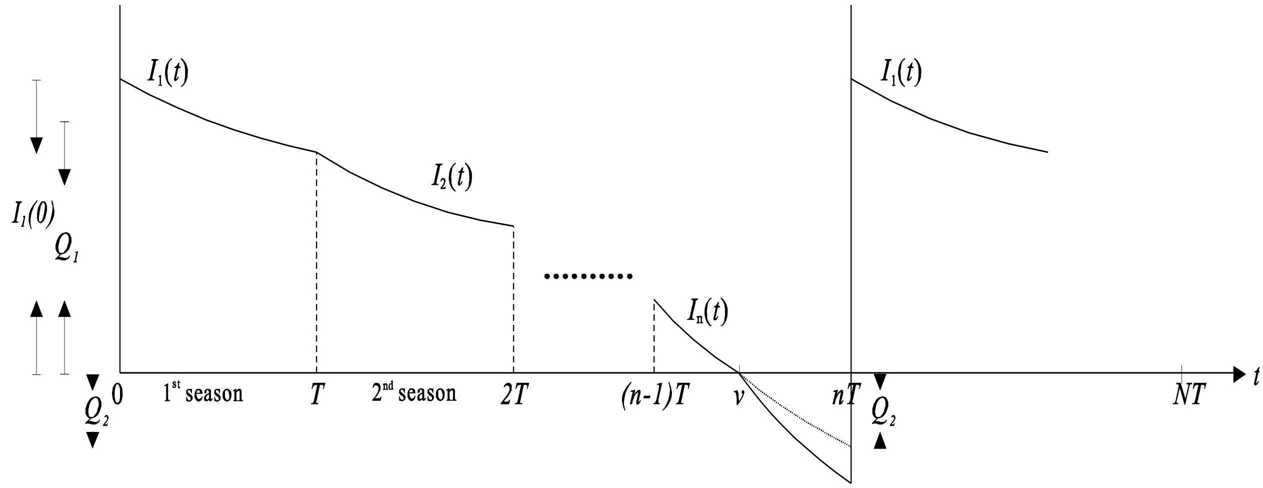

Figure 1. Retailer's inventory system (Hsu et al., 2007).

study is to maximize the unit time profit by determining (1) the retailer's replenishment cycle (2) the duration of the shortages, and (3) the retailer's order quantity *Q*. Two polices are developed to illustrate our study: (i) Without coordination (ii) With coordination.

The inventory system of the retailer during a given cycle is depicted in Figure 1. Suppose the retailer's replenishment cycle is set at *nT*. The study derives the model by backward deduction. Let $I_n(t), n \le N$, be the inventory level during the *n*th period. The differential equation governing the transition of the system during the period interval is;

$$\frac{dI_n(t)}{dt} = -d_1, \quad \nu \le t \le T .$$

(In the *n*th period, since the customers who are willing to backorder will need new items, therefore, the demand rate in the *n*th period divides into two parts, that is, d_n in $0 \le t \le \nu$, and d_1 in $\nu \le t \le T$.)

$$\frac{dI_n(t)}{dt} = -kI_n(t) - dw(n), \quad 0 \le t \le \nu . \qquad (1)$$

with initial condition $I_n(\nu) = 0$. From (1), one has

$$I_n(t) = \frac{dw(n)}{k}\left[e^{k(\nu - t)} - 1\right], \quad 0 \le t \le \nu . \qquad (2)$$

Let $I_{n-1}(t)$ be the inventory level during the (*n*-1)th period, then

$$\frac{dI_{n-1}(t)}{dt} = -kI_{n-1}(t) - dw(n-1), \qquad (3)$$

with initial condition $I_{n-1}(T) = I_n(0)$. From (3), one has

$$I_{n-1}(t) = \frac{d}{k}\left\{w(n-1)[e^{k(T-t)} - 1] + w(n)[e^{k\nu} - 1]e^{k(T-t)}\right\}, \quad 0 \le t \le T . \qquad (4)$$

Similarly, the inventory level during the *j*th period is

$$I_j(t) = \frac{d}{k}\{w(j)[e^{k(T-t)} - 1] + \sum_{i=j+1}^{n-1} w(i)[e^{kT} - 1]e^{k[(i-j)T-t]} + w(n)(e^{k\nu} - 1)e^{k[(n-j)T-t]}\}, \quad 0 \le t \le T, \; j = 1, 2, \ldots, n-1. \qquad (5)$$

Next, the study deduces the retailer's and the supplier's objective functions. The objective functions include:

The sales revenues $R(n,\nu)$,
The purchasing cost $C(n,\nu)$,
The lost sale cost $L(n,\nu)$,
The processing cost $B(n,\nu)$ (including inventory and deteriorated items),
The inventory holding cost $H(n,\nu)$, and
The ordering cost, c_o.
The replenishment cycle and shortage length are set at *nT* and *T*-*v* units of time, respectively. Then
Retailer's unit time profit

$= \frac{1}{nT}$ [sales revenue-purchasing cost-lost sale cost -processing cost-ordering cost -inventory holding cost]

$$= \frac{1}{nT}\left[R(n,\nu) - C(n,\nu) - L(n,\nu) - B(n,\nu) - c_o - H(n,\nu)\right], \quad 0 < \nu \le T, n \le N. \tag{6}$$

Supplier's unit time profit

$= \frac{1}{nT}$ [sales revenue- production cost-shipment cost]

$$= \frac{1}{nT}\{[I_1(0) + Q_2]c - [I_1(0) + Q_2]c_m - s\}, \quad 0 < \nu \le T, n \le N. \tag{7}$$

Where

$$R(n,\nu) = pQ_1 + \lambda p Q_2. \tag{8}$$

$$Q_1 = \begin{cases} \sum_{j=1}^{n-1}\int_0^T d_j dt + \int_0^{\nu} d_n dt = \frac{d(n-1)(2N+2-n)T}{2N} + \frac{d(N+1-n)\nu}{N}, n \ge 2, \\ \int_0^{\nu} d_1 dt = d\nu, \quad n = 1. \end{cases} \tag{9}$$

