Review

Accounting, taxation, and the cost of capital

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Received 6 September, 2011; Accepted 26 December, 2014

This paper investigates the link between accounting and taxation and its implications for the cost of equity capital. Using a simple model, we characterize the determinants of the cost of capital in a setting where reporting rules combine accounting and taxation estimations. Accounting and tax rules usually result in different estimates of true earnings, each one with its own estimation error. The correlation between these errors and the rule of combination of accounting and tax estimates characterizes the degree of connection between accounting and taxation. These two variables determine the overall precision of the public reports issued by the companies and, among other things, influence the cost of capital. The paper characterizes how the cost of capital varies with precision of accounting and tax estimates, with the correlation of estimation errors and with the rule of combination between accounting and tax estimates. The most interesting result is that for low enough or negative levels of the correlation between estimation errors, more precise accounting/tax estimation principles may result in higher cost of capital.

Key words: Cost of capital, information, precision, accounting, taxation.

INTRODUCTION, LITERATURE REVIEW AND ORGANIZATION OF THE PAPER

Conventional wisdom predicts that the cost of equity capital declines when risk-averse investors have more precise information. The argument in favor of this conclusion is that higher information precision lowers the assessed variance of future cash-flows (the estimation risk component of the cost of capital). In turn, this lowers the risk premium required by investors and hence it lowers the cost of equity capital. A second argument is that higher quality information decreases the information asymmetry on the market, increases market liquidity and the share prices and decreases the cost of capital (the information asymmetry component). Given these lines of thought, one may expect corporations to prefer reporting rules that induce the highest possible precision, so that the cost of capital declines. And, indeed, the reduction in the cost of capital seems to be one of the economic effects that major accounting standard setters (e.g. International Accounting Standard Board – IASB and Financial Accounting Standards Board – FASB) have in mind when they issue reporting standards. However, even if a single or just a few sets of reporting standards are to be used world-wide, the application of such standards is not uniform but jurisdiction dependent. A wide network of country-specific institutional factors shapes the application of accounting standards (Ball et al., 2000). In this paper, we study analytically the
implications for the cost of capital of one such institutional factor, namely the link between accounting and taxation. It is well known that accounting and tax principles are different in most, if not all, jurisdictions. The two sets of principles yield different estimates with different precisions of the economic or true earnings. Our paper proves that, in jurisdictions where the two systems interact, the combination between accounting and tax estimations affects the overall precision of reported earnings and the cost of capital in a non-trivial manner. Understanding the relationship between accounting and taxation and the implications of this relationship for the cost of capital is thus essential. Our paper takes some first steps in accomplishing this. It develops this relationship mathematically and provides analytical results about some of the institutional determinants of the cost of capital. Briefly, the paper describes how the cost of capital varies with the degree of inclusion of accounting and tax estimates in the public earnings report, with the precision of accounting and tax estimates as well as with the correlation between the error terms of these estimates. While some classical results still hold in our model (e.g. the cost of capital increases in the volatility of future cash-flows), some of our findings are more surprising and hence interesting. For instance, we prove that, for some features of the link between accounting and taxation, the cost of capital actually increases with the precision of accounting or tax estimates. This result stands in contrast with conventional wisdom regarding the relationship between information precision and the cost of capital. Overall, our paper proves that a careful analysis of the institutional factors involved in a certain setting is required in order to have a clear picture of how information precision influences the cost of capital.

The literature on the relationship between disclosure, information quality and the cost of capital is rich and growing. The topic is studied both empirically and analytically. Given the breadth of the literature we do not attempt here a comprehensive review. One such review may be found in Botosan (2006). Instead, we only focus on those papers that are most relevant to our analysis and highlight the ties between our work and prior literature.