Since the retailer is willing to wait for backorders of new items during stockout, the demand is assumed to be d_1.

$$Q_2 = \int_{\nu}^{T} d_1\theta(T-t)dt = \frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right). \tag{10}$$

The order quantity at each replenishment is $Q = I_1(0) + Q_2$. One has

$$C(n,\nu) = [I_1(0) + Q_2]c. \tag{11}$$

The lost sale amount is $\int_{\nu}^{T} d_1[1 - \theta(T-t)]dt$, one has

$$L(n,\nu) = r\int_{\nu}^{T} d_1[1 - \theta(T-t)]dt. \tag{12}$$

And

$$B(n,\nu) = n\mu. \tag{13}$$

$$H(n,\nu) = \sum_{j=1}^{n-2} H_j^T + H_{n-1}^T + H_n^{\nu}, \quad n \ge 3. \tag{14}$$

$$H(1,\nu) = \frac{dh}{k}\left(\frac{e^{k\nu} - 1}{k} - \nu\right). \tag{15}$$

$$H(2,\nu) = \frac{dh}{k}\left(\frac{e^{kT} - 1}{k} - T + \frac{N-1}{N}\left(e^{k\nu} - 1\right)\frac{e^{kT} - 1}{k}\right) + \frac{dh}{k}\frac{N-1}{N}\left(\frac{e^{k\nu} - 1}{k} - \nu\right) \tag{16}$$

where (Appendix A)

$$H_j^T = h\int_0^T I_j(t)dt. \tag{17}$$

$$H_{n-1}^T = h\int_0^T I_{n-1}(t)dt. \tag{18}$$

$$H_n^{\nu} = h\int_0^{\nu} I_n(t)dt. \tag{19}$$

For $k\nu << 1$, $e^{k\nu}$ is replaced by $1 + k\nu + {k^2\nu^2}/{2}$ (Taylor series approximation), One has

$$F_R(1,\nu) = \frac{1}{nT}\Big\{pd\nu + \lambda p\left(\frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right)\right) - c\left[\frac{d}{k}\left(k\nu + \frac{k^2\nu^2}{2}\right) + \frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right)\right]$$

$$- rd\left(\frac{T}{2} + \frac{\nu^2}{2T} - \nu\right) - \mu - c_o - \frac{hd}{k}\left(\frac{k\nu^2}{2}\right)\Big\}, \quad 0 < \nu \le T. \tag{20}$$

$$F_R(2,\nu) = \frac{1}{nT}\Big\{p\left(dT + \frac{d(N-1)\nu}{N}\right) + \lambda p\left(\frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right)\right) - c\left[\frac{d}{k}\{ w(1)(e^{kT} - 1)\right.$$

$$\left. + \frac{N-1}{N}\left(k\nu + \frac{k^2\nu^2}{2}\right)e^{kT}\} + \frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right)\right] - rd\left(\frac{T}{2} + \frac{\nu^2}{2T} - \nu\right) - 2\mu - c_o$$

$$- \frac{hd}{k}\left(\frac{e^{kT} - 1}{k} - T + \frac{N-1}{N}\left(k\nu + \frac{k^2\nu^2}{2}\right)\frac{e^{kT} - 1}{k}\right) + \frac{dh}{k}\frac{N-1}{N}\left(\frac{k\nu^2}{2}\right)\Big\}, \quad 0 < \nu \le T. \tag{21}$$

$$F_R(n,\nu) = \frac{1}{nT}\left\{ p(\frac{d(n-1)(2N+2-n)T}{2N} + \frac{d(N+1-n)\nu}{N}) + \lambda p(\frac{d}{T}(\frac{T^2}{2} - \frac{\nu^2}{2}))\right.$$

$$-c[\frac{d}{k}\{ \alpha_1 + \frac{N+1-n}{N}(k\nu + \frac{k^2\nu^2}{2})e^{k[(n-1)T]}\} + \frac{d}{T}(\frac{T^2}{2} - \frac{\nu^2}{2})]$$

$$- rd(\frac{T}{2} + \frac{\nu^2}{2T} - \nu) - n\mu - c_o - \frac{hd}{k}\{\alpha_2 + \frac{N-n+1}{N}\left(k\nu + \frac{k^2\nu^2}{2}\right)\frac{e^{(n-1)kT} - e^{kT}}{k}$$

$$\left. + \frac{N-n+2}{N}(\frac{e^{kT}-1}{k} - T) + \frac{N-n+1}{N}\left(k\nu + \frac{k^2\nu^2}{2}\right)\frac{e^{kT}-1}{k} + \frac{N-n+1}{N}(\frac{k\nu^2}{2})\}\right\}, \quad 0 < \nu \le T, n \ge 3.$$

Where

$\alpha_1 = w(1)(e^{kT} - 1) + \sum_{i=2}^{n-1} w(i)(e^{kT} - 1)e^{k[(i-1)T]}$, and

$$\alpha_2 = \left[\frac{e^{kT}-1}{k} - T\right]\frac{(n-2)(2N+3-n)}{2N} + \frac{e^{kT}-1}{N}\frac{1-e^{-kT}}{k}. \quad (22)$$

$$F_S(1,\nu) = \frac{1}{nT}\{\frac{d}{k}[(k\nu + \frac{k^2\nu^2}{2}) + \frac{d}{T}(\frac{T^2}{2} - \frac{\nu^2}{2})](c-c_m)-s\}. \quad (23)$$

$$F_S(2,\nu) = \frac{1}{nT}\{\frac{d}{k}[(e^{kT}-1) + \frac{N-1}{N}(k\nu + \frac{k^2\nu^2}{2})e^{kT} + \frac{d}{T}(\frac{T^2}{2} - \frac{\nu^2}{2})](c-c_m)-s\}. \quad (24)$$

$$F_S(n,\nu) = \frac{1}{nT}\{\frac{d}{k}[\alpha_1 + \frac{N+1-n}{N}(k\nu + \frac{k^2\nu^2}{2})e^{k[(n-1)T]}](c-c_m)-s\}, \quad 0 < \nu \le T, n \ge 3. \quad (25)$$

Without coordination

When the retailer determines the order quantity independently, the retailer's optimization can be formulated as:

Max: $F_R(n,\nu)$

Subject to: $1 \le n \le N, \quad 0 < \nu \le T$. (26)

From Equations 20 to 22, the retailer's unit time profit $F_R(n, \nu)$ is a function of two variables n and ν, where ν is a real number and n is a discrete variable.