The empirical side of the literature provides results regarding the association between the level of disclosure and the cost of capital. Botosan (1997) and Leuz and Verrecchia (2000) provide evidence that increased levels of disclosure decrease the cost of equity capital. These results proved to be quite robust and over time the literature moved the center of interest from the level of disclosure to the information quality. Francis et al. (2004) study the relationship between earnings quality and the cost of capital. Their evidence supports the idea that higher information quality decreases the cost of capital. However, their aim is to investigate a relative ranking between several measures of earnings attributes and document how these measures relate to the cost of capital. They find that accounting-based measures of earnings quality such as accrual quality, persistence, predictability and smoothness have the highest effect on cost of capital. Francis et al. (2008) study the relationship between voluntary disclosure, earnings quality and the cost of capital. They find that firms with good earnings quality also have strong voluntary disclosure systems and that disclosure levels are negatively correlated with cost of capital. However, after controlling for earnings quality, the effect of disclosure disappears. Their findings are interesting because they prove that when both disclosure levels and earnings quality measures are present in a regression, the negative association with the cost of capital is picked-up by the latter. The empirical paper closest to our work is Botosan et al. (2004). This paper studies the effect of information precision on the cost of capital. In line with prior literature, the authors find that precision of public information is negatively correlated with the cost of capital. However, the precision of private information is positively associated with the cost of capital because it increases the information asymmetry on the market. The influence of asymmetric information on the cost of capital is also studied in analytical papers such as Armstrong et al. (2010) and Hughes et al. (2007). These papers present conditions under which asymmetric information affects the cost of capital. Our paper studies analytically only the effect of the precision of public reports on the cost of capital and does not relate to the information asymmetry problem. In addition, we study the relationship between the effect of precision in accounting and taxation rules and how their interaction affects the precision of the final public earnings report. Contrary to the results reported by Botosan et al. (2004), our paper predicts that the cost of capital may under certain circumstances increase with the precision of accounting estimates.

Surprisingly, Daske (2006) did not document a negative relationship between the cost of capital and adoption of high quality financial reporting standards (such as the International Financial reporting Standards – IFRS - and the American Generally Accepted Accounting Principles – the US-GAAP). However, in later work, Daske et al. (2008) found that, under certain circumstances, positive economic consequences (improved liquidity and lower cost of capital) are associated with IFRS adopters. However, this study points out that the capital market benefits (liquidity and low cost of capital) appear exclusively in countries with strong incentives for transparency and strong legal enforcement. Their results add to the list of institutional factors investigated by Hail and Leuz (2006). These authors show that the cost of equity capital is lower in jurisdictions with extensive disclosure requirements and strong securities regulations. Relative to these findings, our paper identifies the link...
between accounting and taxation as a different institutional factor that may explain differences in the level of cost of capital across jurisdictions.

The analytical side of this literature studies how the share price and risk premia are determined in equilibrium and how equilibrium cost of capital varies with its determinants. Our paper takes a similar tack. Easley and O'Hara (2004) consider both the estimation risk and information asymmetry in the formulation of an equilibrium price. They describe how information affects equilibrium prices and the cost of capital. Lambert et al. (2007) also study the effect of accounting information on the cost of capital. Unlike Easley and O'Hara (2004) they use a Capital Asset Pricing Model (CAPM) approach and focus on how accounting reports help investors assess the variance of the firm’s cash-flows as well as the covariation of the firm’s cash-flows with the cash-flows of other firms on the market. Our paper is very close to Lambert et al. (2007) because it is analytically tracking the properties of accounting information (like precision) to the formula of the cost of capital.

A recent trend in the literature is to study the effect of disclosure and information quality on the cost of capital when decisions about the level of disclosure and precision are made simultaneously with other decisions such as investment and capital structure decisions. For instance, Li et al. (2011) study how different informational settings affect both the cost of capital and investment decisions when they are jointly determined in equilibrium. Also, Bertomeu et al. (2011) point out that the relationship between information and cost of capital is more subtle. While their model predicts a negative association between the cost of capital and the extent of voluntary disclosure, they cannot find a causal relation between the two. Instead, they show how exogenous mandatory disclosure requirements and endogenous capital structure decisions also influence the cost of capital. Finally, Gao (2010) studies the relationship between disclosure quality, investor welfare and cost of capital in production economies with perfect competition among investors. One of his findings is that, under certain conditions, the cost of capital may increase with disclosure quality. Our paper is closest to Gao’s paper in that it predicts a positive correlation between cost of capital and quality of information. However, our paper posits a different reason for this positive association, namely the link between accounting and taxation.

The paper is organized as follows. Section 2 introduces a basic model of the cost of capital. It also characterizes the effects of precision of the public earnings reports on the cost of capital. Section 3 introduces our modeling. It adds further detail to the information structure described in section 2 and describes in mathematical terms what we mean, from an informational perspective, by “the link between accounting and taxation”. Section 4 contains our results. It includes a static analysis regarding the variation of the cost of capital with the variables that determine the overall precision of the public reporting system. Since the underlying mathematics is accessible to any reader we included short proofs of our statements in the body of the paper. However, longer and more detailed proofs are available on request from the author. Section 5 reviews our results and discusses limitations.

**Basic model of cost of capital**

This section presents a simple model of the cost of capital. It also discusses how the precision of a reporting system influences the cost of capital. Since the model is well known in the literature, our exposition is kept short and concise. We only present and emphasize those features of the model that prove useful in the preparation of our own modeling in section 3. In addition, unlike Easley and O’Hara (2004), our model only considers the estimation risk component of cost of capital. It does not touch on the information asymmetry problem. In this sense, our baseline model of cost of capital follows the arguments in Li et al. (2011) but a similar formula for the cost of capital can also be derived by following the arguments in Lambert et al. (2007) and those in Gao (2010).

**Cost of capital**

Consider an entrepreneur who owns a firm with a terminal cash-flow $\xi$. The cash-flow is a random variable which is realized at a certain point in the future. It is assumed to be normally distributed with mean $\mu$ and variance $\sigma^2_x$. In shorthand notation (which will be used from now on) $\xi \sim N(\mu, \sigma^2_x)$. At an interim data, prior to the realization of the terminal cash-flow, the entrepreneur must sell (say for consumption purposes) a fraction $\alpha$ of the firm. The firm is priced by risk-averse and rational investors.

To influence investors’ perceptions about the cash-flow, the entrepreneur issues a public report $\bar{\xi}$ whose realization we denote simply as $\tau$. We assume investors do not search for private information but only rely on this public report. For tractability reasons, investors are assumed to have constant absolute risk aversion utility functions characterized by risk-aversion coefficient $\theta$. The expression of such a function is $U(w) = -e^{-\theta w}$ where $w$ is the wealth of a representative investor. Also, investors are uniformly distributed over the unity interval. If a fraction $\alpha$ is to be sold to these investors then Li et al. (2011) prove that the cost of capital has the following formula:

**Lemma 1** The cost of capital (C) is a multiple of the cash-flow variance conditional on all available information on the market.
\[ C = a \theta \text{Var}[\tilde{x}|r] \]

**Proof:** As in Li et al. (2011), a representative investor’s potential wealth is given by \( \tilde{w} = (\tilde{x} - p)\delta \). In this expression \( p \) gives the equilibrium market price and \( \delta \) is the demand of the representative investor. The expression reflects the fact that the investor pays the equilibrium price \( p \) per share and expects to receive uncertain cash-flow \( \tilde{x} \). It is well known (Christensen and Fetham, 2002) that when investors have CARA utility functions and their prospective wealth is normally distributed then maximization of expected utility reduces to the maximization of the \( E[\tilde{w}|r] - \frac{1}{2} \theta \text{Var}[\tilde{w}|r] \). Since \( \text{Var}[\tilde{w}|r] = \text{Var}[(\tilde{x} - p)\delta] = \delta^2 \text{Var}[\tilde{x}|r] \) and \( E[\tilde{w}|r] = (E[\tilde{x}|r] - p)\delta \), it then follows that the investor chooses demand \( \delta \) that maximizes function \( E[\tilde{x}|r] - p)\delta - \delta^2 \text{Var}[\tilde{x}|r] \). Taking the first order derivative with respect to \( \delta \) and solving for \( \delta \) we obtain the demand of a representative investor:

\[ \delta = \frac{E[\tilde{x}|r] - p}{\theta \text{Var}[\tilde{x}|r]} \]

Given the uniform distribution assumption about the investors then market clearing condition which requires total supply \( \alpha \) to equal total demand \( \int_0^1 \delta \, di = \delta \) implies that \( p = E[\tilde{x}|r] - a \theta \text{Var}[\tilde{x}|r] \) which further yields \( C = E[\tilde{x}|r] - p = a \theta \text{Var}[\tilde{x}|r] \)

The cost of capital is equal to the risk premium the risk-averse investors require to invest in the firm.

**Basic information structure**

The entrepreneur can influence the cost of capital by issuing a public report \( \tilde{r} \) which changes investors’ assessment of the cash-flows variance. As it is common in the literature, we assume that report \( \tilde{r} \) is an unbiased estimate of the cash-flow \( \tilde{x} \). Thus, by assumption, \( \tilde{r} = \tilde{x} + \tilde{\omega} \) with \( \tilde{\omega} \sim N(0, \sigma_\omega^2) \) and \( \text{Cov}(\tilde{x}, \tilde{\omega}) = 0 \). Given these assumptions about the basic information structure, the following lemma holds:

**Lemma 2** The cost of capital increases in the volatility of the terminal cash-flow \( \sigma_{\tilde{x}}^2 \) and in the noise in the information system, \( \sigma_\omega^2 \).