Theorem 1

$F_R(n,\nu)$ is concave in ν.

Proof (Appendix B)

From Theorem 1, for given n, we can derive the optimal $\nu_R^*(n)$ by solving the equation, $\partial F_R(n,\nu)/\partial\nu = 0$.

$$\partial F_R(n,\nu)/\partial\nu = \frac{1}{nT}[\frac{d}{T}(c - \lambda p - r)\nu - (cd\frac{N-n+1}{N}e^{(n-1)kT} + \frac{dh}{k}\frac{N-n+1}{N}e^{(n-1)kT})(1+k\nu) + pd\frac{N-n+1}{N} + dr + \frac{dh}{k}\frac{N-n+1}{N}]. \quad (27)$$

Equating Equation 27 with respect to ν to zero, one can derive the retailer's critical time

$$v_R(n) = \frac{\frac{N-n+1}{N}[pd + \frac{dh}{k} - (cd + \frac{dh}{k})e^{(n-1)kT}] + dr}{\frac{N-n+1}{N}(cd + \frac{dh}{k})ke^{(n-1)kT} - \frac{d(c - \lambda p - r)}{T}} \quad (28)$$

Let $v_R^*(n) = \min\{v_R(n), T\}$. Since the integer variable n cannot be found by an analytic method, the following solution search procedure is used.

Solution search procedure

Step 1. Set n =1, n_R^* = 0, v_R^*= 0 and F_R^*= 0.
Step 2. While $n \le N$ do Step 3-5.
Step 3. Solve $v_R(n)$ and $v_R^*(n)$ using (28).
Step 4. Calculate $F_R(n, v_R^*(n))$ using (20)-(22).
Step 5. If $F_R(n, v_R^*(n)) > F_R^*$, let $F_R^*=F_R(n, v_R^*(n))$, $n_R^*= n$ and $v_R^*= v_R^*(n)$.
Step 6. Stop.

From the solution search procedure, if the retailer's optimal solution is $(n_R^*, v_R^*(n_R))$, then the retailer's optimal unit time profit is $F_R(n_R^*, v_R^*(n_R))$, the supplier's unit time profit is $F_S(n_R^*,v_R^*(n_R))$, and the unit time system profit is

$$F(n_R^*, v_R^*(n_R)) = F_R(n_R^*, v_R^*(n_R))+F_S(n_R^*, v_R^*(n_R)). \quad (29)$$

Example 1

The preceding theory can be illustrated by the following numerical example with the following parameters:

Length of a periodic interval, T=10 h

Expiration date, N=10, NT=100 h

Retailer's selling price per unit, p=$30

Backorder price, $p_b=\lambda p$=21, where λ=0.7

Retailer's unit inventory holding cost per unit time, h=$0.05

Retailer's wholesale purchase price per unit, c=$12

Retailer's penalty cost per unit of a lost sale including loss of profit, r=$5

Processing cost per period, μ=$20

Retailer's ordering cost per replenishment cycle, c_o=$800

Demand rate, $d_j = dw(j), j = 1,2,...10$, where $w(j) = (11 - j)/10$, d=8.

With the deterioration rate k=0.008, using the mathematical software MATHCAD and MAPLE 7, the optimal decision is obtained and the results are as follows: (Table 1) n_R^*=3, v_R^*=10, the optimal replenishment cycle is n_R^*T=30, the length of shortage is 0, the optimal order quantity $Q=I_1(0)+Q_2$=242.597, the sales amount Q_1+Q_2=216, the deteriorated quantity per period is 26.597, the optimal unit profit of retailer is F_R^*=$84.74, the unit profit of supplier is F_S=$51.606, and the unit time system profit is $F_R^*+F_S$=$136.346.

Sensitivity analysis

Sensitivity analysis is carried out when a parameter of the fixed set of parameter values Φ={k, N, p, d, T, λ, h, c, r, μ, and c_o} changes 10, 20 and 30%. The results are shown in Tables 2 to 13. The main conclusions drawn from the sensitivity analysis are as follows:

1) The parameters p, d and c are very sensitive to *PPC*, the parameters k, N, T and c_o have medium degree sensitivity to *PPC*, the parameters λ, h, r, and μ have low degree sensitivity to *PPC*.
2) When the values of p, d and N increase, *PPC* increases.
3) When the values of c, k, and c_o increase, *PPC* decreases.
4) When T increases, *PPC* does not increase because the replenishment cycle increases to counteract the profit effect.
5) When λ and r increases, *PPC* maintains nearly constant because the shortage is little.

With coordination

If the retailer and the supplier coordinate to determine their order quantity by sharing their production and demand information, that is, to determine n_J and $v_J(n_J)$, then the unit time system profit is

$$F(n_J, v_J(n_J)) = F_R(n_J, v_J(n_J))+F_S(n_J, v_J(n_J)). \quad (30)$$

Theorem 2

$F(n,v)$ is concave in v .