The proof is simple and follows from the well known result that (given normality of \( \tilde{r} \) as above) \( \text{Var}[\tilde{x}|r] = \frac{\sigma_\omega^2}{\sigma_{\tilde{x}}^2 + \sigma_\omega^2} = \frac{1}{\sigma_{\tilde{x}}^2 + \sigma_\omega^2} \). The variation of \( \text{Var}[\tilde{x}|r] \) and hence that of the cost of capital with \( \sigma_{\tilde{x}}^2 \) and \( \sigma_\omega^2 \) is then clear.

Lemma 2 establishes that the cost of capital moves in the same direction with the variance of the error in the public report. We use this lemma later in paper to ease the exposition of our results. All results that hold for the variance of the error in the public report also hold for the cost of capital.

**Modeling the link between accounting and taxation**

In this section, we maintain the notation so far and add extra structure to the general information system described in section 2. The aim is to make precise what we mean by “the link between accounting and taxation”. Our approach is purely informational in the sense that the set of tax principles is viewed as an earnings estimator much like the accounting one. Three ideas are key to our modeling. First, the accounting and tax estimates of cash-flows have different precisions. Thus, accounting and taxation systems induce two different estimators or signals:

The accounting signal \( \tilde{y} = \tilde{x} + \tilde{\epsilon} \) with \( \tilde{\epsilon} \sim N(0, \sigma_\epsilon^2) \)

The taxation signal \( \tilde{z} = \tilde{x} + \tilde{\tau} \) with \( \tilde{\tau} \sim N(0, \sigma_\tau^2) \)

We assume that the error terms are not correlated with the cash-flow. Formally, \( \text{Cov}(\tilde{x}, \tilde{\epsilon}) = \text{Cov}(\tilde{x}, \tilde{\tau}) = 0 \). We make no assumption on which of the two estimators yields more precise earnings estimates.

Second, while different, the error terms in the accounting and taxation estimation functions are assumed to be related. We assume the error terms \( \tilde{\epsilon} \) and \( \tilde{\tau} \) exhibit correlation and we allow this correlation to be either positive or negative depending on how accounting and tax estimation principles are set-up. Formally, estimation errors \( \tilde{\epsilon} \) and \( \tilde{\tau} \) are assumed to have a bivariate normal distribution characterized by \( N(0, \sigma_\epsilon^2, \sigma_\tau^2, \rho) \). The degree of correlation between the error terms of the two estimates, \( \rho \) represents one feature of the link between accounting and taxation. That is we allow for accounting and tax rules to be framed in a wide variety of ways such that the correlation between the error terms that they induce can be either positive, negative or zero.

Third, we conceive the public reported signal \( \tilde{r} \), as a linear combination between the accounting and tax signals. Thus in our modeling, the reporting rule combines a purely accounting estimate with a tax estimate. The weight placed on each of the two signals captures the second feature of the link between accounting and taxation. Denote \( \varphi \) the weight place on the accounting estimate \( \tilde{y} \). Then \( 1 - \varphi \) is the weight placed on the tax estimate \( \tilde{z} \). With this notation, the accounting report \( r \) can be written as:

\[ \tilde{r} = \varphi \tilde{y} + (1 - \varphi)\tilde{z} \]

Variable \( \varphi \) captures the relative dominance of accounting and tax rules in the public report. When \( \varphi = 1 \),
accounting estimations dominate and public reporting is completely detached from tax principles. Assuming investors know the informational properties of signals $\bar{y}$ and $\bar{\epsilon}$ (like we do in this model), a combination rule that places all weight on the accounting estimate renders the tax estimate useless for reporting purposes. When $\varphi = 0$, tax principles dominate public reporting. Any $\varphi \in (0,1)$ reflects a non-trivial link between accounting and taxation in the set-up of the public report. To make sure the random variable $\bar{\epsilon}$ associated with the public report is normally distributed as in section 2 above, we needed to make the further assumption that $\bar{\epsilon}$ and $\bar{\epsilon}$ are jointly normal. This assumption was needed because both $\bar{\epsilon}$ and $\bar{\epsilon}$ were assumed to be dependent and, in general, a linear combination of normally but not independently distributed random variables may not be normal. However, assuming joint normality of $\bar{\epsilon}$ and $\bar{\epsilon}$ ensures that $\bar{\epsilon}$ is a normally distributed random variable and preserves the validity of lemma 2 mentioned before.

Analytically, the pair $(\alpha^2,\beta^2)$ reflects the volatility of the accounting and tax estimations. The inverse of the volatility is usually associated with precision of the estimates induced by the application of accounting and tax principles. The pair $(\rho,\varphi)$ captures the notion of the link between accounting and taxation. These four variables $(\alpha^2,\beta^2,\rho,\varphi)$ represent the determinants of the variance of the public report and hence the determinants of the cost of capital in our model. The following lemma shows how these variables affect the variance of the public report:

**Lemma 3**

\[
\text{Var}[\bar{\epsilon}] = \alpha^2 + \varphi^2\beta^2 + (1-\varphi)^2\beta^2 + 2\rho\varphi(1-\varphi)\alpha\beta.
\]

**Proof:** From lemma 3 it follows that the earnings variance is an increasing linear function of $\rho$. Coefficient of $\rho$ in the formula of $\text{Var}[\bar{\epsilon}]$ is $2\rho(1-\varphi)\alpha\beta$, which is positive.

One consequence of proposition 4 is that taking the other variables as given, the cost of capital is at its minimum when $\rho$ is minimum ($\rho = -1$). Another interesting result about the correlation is captured in the following proposition:

**Proposition 4** – The cost of capital unambiguously increases with increases in the correlation coefficient between the error terms in the accounting and taxation estimators.

**Proof:** From lemma 3 it follows that the earnings variance is an increasing linear function of $\rho$. Coefficient of $\rho$ in the formula of $\text{Var}[\bar{\epsilon}]$ is $2\rho(1-\varphi)\alpha\beta$, which is positive.

One consequence of proposition 4 is that taking the other variables as given, the cost of capital is at its minimum when $\rho$ is minimum ($\rho = -1$). Another interesting result about the correlation is captured in the following proposition:

**Proposition 5** – There exists levels of correlation between accounting and tax estimates $(\rho)$ and combination rules $\varphi$ such that the variance of an earnings report (and hence the cost of capital) with accounting and taxation estimates is lower than the variance of the earnings report (and cost of capital) that relies only on accounting estimates.

**Proof:** The variance of the error term in the public report when accounting and taxation interact is $\varphi^2\beta^2 + (1-\varphi)^2\beta^2 + 2\rho\varphi(1-\varphi)\alpha\beta$, while the variance of the error term in the public report when accounting estimations dominate is simply $\beta^2$. For the mixed reporting setting (accounting and tax estimations) to dominate the purely accounting setting we need to have:

\[
\varphi^2\beta^2 + (1-\varphi)^2\beta^2 + 2\rho\varphi(1-\varphi)\alpha\beta < \beta^2
\]

Working out this inequality one obtains the cut-off point

\[
\rho < \frac{(1+\varphi)\beta^2 - (1-\varphi)\beta^2}{2\rho\varphi\alpha\beta}
\]

Further, the term on the right-hand side of the inequality is well behaved (is between -1 and 1) if and only if $\frac{\alpha - \beta}{\alpha + \beta} < \varphi < \frac{\alpha + \beta}{\alpha - \beta}$. It can be easily seen that when the variance in the accounting estimate is bigger than the variance in the tax estimate ($\beta > \alpha$) the inequality above holds true for any $\rho$. This is hardly surprising because adding to the mix an estimate with lower variance (bigger precision) decreases the overall earnings report variance and with it, decreases the cost of capital. The more interesting case is when the variance in the tax estimate is bigger than the variance of the accounting estimate ($\alpha > \beta$). In this case, it is still possible that the mixed earnings report dominates the pure accounting report provided $\frac{\alpha - \beta}{\alpha + \beta} < \varphi$ and

\[
\rho < \frac{(1+\varphi)\beta^2 - (1-\varphi)\beta^2}{2\rho\varphi\alpha\beta}
\]

**Results on correlation ($\rho$)**

**Proposition 4** – The cost of capital unambiguously increases with increases in the correlation coefficient between the error terms in the accounting and taxation estimators.

**Proof:** From lemma 3 it follows that the earnings variance is an increasing linear function of $\rho$. Coefficient of $\rho$ in the formula of $\text{Var}[\bar{\epsilon}]$ is $2\rho(1-\varphi)\alpha\beta$, which is positive.