Proof (Appendix C)

Applying the solution search procedure and Theorem 2, the optimal solution of n_J^* and $v_J^*(n_J)$ can be derived. With

Table 1. Solution search results without coordination.

k=0.008, N=10, p=30, d=8, T=10, λ=0.7, h=0.05, c=12, r=5, μ=20, c_o=800.								
n	$v_R^*(n)$	$F_R(n,v_R^*(n))$	$R(n,v)/nT$	$C(n,v)/nT$	$L(n,v)/nT$	c_o/nT	$B(n,v)/nT$	$H(n,v)/nT$
1	10	56.16	240	99.84	0	80	2	2
2	10	83.482	228	98.642	0	40	2	3.876
3*	10*	84.74*	216	97.039	0	26.667	2	5.555
4	9.239	80.08	203.878	95.022	0.029	20	2	6.748
5	7.675	73.933	192.208	92.525	0.216	16	2	7.533
6	6.304	67.373	181.044	89.657	0.455	13.333	2	8.226
7	5.154	60.823	170.166	86.454	0.671	11.429	2	8.79
8	4.26	54.542	159.428	82.878	0.824	10	2	9.184
9	3.661	48.733	148.702	78.831	0.893	8.889	2	9.356
10	3.41	43.579	137.842	74.153	0.869	8	2	9.241

Table 2. Sensitivity analysis for sensitive parameter *k.*

k	n_R^*	v_R^*	F_R^*	*PPC* (%)
0.0056	3	10	88.233	4.1
0.0064	3	10	87.086	2.8
0.0072	3	10	85.922	1.4
{0.008}	3	10	84.74	--
0.0088	3	10	83.54	-1.4
0.0096	3	10	82.322	-2.9

1. *PPC* denotes percent profit change. 2. The value in {} is the parameter of Example 1.

Table 3. Sensitivity analysis for sensitive parameter *N.*

N	n_R^*	v_R^*	F_R^*	*PPC* (%)
7	2	10	80.792	-4.7
8	2	10	81.913	-3.3
9	3	10	83.448	-1.5
{10}	3	10	84.74	--
11	3	10	85.797	1.2
12	3	10	86.678	2.3
13	3	10	87.423	3.2

Table 4. Sensitivity analysis for sensitive parameter *p.*

p	n_R^*	v_R^*	F_R^*	*PPC* (%)
21	3	10	19.94	-76.5
24	3	10	41.54	-51
27	3	10	63.14	-25.5
{30}	3	10	84.74	--
33	3	10	106.34	25.5
36	2	10	129.082	52.3
39	2	10	151.882	79.2

Table 5. Sensitivity analysis for sensitive parameter *d.*

d	n_R^*	v_R^*	F_R^*	***PPC (%)***
5.6	3	10	50.718	-40.1
6.4	3	10	62.059	-26.8
7.2	3	10	73.399	-13.4
{8}	3	10	84.74	--
8.8	3	10	96.081	13.4
9.6	2	10	108.578	28.1
10.4	2	10	121.126	42.9

Table 6. Sensitivity analysis for sensitive parameter *T.*

T	n_R^*	v_R^*	F_R^*	***PPC (***%)
7	3	8.293	78.249	-7.7
8	3	9.245	81.495	-3.8
9	3	10	83.604	-1.3
{10}	3	10	84.74	--
11	3	10	85.162	0.5
12	3	10	85.125	0.45
13	3	10	84.783	0.05

Table 7. Sensitivity analysis for sensitive parameter *λ*.

λ	n_R^*	v_R^*	F_R^*	***PPC (%)***
0.49	3	10	84.74	0
0.56	3	10	84.74	0
0.63	3	10	84.74	0
{0.7}	3	10	84.74	--
0.77	3	9.654	84.769	0.03
0.84	3	8.618	85.243	0.6
0.91	3	7.783	86.167	1.7

Table 8. Sensitivity analysis for sensitive parameter *h.*

***h*=0.05**	n_R^*	v_R^*	F_R^*	***PPC (***%)
0.035	3	10	86.406	2
0.04	3	10	85.851	1.3
0.045	3	10	85.295	0.65
{0.05}	3	10	84.74	--
0.055	3	10	84.184	-0.66
0.06	3	10	83.629	-1.3
0.065	3	10	83.074	-2

Table 9. Sensitivity analysis for sensitive parameter *c*.

c	n_R^*	v_R^*	F_R^*	*PPC* (%)
8.4	3	10	113.852	34.4
9.6	3	10	104.148	22.9
10.8	3	10	94.444	11.5
{12}	3	10	84.74	--
13.2	3	10	75.036	-11.5
14.4	3	10	65.332	-22.9
15.6	3	10	55.628	-34.4

Table 10. Sensitivity analysis for sensitive parameter *r*.

r	n_R^*	v_R^*	F_R^*	*PPC* (%)
3.5	3	10	84.74	0
4	3	10	84.74	0
4.5	3	10	84.74	0
{5}	3	10	84.74	--
5.5	3	10	84.74	0
6	3	10	84.74	0
6.5	3	10	84.74	0

Table 11. Sensitivity analysis for sensitive parameter *μ*.

μ	n_R^*	v_R^*	F_R^*	*PPC* (%)
14	3	10	85.34	0.71
16	3	10	85.14	0.47
18	3	10	84.94	0.24
{20}	3	10	84.74	--
22	3	10	84.54	-0.24
24	3	10	84.34	-0.47
26	3	10	84.14	-0.71

Table 12. Sensitivity analysis for sensitive parameter c_o.

c_o	n_R^*	v_R^*	F_R^*	*PPC* (%)
560	2	10	95.482	12.7
640	2	10	91.482	8
720	2	10	87.482	3.2
{800}	3	10	84.74	--
880	3	10	82.073	-3.1
960	3	10	79.407	-6.3
1040	3	10	76.74	-9.4

Table 13. Sensitivity analysis of PPI when parameter changes.

Parameter	-30%	-20%	-10%	+10%	+20%	+30%	Degree of sensitivity
k	4.1	2.8	1.4	-1.4	-2.9	-4.3	Medium
N	-4.7	-3.3	-1.5	1.2	2.3	3.2	Medium
p	-76.5	-51	-25.5	25.5	52.3	79.2	High
d	-40.1	-26.8	-13.4	13.4	28.1	42.9	High
T	-7.7	-3.8	-1.3	0.5	0.45	0.05	Medium
λ	0	0	0	0.03	0.6	1.7	Low
h	2	1.3	0.65	-0.66	-1.3	-2	Low
c	34.4	22.9	11.5	-11.5	-22.9	-34.4	High
r	0	0	0	0	0	0	Low
μ	0.71	0.47	0.24	-0.24	-0.47	-0.71	Low
c_o	12.7	8	3.2	-3.1	-6.3	-9.4	Medium

$$e^{k\nu} \approx 1 + k\nu + \frac{k^2\nu^2}{2},$$

$$\partial F(n,\nu)/\partial \nu = \frac{1}{nT}[\frac{d}{T}(c_m - \lambda p - r)\nu - (c_m d\frac{N-n+1}{N}e^{(n-1)kT} + \frac{dh}{k}\frac{N-n+1}{N}e^{(n-1)kT})(1+k\nu)$$
$$+ pd\frac{N-n+1}{N} + dr + \frac{dh}{k}\frac{N-n+1}{N}]. \quad (31)$$

Equating Equation 31 with respect to ν to zero, one can derive the joint critical time

$$\nu_J(n) = \frac{\frac{N-n+1}{N}[pd + \frac{dh}{k} - (c_m d + \frac{dh}{k})e^{(n-1)kT}] + dr}{\frac{N-n+1}{N}(c_m d + \frac{dh}{k})ke^{(n-1)kT} - \frac{d(c_m - \lambda p - r)}{T}} \quad (32)$$

Let $\nu_J^*(n) = \min\{\nu_J(n), T\}$. If the joint optimal solution is (n_J^*, $\nu_J^*(n_J)$), then the optimal unit time system profit is

$$F(n_J^*, \nu_J^*(n_J)) = F_R(n_J^*, \nu_J^*(n_J)) + F_S(n_J^*, \nu_J^*(n_J)). \quad (33)$$

It is obviously the optimal unit time system profit (33) is better than that of the unit time system profit (29) (Table 14). The coordination policy can be illustrated by the following Example 2.

Example 2

N=10, p=30, d=1, T=100, λ=0.9, h=0.005, c=10,

c_m=5, r=25, μ=20, s=150, and c_o=1200. With the deterioration rate k=0.0025, using the mathematical software MATHCAD and MAPLE 7, the optimal decision is obtained and the results are as follows (Table 14):

Sensitivity analysis

Sensitivity analysis with different deterioration rate k is carried out in Tables 15 and 16, Figures 2 and 3. The main conclusions drawn are as follows:

(1) Table 15 and Figure 2 show the changes in n_R^*, ν_R^*, the deteriorated quantity per period ($I_1(0) - Q_1$)/n_R^* and the ordering quantity $I_1(0) + Q_2$ for variable k. It is shown that as k increases, the replenishment cycle n_R^*T decreases, but ($I_1(0) - Q_1$)/n_R^* increases.
(2) Table 16 and Figure 3 show the changes in the retailer's unit time profit F_R, the supplier's unit time profit F_S and the unit time system profit F_R+F_S for variable k. Itis shown that as k increases, F_R, F_R+F_S decreases, but F_S increases.

From the afore-mentioned analysis, it can be shown that higher deterioration rate leads to lower system profit. However, if the retailer and the supplier coordinate to determine their order quantity, the system profit increases significantly. Even though the overall profit is better, the

Table 14. Optimal ordering decision with v.s. without coordination.

Item	Without coordination	Coordination
n_R^*	2	3
v_R^*	87.828	83.895
order quantity, $Q=I_1(0)+Q_2$	237.68	382.824
sales amount, Q_1+Q_2	190.476	271.924
deteriorated quantity per period, $(I_1(0)-Q_1)/n_R^*$	23.602	36.967
unit profit of retailer, F_R	\$9.751	\$9.268
unit profit of supplier, F_S	\$5.192	\$5.866
unit time system profit, F_R+F_S	\$14.943	\$15.134

Table 15. Optimal ordering decision without coordination for various deterioration rates.

N=10, p=30, d=1, T=100, λ=0.9, h=0.005, c=10, r=25, μ=20, s=150, c_o=1200.							
k	n_R^*	v_R^*	$I_1(0)$	Q_1	Deteriorated quantity per period = $(I_1(0)-Q_1)/n_R^*$	Q_2	Ordering quantity = $I_1(0)+Q_2$
$1.563*10^{-4}$	3	93.869	270.972	265.095	1.959	5.943	276.916
$6.25*10^{-4}$	3	90.848	286.791	262.678	8.038	8.733	295.524
0.0025	2	87.828	226.249	179.045	23.602	11.431	237.68
0.01	1	85.714	122.448	85.714	36.734	13.266	135.714

Table 16. The effect of system profit increase with various deterioration rates.

N=10, p=30, d=1, T=100, λ=0.9, h=0.005, c=10, c_m=5, r=25, μ=20, s=150, c_o=1200.													
Parameter	Without coordination					Coordination					Percent profit increase		
k	n_R^*	v_R^*	F_R^*	F_S	$F_R^*+F_S$	n_J^*	v_J^*	F_R	F_S	$(F_R+F_S)^*$	F_R (%)	F_S (%)	F_R+F_S (%)
$1.563*10^{-4}$	3	93.869	12.972	4.115	17.087	3	92.803	12.971	4.117	17.088	-0.006	0.04	0.005
$6.25*10^{-4}$	3	90.848	12.327	4.424	16.751	3	91.367	12.327	4.425	16.752	-0.002	0.009	0.001
0.0025	2	87.828	9.751	5.192	14.943	3	83.895	9.268	5.866	15.134	-5	13	1.3
0.01	1	85.714	3.086	5.285	8.371	2	64.501	-0.38	9.493	9.113	-112.3	79.6	8.9

Percent profit increase = [(unit time profit of coordination / unit time profit without coordination)-1]*100 %.

retailer may not gain profit under coordination. Therefore, in order to entice the retailer to co-operate, a compensation mechanism must be incorporated.