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\[
\rho < \frac{(1+\varphi)\beta^2 - (1-\varphi)\beta^2}{2\rho\varphi\alpha\beta}
\]
Thus, for high enough $\varphi$ but low enough levels of correlation $\rho$, a reporting system that combines accounting and tax estimates yields a lower cost of capital than a reporting system where accounting alone dominates. But in this second case, the condition derived in proposition 5 that the level of correlation should be low enough is essential. The low or negative correlation outweighs the larger volatility induced by the tax estimation and induces a smaller total variance of the public report.

**Results on precision**

This section looks at how precision of accounting and tax estimates as captured by the inverses of their variances ($\sigma_t^2, \sigma_e^2$) manifests in the cost of capital. As it becomes clear from the proposition below, the nature (the sign) of the correlation between accounting and tax estimation error is essential in the analysis.

**Proposition 6**

a) For positively correlated accounting and taxation estimation errors ($\rho > 0$), the cost of capital unambiguously decreases when their precision increases.

b) For negatively correlated accounting and taxation estimation errors ($\rho < 0$) and, for small enough standard deviations (large enough precisions), the cost of capital increases when precision increases.

**Proof:** Taking the first-order derivative of the earnings variance with respect to $\sigma_e$ and $\sigma_t$ respectively we find:

\[
\frac{\partial \text{Var}[\hat{y}]}{\partial \sigma_e} = 2(1 - \varphi)^2 \sigma_e + 2\varphi(1 - \varphi)\rho \sigma_e \\
\frac{\partial \text{Var}[\hat{y}]}{\partial \sigma_t} = 2\varphi^2 \sigma_e + 2\varphi(1 - \varphi)\rho \sigma_t
\]

If $\rho > 0$ then both derivatives are positive so the earnings variance and the cost of capital increase as $\sigma_e$ and $\sigma_t$ increase (or, alternatively, increase as the precision of the accounting and taxation estimates decrease) which proves part a.)

However, if $\rho < 0$, each of the two derivatives above has a unique root:

\[
\sigma_t^* = -\frac{\varphi}{1 - \varphi} \rho \sigma_e \\
\sigma_e^* = -\frac{\varphi}{1 - \varphi} \rho \sigma_t
\]

It follows that for $\sigma_t < \sigma_t^*$ and $\sigma_e < \sigma_e^*$ the two derivatives are negative. Mathematically, taking $\sigma_t^*, \varphi, and \rho < 0$ as given, this triple defines a cut-off point $\sigma_e^* = -\frac{1 - \varphi}{\varphi} \rho \sigma_t$ for the volatility of the accounting estimate. Below this point, decreases in the volatility of accounting estimates have the effect of increasing the cost of capital. Similarly, taking $\sigma_t^*, \varphi, and \rho < 0$ as given, this triple defines a cut-off point $\sigma_t^* = -\frac{1 - \varphi}{\varphi} \rho \sigma_e$ for the volatility of the tax estimate. Below this point, decreases in the volatility of tax estimates have, again, the effect of increasing the cost of capital.

Economically, this means that, other things being equal, for small enough estimation variances, adding a bit of extra noise could actually decrease the variance of the public report and hence the cost of capital. This holds true for both the accounting and tax estimate variances. Put differently, starting at high levels of the estimation variances, reduction in these variances decreases the cost of capital but only up to some level. Decreasing the variances below this level starts increasing the variance in the public report and cost of capital. In short, proposition 6 proves that in some cases (negative correlation between accounting and tax estimates) what is beneficial to the cost of capital is more but not unbounded information precision. This result contradicts conventional wisdom regarding the relationship between information precision and the cost of capital. The reason for this result is the negative correlation between the estimation errors generated by the application of accounting and tax principles. Positive or zero correlations render the first order derivatives strictly positive and take us back to the conventional wisdom of the negative relationship between the cost of capital and information precision. However, negative correlation changes that relationship. When the link between accounting and taxation is characterized by negative correlations between accounting and tax estimates, increases in precision (decrease in estimation variance) of either accounting or tax estimates is desired but only up to a level. Beyond that level, increasing precision actually increases the cost of capital.

**Results on the rule of combination between accounting and tax estimations ($\varphi$)**

This section looks at how the rule of combination, $\varphi$, of accounting and tax estimates influences the cost of capital. Like in the analysis of the previous propositions, the results in this section depend on the degree of correlation $\rho$. In addition, the relationship between the degree of correlation $\rho$ and the relative precision of accounting and tax estimates ($\sigma_t^*/\sigma_e^*$) also influences the analysis.