Conclusion

This study focuses on how to determine the optimal ordering policy for fast deteriorating items with expiration date. The customers' demand will decrease due to nearness to expiration date. Facing the deterioration and expiration date, how to decide an optimal order quantity is vital to retailer. The study develops a maximum profit model for coordinating the retailer and the supplier. The numerical examples show that the higher deterioration rate results in less the profit. However, the coordination policy between the retailer and the supplier will improve the efficiency of the ordering policy. When the deterioration rate increases, coordination should be considered since the percent profit increases significantly. However, in order to entice the retailer to co-operate, compensation mechanism must be incorporated (Zimmer, 2002). The results of this study give managerial insights to decision maker developing an optimal ordering decision for deteriorating product with expiration date.

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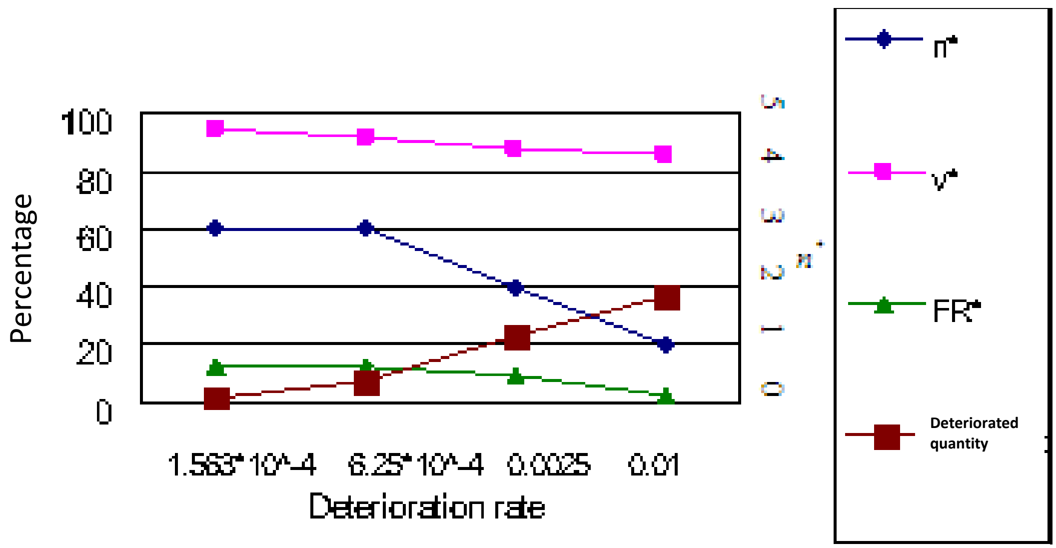

Figure 2. Optimal ordering decisions with various deterioration rates for the retailer.

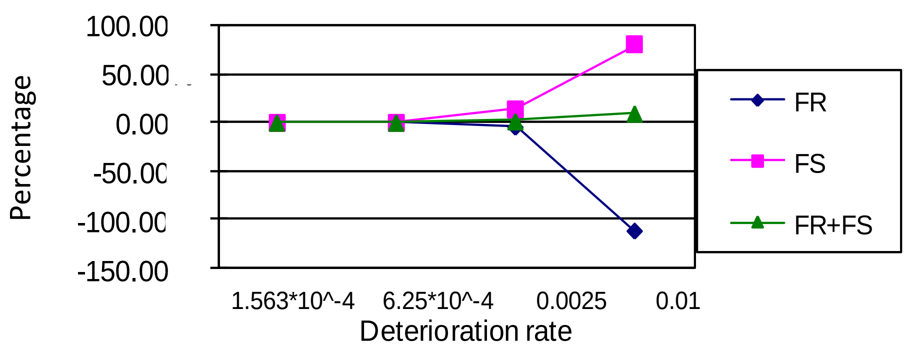

Figure 3. Percent profit increase due to coordination.

REFERENCES

Abad PL (2001). Optimal price and order size for a reseller under partial backordering. Comput. Oper. Res. 28:53–65.

Aggarwal SP (1978). A note on an order-level inventory model for a system with constant rate of deterioration. Opsearch 15:184–187.

Bhunia AK, Maiti M (1999). An inventory model of deteriorating items with lot-size dependent replenishment cost and a linear trend in demand. Appl. Math. Model. 23:301–308.

Bhunia AK, Maiti M (1998). Deterministic inventory model for deteriorating items with finite rate of replenishment dependent on inventory level. Comput. Oper. Res. 25(11):997–1006.

Chakrabarty T, Giri BC, Chaudhuri KS (1998). An EOQ model for items with weibull distribution deterioration, shortages and trended demand: an extension of philip's model. Comput. Oper. Res. 25(7–8):649–657.

Dye CY, Ouyang LY, Hsieh TP (2007). Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate. Eur. J. Oper. Res. 178(3):789-807.

Fites D (1996). Make your dealers your partners. Harvard Bus. Rev. 74:84-95.

Ghare PM, Schrader GF (1963). A model for exponentially decaying inventory. J. Ind. Eng. 14:238-243.

Goyal SY, Gupta YP (1989). Integrated inventory models, The buyer–vendor coordination. Eur. J. Oper. Res. 41:261-269.

He Y, Wang SY, Lai KK (2010). An optimal production-inventory model for deteriorating items with multiple-market demand. Eur. J. Oper. Res. 203(3):593-600.

Ho JC, Solis AO, Chang YL (2007). An evaluation of lot-sizing heuristics for deteriorating inventory in material requirements planning systems. Comput. Oper. Res. 34(9):2562-2575.

Hsieh CC, Lee ZZ (2005). Joint determination of production run length and number of standbys in a deteriorating production process. Eur. J. Oper. Res. 162(2):359-371.

Hsu PH, Wee HM, Teng HM (2007). Optimal Ordering Decision for Deteriorating Items with Expiration Date and Uncertain Lead Time. Comput. Ind. Eng. 52:448-458.

Hsu PH, Wee HM, Teng, HM (2010). Preservation Technology Investment for Deteriorating Inventory. Int. J. Prod. Econ. 124(2):388–394.

Mahapatra NK (2005). Decision process for multiobjective, multi-item production-inventory system via interactive fuzzy satisficing technique. Comput. Math. Appl. 49:805–821.

Mukhopadhyay S, Mukherjee RN, Chaudhuri KS (2004). Joint pricing and ordering policy for a deteriorating inventory. Comput. Ind. Eng., 47: 339–349.

Padmanabhana G, Vratb P (1995). EOQ models for perishable items under stock dependent selling rate. Eur. J. Oper. Res. 86(2):281–292.

Rau H, Wu MY, Wee HM (2003). Integrated inventory model for deteriorating items under a multi-echelon supply chain environment.

Int. J. Prod. Econ. 86(2):155-168.
Shah YK, Jaiswal MC (1977). An order-level inventory model for a system with constant rate of deterioration. Opsearch. 14: 174–184.
Sucky E (2005). Inventory management in supply chains: A bargaining problem. Int. J. Prod. Econ. 93-94:253-262.
Wee HM (1995). Joint pricing and replenishment policy for deteriorating inventory with declining market. Int. J. Prod. Econ. 40:163-171.
Wee HM (1999). Deteriorating inventory model with quantity discount, pricing and partial backordering. Int. J. Prod. Econ. 59:511–518.
Weng ZK (1995). Channel coordination and quantity discounts. Manage. Sci. 41:1509-1522.
Yang PC (2004). Pricing strategy for deteriorating items using quantity discount when demand is price sensitive. Eur. J. Oper. Res. 157:389-397.
You PS (2005). Optimal replenishment policy for product with season pattern demand. Oper. Res. Lett., 33(1):90-96.
Zimmer K (2002). Supply chain coordination with uncertain just-in-time delivery. Int. J. Prod. Econ. 77(1):1-15.

Appendix A

$$Q_1 = \sum_{j=1}^{n-1}\int_0^T d_j dt + \int_0^\nu d_n dt = \frac{d(n-1)(2N+2-n)T}{2N} + \frac{d(N+1-n)\nu}{N}. \quad \text{(A-1)}$$

$$Q_2 = \int_\nu^T d_1\theta(T-t)dt = \frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right). \quad \text{(A-2)}$$

$$C(n,\nu) = [I_1(0) + Q_2]c.$$

$$=(\frac{d}{k}\{w(1)(e^{kT}-1) + \sum_{i=2}^{n-1} w(i)(e^{kT}-1)e^{k[(i-1)T]}$$

$$+ w(n)(k\nu + k^2\nu^2/2)e^{k[(n-1)T]}\} + \frac{d}{T}\left(\frac{T^2}{2} - \frac{\nu^2}{2}\right))c$$

$$\approx [\frac{d}{k}\{\ \alpha_1 + \frac{N+1-n}{N}(k\nu + k^2\nu^2/2)e^{k[(n-1)T]}\} + \frac{d}{T}(\frac{T^2}{2} - \frac{\nu^2}{2})\ ]c$$

$, \quad 0 < \nu \le T, n \ge 3,$

where $\alpha_1 = w(1)(e^{kT}-1) + \sum_{i=2}^{n-1} w(i)(e^{kT}-1)e^{k[(i-1)T]}$, and

(A-3)

$$I_1(0) = \begin{cases} \frac{d}{k}(k\nu + k^2\nu^2/2), & n = 1. \\ \frac{d}{k}\{(e^{kT}-1) + \frac{N-1}{N}(k\nu + k^2\nu^2/2)e^{kT}\}, & n = 2. \\ \frac{d}{k}\{w(1)(e^{kT}-1) + \sum_{i=2}^{n-1} w(i)(e^{kT}-1)e^{k[(i-1)T]} + w(n)(k\nu + k^2\nu^2/2)e^{k[(n-1)T]}\}, & n \ge 3. \end{cases} \quad \text{(A-4)}$$

$$L(n,\nu) = r\int_\nu^T d_1[1-\theta(T-t)]dt = rd\left(\frac{T}{2} + \frac{\nu^2}{2T} - \nu\right). \quad \text{(A-5)}$$

$$H_j^T = h\int_0^T I_j(t)dt$$

$$= \frac{dh}{k}\left\{\frac{N-j+1}{N}(\frac{e^{kT}-1}{k} - T) + \frac{e^{kT}-1}{N}\frac{1-e^{-kT}}{k}[(N-j+1)\frac{e^{kT}[1-e^{(n-j-1)kT}]}{1-e^{kT}}\right.$$

$$\left. - \frac{e^{kT} - (n-j)e^{(n-j)kT} + (n-j-1)e^{(n-j+1)kT}}{(1-e^{kT})^2}] + \frac{N-n+1}{N}(e^{k\nu}-1)e^{k(n-j)T}\frac{1-e^{-kT}}{k}\right\} \quad \text{(A-6)}$$

$, j = 1, 2 \cdots, n-2.$

$$\sum_{j=1}^{n-2} H_j^T = \frac{dh}{k}\Bigg\{(\frac{e^{kT}-1}{k}-T)\frac{(n-2)(2N+3-n)}{2N}+\frac{e^{kT}-1}{N}\frac{1-e^{-kT}}{k}$$

$$\times\left(\left[\frac{e^{kT}}{1-e^{kT}}\frac{(n-2)(2N+3-n)}{2}\right]-\frac{e^{kT}}{1-e^{kT}}\left[\frac{(N+1)(e^{(n-2)kT}-1)}{1-e^{-kT}}-\frac{e^{(n-2)kT}-(n-1)+(n-2)e^{-kT}}{\left(1-e^{-kT}\right)^2}\right]\right.$$