**Proposition 7 –** Given a triple $(\rho, \sigma_t^*, \sigma_e^*)$ then,
(a) if the tax estimation dominates in precision the accounting estimation \((\frac{\sigma_\epsilon}{\sigma_\eta} < 1)\) and the correlation coefficient is positive and large enough \((\rho \in \left[\frac{\sigma_\eta}{\sigma_\epsilon}, 1\right])\) then the earnings report variance and the cost of capital increases in \(\varphi\).

(b) if the accounting estimation dominates in precision the tax estimation \((\frac{\sigma_\eta}{\sigma_\epsilon} > 1)\) and the correlation coefficient is positive and large enough \((\rho \in \left[\frac{\sigma_\eta}{\sigma_\epsilon}, 1\right])\) then the earnings report variance and the cost of capital decreases in \(\varphi\).

(c) in all other cases, there exists a cut-off point \(\varphi^* = \frac{\sigma_\epsilon^2 - \rho \sigma_\epsilon \sigma_\eta}{\sigma_\eta^2 + \rho \sigma_\epsilon \sigma_\eta - 2 \rho \sigma_\epsilon \sigma_\eta}\) such that the cost of capital strictly decreases in \(\varphi\) if \(\varphi < \varphi^*\) and strictly increases in \(\varphi\) if \(\varphi > \varphi^*\).

Proof: Taking the first order derivative of the report variance in respect to \(\varphi\)

\[
\frac{\partial \text{Var}[\hat{\epsilon}]}{\partial \varphi} = 2 \varphi \sigma_\epsilon^2 - 2(1 - \varphi) \sigma_\eta^2 + 2(1 - 2 \varphi) \rho \sigma_\epsilon \sigma_\eta
\]

and solving \(\frac{\partial \text{Var}[\hat{\epsilon}]}{\partial \varphi} = 0\) for \(\varphi\), we obtain the cut-off point \(\varphi^*\) as stated in the proposition. The second order derivative of the report variance in respect to \(\varphi\) is:

\[
\frac{\partial^2 \text{Var}[\hat{\epsilon}]}{\partial \varphi^2} = 2(\sigma_\epsilon^2 + \sigma_\eta^2 - 2 \rho \sigma_\epsilon \sigma_\eta)
\]

This is always positive, since \(\rho \in [-1, 1]\). This means that, when well behaved, the cut-off point \(\varphi^*\) as in part (c) of proposition 7 gives a point of minimum. The three cases above, (a) through (c) are then obtained by analyzing conditions under which \(\varphi^*\) is well behaved (i.e. \(\varphi^* \in [0, 1]\)).

To prove part (a), assume \(\frac{\sigma_\epsilon}{\sigma_\eta} < 1\) and \(\rho \in \left[\frac{\sigma_\eta}{\sigma_\epsilon}, 1\right]\). This implies \(0 < -\sigma_\epsilon^2 + \rho \sigma_\epsilon \sigma_\eta\). But then, since \((\sigma_\epsilon^2 + \sigma_\eta^2 - 2 \rho \sigma_\epsilon \sigma_\eta > 0)\) it follows that \(0 < -\sigma_\epsilon^2 + \rho \sigma_\epsilon \sigma_\eta < \varphi(\sigma_\epsilon^2 + \sigma_\eta^2 - 2 \rho \sigma_\epsilon \sigma_\eta) - \sigma_\epsilon^2 + \rho \sigma_\epsilon \sigma_\eta\). Therefore \(\frac{\partial \text{Var}[\hat{\epsilon}]}{\partial \varphi} > 0\) and hence the variance of the earnings report and the cost of capital increase in \(\varphi\). A similar proof follows for part (b). Part (c) follows from solving \(\frac{\partial \text{Var}[\hat{\epsilon}]}{\partial \varphi} = 0\).

Setting aside the mathematics underlying the argument, what parts (a) and (b) of proposition 7 say is that it is only when the correlation coefficient is positive and large enough then, by placing more weight on the more precise (lower variance) signal strictly lowers the cost of capital. If the aim is to lower the cost of capital then, taking the positive correlation and the estimation variances \((\sigma_\epsilon^2, \sigma_\eta^2)\) as given, the reporting rule should place more weight on the most precise estimation. Such a conclusion is in line with conventional wisdom about precision and the cost of capital. However, part (c) of the proposition proves that when the coefficient of correlation is positive but sufficiently low, the relationship between \(\varphi\) and the cost of capital becomes blurred. In particular, part (c) identifies an interior solution \(\varphi^*\) which limits the extent to which emphasis should be placed on the more precise (lower variance) estimate.