$$-\frac{(n-2)e^{kT}}{(1-e^{kT})^2}+\frac{1}{(1-e^{kT})^2}\left[\frac{n(e^{(n-1)kT}-e^{kT})}{1-e^{-kT}}-\frac{e^{(n-1)kT}-(n-1)e^{kT}+(n-2)}{(1-e^{-kT})^2}\right]$$

$$\left.-\frac{1}{(1-e^{kT})^2}\left[\frac{(n-1)(e^{nkT}-e^{2kT})}{1-e^{-kT}}-\frac{e^{nkT}-(n-1)e^{2kT}+(n-2)e^{kT}}{(1-e^{-kT})^2}\right]\right)$$

$$+\frac{N-n+1}{N}(e^{k\nu}-1)\frac{e^{(n-1)kT}-e^{kT}}{k}\Bigg\}, \quad n\ge 3,$$

$$\approx \frac{dh}{k}\{\alpha_2+\frac{N-n+1}{N}\left(k\nu+{}^{k^2\nu^2}\!/\!_2\right)\frac{e^{(n-1)kT}-e^{kT}}{k}\},$$

where

$$\alpha_2=\left[\frac{e^{kT}-1}{k}-T\right]\frac{(n-2)(2N+3-n)}{2N}+\frac{e^{kT}-1}{N}\frac{1-e^{-kT}}{k}.$$

$$\times\left(\left[\frac{e^{kT}}{1-e^{kT}}\frac{(n-2)(2N+3-n)}{2}\right]-\frac{e^{kT}}{1-e^{kT}}\left[\frac{(N+1)\left(e^{(n-2)kT}-1\right)}{1-e^{-kT}}-\frac{e^{(n-2)kT}-(n-1)+(n-2)e^{-kT}}{\left(1-e^{-kT}\right)^2}\right]\right.$$

$$-\frac{(n-2)e^{kT}}{\left(1-e^{kT}\right)^2}+\frac{1}{\left(1-e^{kT}\right)^2}\left[\frac{n[e^{(n-1)kT}-e^{kT}]}{1-e^{-kT}}-\frac{e^{(n-1)kT}-(n-1)e^{kT}+(n-2)}{\left(1-e^{-kT}\right)^2}\right]$$

$$\left.-\frac{1}{\left(1-e^{kT}\right)^2}\left[\frac{(n-1)\left[e^{nkT}-e^{2kT}\right]}{1-e^{-kT}}-\frac{e^{nkT}-(n-1)e^{2kT}+(n-2)e^{kT}}{\left(1-e^{-kT}\right)^2}\right]\right).$$

(A-7)

$$H_{n-1}^T = h\int_0^T I_{n-1}(t)dt$$

$$=\frac{dh}{k}\left[\frac{N-n+2}{N}\left(\frac{e^{kT}-1}{k}-T\right)+\frac{N-n+1}{N}\left(e^{k\nu}-1\right)\frac{e^{kT}-1}{k}\right],$$

$$\approx\frac{dh}{k}\left[\frac{N-n+2}{N}\left(\frac{e^{kT}-1}{k}-T\right)+\frac{N-n+1}{N}\left(k\nu+{}^{k^2\nu^2}\!/\!_2\right)\frac{e^{kT}-1}{k}\right].$$

(A-8)

$$H_n^{\nu} = h\int_0^{\nu} I_n(t)dt = \frac{dh}{k}\frac{N-n+1}{N}\left(\frac{e^{k\nu}-1}{k}-\nu\right).$$

$$\approx \frac{dh}{k}\frac{N-n+1}{N}\left(k\nu^2/2\right). \tag{A-9}$$

$$H(1,\nu) = \frac{dh}{k}\left(\frac{e^{k\nu}-1}{k}-\nu\right)$$

$$\approx \frac{dh}{k}\left(k\nu^2/2\right). \tag{A-10}$$

$$H(2,\nu) = \frac{dh}{k}\left(\frac{e^{kT}-1}{k}-T+\frac{N-1}{N}\left(e^{k\nu}-1\right)\frac{e^{kT}-1}{k}\right)+\frac{dh}{k}\frac{N-1}{N}\left(\frac{e^{k\nu}-1}{k}-\nu\right)$$

$$\approx \frac{dh}{k}\left(\frac{e^{kT}-1}{k}-T+\frac{N-1}{N}\left(k\nu+k^2\nu^2/2\right)\frac{e^{kT}-1}{k}\right)+\frac{dh}{k}\frac{N-1}{N}\left(k\nu^2/2\right). \tag{A-11}$$

Appendix B: Proof of Theorem 1.

$$\frac{\partial^2}{\partial\nu^2}F_R(n,\nu) = \frac{1}{nT}\left[\frac{d(c-\lambda p-r)}{T}-(cd+\frac{hd}{k})\frac{N+1-n}{N}e^{(n-1)kT}k\right]. \tag{B-1}$$

Since $c < \lambda p$, one has $c-\lambda p-r$ <0, and $(cd+\frac{hd}{k})\frac{N+1-n}{N}e^{(n-1)kT}k$ >0, which leads to $\partial^2 F_R(n,\nu)/\partial\nu^2 < 0$, this completes the proof.

Appendix C: Proof of Theorem 2.

$$F(n,\nu) = \frac{1}{nT}\left[R(n,\nu)-(I_1(0)+Q_2)c_m-L(n,\nu)-B(n,\nu)-H(n,\nu)-c_o-s\right], \quad 0<\nu\le T, n\le N. \tag{C-1}$$

$$\frac{\partial^2}{\partial\nu^2}F(n,\nu) = \frac{1}{nT}\left[\frac{d(c_m-\lambda p-r)}{T}-(c_m d+\frac{hd}{k})\frac{N+1-n}{N}e^{(n-1)kT}k\right]. \tag{C-2}$$

Note $c_m < c < \lambda p$, one has $c_m-\lambda p-r$ <0, and $(c_m d+\frac{hd}{k})\frac{N+1-n}{N}e^{(n-1)kT}k$ >0, which leads to $\partial^2 F(n,\nu)/\partial\nu^2 < 0$, this completes the proof.