In particular, part (c) says that when \(\frac{\sigma_\epsilon}{\sigma_\eta} < 1\) but \(\rho < \frac{\sigma_\eta}{\sigma_\epsilon}\) (i.e., the tax estimation is more precise and correlation is low enough) then, decreasing the weight of the accounting signal beyond the level of \(\varphi^*\), starts increasing the variance of the earnings report and the cost of capital. Part (c) also says that, for instance, when \(\frac{\sigma_\eta}{\sigma_\epsilon} > 1\) but \(\rho < \frac{\sigma_\eta}{\sigma_\epsilon}\) (that is, the accounting signal is more precise but there is low correlation) then, increasing the weight of the accounting signal \((\varphi)\) is beneficial to the cost of capital only up to the level of \(\varphi^*\). Beyond this level, the variance of the earnings report and the cost of capital start increasing. That is, even if the accounting estimate is more precise than the tax estimate, placing a weight on the accounting estimate that exceeds the level of \(\varphi^*\) increases the cost of capital. If the aim is to diminish the cost of capital then, taking precisions and the degree of correlation as given, the cost of capital is at its minimum when the reporting rule follows the cut-off point \(\varphi^*\). Placing all weight on the most precise estimate is not necessarily conducive to lower cost of capital. As in proposition 5 above, the reason is, partly, the low enough correlation. However, unlike the case of proposition 5 where results depend entirely on the negative correlation \((\rho < 0)\), in proposition 7 a non-trivial interior point \(\varphi^{**} = \frac{\sigma_\epsilon^2}{\sigma_\eta^2 + \sigma_\epsilon^2}\) is obtained even with no correlation \((\rho = 0)\). This means, that the relative weight of accounting and tax estimates \((\varphi)\) has a role of its own independent on the correlation coefficient.

**DISCUSSION AND CONCLUSION**

This paper analytically explores the relationship between accounting and taxation and its implications for the cost of capital. The approach was purely informational. That is, we looked at taxation as at another measurement device which conveys information about the true earnings of a company. Depending on the jurisdiction, the information in the tax estimation may be more or less precise then the information in an accounting estimation. Essential for our analysis is that accounting and tax estimations may be correlated. The degree of correlation between the two estimates is viewed as one feature of the link between accounting and taxation. The other feature is the rule of combination (the reporting rule) of accounting and tax estimates in the public report. The precisions of the two estimates, their correlation and the
combination rule represent the informational determinants of the cost of capital in a setting where accounting and taxation rules coexist. Key for our results is the coefficient of correlation between the errors in the accounting and tax estimates. We found that the cost of capital unambiguously increases in this coefficient. When this coefficient is given, positive and high enough, we found that many of the classical results about information and cost of capital hold in our model too: cost of capital decreases in the precision of information. Hence, reporting rules should place more weight on the estimate that is most precise. However, when accounting and tax estimates exhibit either low but positive or negative correlations we found interior solutions for both precisions of estimates and the combination rule. This means that, taking the other determinants as given, there exists limits beyond which increasing precision of estimates (accounting or tax) may actually increase the cost of capital. Such an idea is in sharp contrast with conventional knowledge about information and cost of capital and proves that institutional factors such as the link between accounting and taxation must be considered when analyzing the relationship between public earnings reports and the cost of capital.

A few words about the limitations of our study are in order. First, the paper did not explore how tax rules shift cash-flows between periods. This is the cost we paid for taking a purely informational approach. Second, our analysis is developed in exogenous terms. All determinants of the cost of capital are taken as given. They do not appear as equilibrium results in a certain game or on a certain market. Therefore, most of our results are driven purely by the statistical properties of the public report. Setting aside these limitations, we believe the paper has the merits of exploring theoretically the effects of an institutional factor (the link between accounting and taxation) on the cost of capital. It generates interesting empirically testable propositions in settings where public reports are affected by accounting as well as tax estimates.

Conflict of Interests

The author has not declared any conflict of interests.

ACKNOWLEDGEMENT

The author gratefully acknowledges financial support from research Grant PNCDI II – ID_1840/IDEI II. The benefits of IFRS adoption: an exploratory research regarding the internationalization of Romanian accounting on the cost of capital.

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